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A
FAMILIAR INTRODUCTION
TO
CRYSTALLOGRAPHY;

INCLUDING AN EXPLANATION OF THE
PRINCIPLE AND USE OF THE GONIOMETER.

With an Appendix,

CONTAINING
THE MATHEMATICAL RELATIONS OF CRYSTALS;
RULES FOR DRAWING THEIR FIGURES;
AND AN ALPHABETICAL ARRANGEMENT OF MINERALS,
THEIR SYNONYMES, AND PRIMARY FORMS.

ILLUSTRATED BY NEARLY 400 ENGRAVINGS ON WOOD.

By HENRY JAMES BROOKE,
F. R. S. F. L. S. &c.

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1823.

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TO

THE INVENTOR

OF THE

REFLECTIVE GONIOMETER,

AN INSTRUMENT TO WHICH

CRYSTALLOGRAPHY IS LARGELY INDEBTED

FOR

THE PRECISION OF ITS RESULTS,

THIS INTRODUCTORY TREATISE

IS RESPECTFULLY INSCRIBED BY

THE AUTHOR.

INTRODUCTION.

The immediate purpose of the science of Crystallography, regarded as a branch of Mineralogy, is to teach the methods of determining the species to which a mineral belongs, from the characters of its crystalline forms. But the science itself is also capable of being rendered more extensively useful.

The crystalline forms of pharmaceutical preparations will furnish a certain test of the nature of the crystallised body, although it will not determine its absolute state of purity. In chemical analysis, the forms of crystals will frequently supersede a more rigorous examination of the crystallised matter; and commercial transactions in the more precious mineral productions may frequently be guided by the crystalline form, or by the character of the cleavage planes, of those bodies.

It does not appear in the works hitherto published that the connection between the *crystal* and the *mineral* has been any where so systematically explained as to enable the mineralogical student readily to connect the one with the other.

The Abbé Haüy's works on crystallography are the only ones in which a truly scientific exposition of the theory of crystals is to be found;* but by designating most of their forms by separate names, he has presented those forms to the mind rather as independent individuals, than as parts of such groups as should render their relations to each other, and hence their mineralogical relations, apparent.

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* An interesting volume on Crystallization, founded on the Abbé Hauy's theory, was published in 1819, by Mr. Brochant de Villiers, and will afford the reader a clear view of that theory, connected with other interesting objects relating to the formation of crystals.

I have been induced, therefore, to attempt such an *arrangement* of the various forms of crystals, as will indicate their constant relations to, or differences from, each other, for the purpose of more readily referring from the crystal to the mineral; and this arrangement is contained in the Tables of Modifications which will be found in the following pages.*

The best illustration of the manner in which some of the forms of crystals may be conceived to be allied to others, is afforded by the Abbé Haüy's theory of decrement. That theory appears, however, much encumbered by his adoption of two kinds of molecules, and by the forms which he has assigned to particular molecules of one of those kinds; in consequence of this, I have ventured to propose a new theory, in reference to several of the classes of primary forms, which may in some respects be regarded as more simple, and which forms the subject of the section on molecules.†

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* Since these tables were constructed I have learned from Mr. Konig of the British Museum, that he had for some time entertained an intention of framing a set of tables nearly on the same principle: and he has shewn me a considerable number of drawings of the figures of crystals which were made partly with a view to this object, yet serving at the same time as records of many of the crystalline forms of minerals contained in that rich collection, upon which his attention is so constantly and so assiduously bestowed.

† The theory of spherical molecules which has been entertained by some distinguished philosophers, has not been alluded to in this treatise, as the laws of decrement appeared more readily explicable on the supposition of the molecules of crystals being solids contained within plane surfaces.

I have also attempted to supply some rules for studying the forms of crystals, and for what may be termed reading them; which, although they may not enable the learner to trace at once the relation of the different crystalline forms to each other, they will certainly assist him in his examination of the minerals themselves; and it is from an attentive study of these that he must at last derive his best information.

From the very elementary nature of some of the definitions, it is evident that the reader of the earlier part of the volume is supposed to be unacquainted with the first rudiments of geometry. By being thus elementary I have been inclined to hope that crystallography may be rendered more familiar, and its principles be more easily acquired; and that the young collectors of minerals may be led by these first and easy steps in the path of science, to make their collections subservient to the cultivation of higher sources of amusement.

The description of the principle and of the method of using the reflective goniometer, has been minutely detailed on account of the importance of the instrument to the practical mineralogist, and with a view to remove the impression of its application to the measurement of crystals being difficult.

The Abbé Haüy has used plane trigonometry in his calculations of the laws of decrement. The substitution of spherical for plane trigonometry in this volume, was made at the recommendation of Mr. Levy; from whom I have also received many other valuable suggestions relative to the methods of calculation employed in the section on calculation; which it will be apparent to the reader, is little more than an outline of a method which must frequently be filled up by the exercise of his own judgment. In this as well as in other respects the mathematical part of the volume differs much from the analytical processes contained in the Abbé Haüy's more comprehensive works.*

When I first began to examine the crystalline forms of minerals I was much assisted by a large collection of the drawings of crystals, which was very kindly lent me by my friend Mr. William Phillips. The number and variety of the figures contained in this collection was the immediate cause of the attempt to reduce the forms of crystals into classes, and of the construction of the tables of modifications already alluded to. Since that period, and particularly during the printing of this volume, Mr. Phillips has frequently assisted me by his communications relative to the forms, the cleavages, and the measurements of crystals, which I had not other means of immediately acquiring.

During the course of my investigations I have frequently found it necessary to consult larger collections of minerals than my own. On these occasions I have generally referred to my friend Mr. Heuland, and I am happy in the opportunity of acknowledging the readiness and the liberality with which he has invariably assisted my views, by permitting an access at all times to his large and valuable cabinets, and very frequently by contributing rare and interesting specimens to mine which I could not otherwise have acquired.

I have sometimes sought information from the extensive cabinet in the British Museum, and have always found the utmost facility afforded to research by the habitual urbanity and friendly attention of Mr. Konig. And I have frequently been indebted also to Mr. G. B. Sowerby for illustrative specimens of minerals of unfrequent occurrence.

---

* Mr. Levy is at present engaged in an examination and description of one of the finest collections of minerals in the kingdom, which belongs to C. H. Turner, Esq. The opportunity which this examination will afford him of connecting a knowledge of the forms of crystals with his well-known mathematical attainments, will enable him to convey valuable instruction in this department of science to others, to which object he intends to devote some portion of his time in future.

Since the committal of a considerable portion of these pages to the hands of the compositor, and indeed subsequently to the printing a large part of the volume, a new edition has appeared of the Abbé Haüy's treatise on crystallography; this event was very soon followed by the decease of the learned author; and subsequently to his decease three volumes of a new edition of his treatise on mineralogy have been published: events so intimately connected with the subject of this volume that I cannot well pass them over in silence.

I am perfectly disposed to concur in the public eulogium which has been so deservedly passed upon the deceased philosopher, for having been the first to elevate crystallography to the rank of a science, and to trace out a secure path to its attainment; but I regret that I cannot agree in that unqualified approbation of his recent works which some of his surviving friends have so liberally bestowed upon them. For those works will be found to contain errors of so remarkable a character, as to excite our surprise when we recollect the generally accurate and enlightened judgment of the author.

Upon these, as criticism can no longer reach the ear of the author, I shall offer but few remarks.

One of his sources of error may be discovered in an apparently groundless notion which his theory embraces, that nature has imposed limits to the angles at which the primary planes of crystals incline to each other. And some of the mistakes which originate from this supposition are so important, as to cast a shade of disconfidence over his determinations relative to the primary forms of crystals.

His inaccuracy with respect to the angle of carbonate of lime is a well known example of one of these theoretic errors.

His inaccurate measurements of many of the angles of crystals, have probably been occasioned by the comparatively imperfect instrument with which those measurements were taken. That he should have continued to prefer this, to the more perfect goniometer invented by Dr. Wollaston, may possibly have been owing to the decay of sight incident to his period of life, and to that dislike to change which so frequently accompanies advanced age.

But some of his inaccuracies are independent both of his theory and his goniometer, and it would almost appear that he had occasionally written from the dictates of his fancy, without examining the minerals he has described.

The resemblance he imagines to exist between the crystals of bournonite and those of sulphuret of antimony is an instance of this nature; and on some of his figures, as those of wolfram, and some of those which he has still retained as stilbite, although they belong to a distinct species of mineral to which I have given the name of heulandite, he has placed imaginary planes which have no existence on the crystals themselves.

His persisting in the identity of the angles of the primary forms of carbonate of lime, bitter spar, and carbonate of iron, if he has really been deceived by his goniometer, evinces a carelessness in the use of that instrument which must still further diminish our confidence in his results.

Dr. Wollaston in the year 1812 discovered that these species differed from each other, and noticed their differences in a paper published in the Philosophical Transactions for that year. He found carbonate of lime to measure 105°5′

Bitter spar . . . . . . 106°15′
Carbonate of iron 107°

Notwithstanding the discovery of these facts, which have been so frequently verified by subsequent measurements, the Abbé Haüy has not only continued to insist on the superior accuracy of his own measurements, but discusses through several pages how it could have happened that the *iron* should have displaced the *lime* in the crystals of *carbonate of iron*, which his original error has led him to regard as *pseudomorphous!*

With all the faults, however, which the late Abbé's works contain, many of which we must in justice to his better judgment ascribe to feelings of a personal nature, those works present to the reader truly philosophical views of the sciences of which they treat, and they cannot be perused without frequently affording him both gratification and improvement.

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CORRIGENDA.

The reader is requested to make the following corrections with the pen, most of which are important to the accuracy of the text.

The lines reckoned do not include the running titles, or the word Fig. standing over the diagrams.

Page 39, line 8, for direction of *c*, ... read .... *e*.
40, — 1, — line .... lines.
43, — 21, — tetrahedrons .... tetrahedron.
80, — 7 from the bottom, for class *n* ... class *o*.
92, — 2, for deduced .... derived.
97, — 2, — together with .... and.
5, after symbols .... . add .... will follow the tables of modifications.
107, — 9, for 120°15′52″ .... read .... 125°15′52″
113, — 9, — 70°31′43″ .... 70°31′44″
115, — 15, — 120°15′52″ .... 125°15′52″
177, — 6, — and a rhombic base...... with a rhombic base.
181, — 6 from the bottom, for F .... E.
201, — 6, for F .... E.
207, — 6, after intersect .... add ...... two of.
209, — 10, for *k* .... read .... *l*.
213, — 9 from the bottom, after diagonals, add, or edges.
218, — 4, for *m* .... read .... *o*.
257, — 5, — $<$ .... $>$
260, — 3 from the bottom, for $\overset{p}{\underset{p}{P}}$ .... $\overset{p}{\underset{p}{A}}$
270, — 3, for Gp ...., Gq
— 6, — Bq .... Hq
277, — 2, — Bp B′p .... Bp B′q
278, — 19, — C .... O
298, — 13, — *i f* .... *i d*.
301, — 15, — Ep .... Fp
310, — 10 from the bottom, for B′q B″r... B″qB′r
311, — 9, after = ... add .... R.
— 12, — = ... R.
— 3 from the bottom, for B′qB″r read B″qB′r
315, — 12, for cos. A_8 .... cos. A_3
— 16, afer sin. (120°—A_2) .. add .. :
316, — at the bottom, after = in both formulæ, add R,
317, — 9 from the bottom, for *aiec* read *ab o d*.
319, — 3 ———————— after (I_8—I_1) add :
322, — in both formulæ, after = ... add R.
323, — 10 from the bottom, for *a* ... read *c*.
324, — 2, for *o*, and *o*, ,.................. *d*, and *d*.
326, — 5 from the bottom, for cos. $\frac{1}{2} I_2$ .. cot. $\frac{1}{2} I_2$
349, — 5, for *a* .... *c*.
351, — 6, — 351 .... 353.
— 11, in the formula, for sin. $\frac{1}{2} I_1$... sin. I_1
353, — at bottom, for *a e* .... *a c*.
354, — 1, for *a e* ... . *a c*.
— 5, — R. .... R :
— 6, after 1 . ... add ...... :
377, — at bottom, for tang. ($\frac{1}{2} I = 45°$) read tang. ($\frac{1}{2}$I—45°)
390, — ———— — 39°2′30″= .... 39°2′30″—
391, — 5 from the bottom, for 4°15′32″ .. 4°15′33″
392, — 4, for 4°15′32″ .... 4°15′33″
— 6, — 28°10′26″ .... 28°10′27″

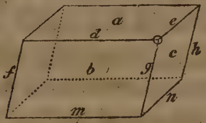
DEFINITIONS.

The object of the science of Crystallography, regarded as a branch of mineralogy, is to trace and to demonstrate those relations and differences between the various crystalline forms of minerals, by means of which we are enabled generally to discriminate the different species of crystallized minerals from each other.

A *crystal*, in mineralogy, is any symmetrical mineral solid, whether *transparent* or *opaque*, contained within plane, or sometimes within curved surfaces.

These surfaces, as *a*, *b*, *c*, fig. 1., are called *planes* or *faces*.

Fig. 1.



The *exterior planes* of a crystal as they occur in nature, are called its *natural planes*.

Crystals may sometimes be split in directions parallel to their natural planes, and frequently in other directions.

The *splitting* a crystal in any direction, so as to obtain a new plane, is termed *cleaving* it, and the crystal is said to have a *cleavage* in the direction in which it may be so split.

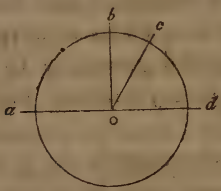
The planes produced by *cleaving* a crystal are called its *cleavage planes*.

An *edge*, as *d* fig. 1, is the line produced by the meeting of two planes.

A *plane angle*, or as it is more commonly termed, an *angle*, is formed by the meeting of any two lines or edges. The angles *d o e*, *d o g*, fig. 1. are formed by the meeting of the lines *d o*, *o e*, and *d o*, *o g*.

A *solid angle* is produced by the meeting of *three* or *more plane angles*, as at *o*, fig. 1.

Fig. 2.



The *measure*, or, as it is sometimes termed, the *value of an angle*, is the number of degrees, minutes, &c. of which it consists; these being determined by the portion of a circle which would be intercepted by the two lines forming the angle, supposing the point of their meeting to be in the centre of the circle.

For the purpose of measuring angles the circle is divided into 360 equal parts, which are called *degrees*; each degree into 60 equal parts, which are called *minutes*; and each minute into 60 *seconds*; and these divisions are thus designated; 360°, 60′, 60″,—the ° signifying degrees, the ′ minutes, the ″ seconds.

If $\frac{1}{4}$ of the circle, or 90°, be intercepted by the two lines *a o*, *o b*, fig. 2, which meet at an angle *a o b* in the centre, those lines are perpendicular to each other, and the angle at which they meet is said to measure 90°, and is termed a *right angle*.

If less than $\frac{1}{4}$ of the circle be so intercepted, as by the lines *o b*, *o c*, the angle *b o c* will measure less than 90°, and is said to be *acute*. If it measure more than 90°, as it would if the angle were formed by the lines *a o*, *o c*, it is called *obtuse*.

The lines *a o*, *o b*, or *a o*, *o c*, or *b o*, *o c*, are sometimes said to *contain* a right, an obtuse, or an acute angle.

In fig. 1, the *plane a*, and that on which the figure is supposed to rest, are called *summits*, or *bases*, or *terminal planes*, and the planes *b* and *c*, with those parallel to them, are termed *lateral* planes.

The edges of the terminal planes, as *d*, *e*, *m*, *n*, fig. 1, are called *terminal edges*.

The edges *f*, *g*, *h*, produced by the meeting of the lateral planes, are termed *lateral edges*.

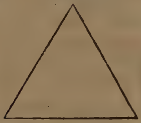
The *planes* of a crystal are said to be *similar* when their corresponding edges are proportional, and their corresponding angles equal.

Edges are *similar* when they are produced by the meeting of planes respectively similar, at equal angles.

Angles are *similar* when they are equal and contained within similar edges respectively.

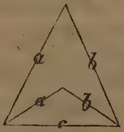
Solid angles are *similar* when they are composed of equal numbers of plane angles, of which the corresponding ones are similar.

Fig. 3.



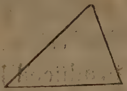
An *equilateral triangle*, fig. 3, is a figure contained within three equal sides, and containing three equal angles.

Fig. 4.



An *isosceles triangle*, fig. 4, has two equal sides, *a*, *b*, which may contain either a right angle, or an acute, or obtuse angle. If the contained angle be less than a right angle, the triangle is called *acute*, but if greater, it is called *obtuse*. The line on which *c* is placed is called the *base* of the triangle.

Fig. 5.

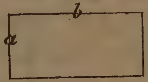


A *scalene triangle*, fig. 5, has three unequal sides, and contains three unequal angles.

Fig. 6.

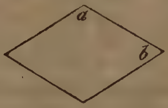
A *square*, fig. 6, has four equal sides, containing four right angles.

Fig. 7.



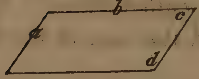
A *rectangle*, fig. 7, has its adjacent sides, *a* and *b*, unequal, the four contained angles being right angles.

Fig. 8.



A *rhomb*, fig. 8, has four equal sides, but its adjacent angles, *a* and *b*, unequal.

Fig. 9.



An *oblique angled parallelogram*,* fig. 9, has its opposite sides parallel, but its adjacent sides *a*, *b*, and its adjacent angles, *c*, *d*, unequal.

Where certain forms of crystals are described with reference to the *rhomb* as the figure of some of their planes, they are termed *rhombic*.†

A *parallelopiped* is any solid contained within three pairs of parallel planes.

Crystals are conceived to be formed by the *aggregation of homogeneous molecules*, which may be again separated from each other mechanically, that is, by splitting or otherwise breaking the crystal.

These *molecules*, which relate properly to the *crystal*, must be carefully distinguished from the *elementary particles* of which the *mineral* itself is composed.

Sulphur and lead are the *elementary particles*, which, by their chemical union, constitute galena; but the *molecules* of galena are portions of the compound crystalline mass, and are therefore to be regarded as *homogeneous*, in reference to the mass itself.

---

* A *parallelogram* is any right lined quadrilateral plane figure, whose *opposite* sides are equal and parallel.

† What is here called *rhombic*, most writers on this subject have, in imitation of the French idiom, denominated *rhomboidal;* but as the term *rhomboid* has been used in works on geometry to signify an oblique angled parallelogram, and as the same term has also been already appropriated in crystallography to a solid contained within six equal rhombic planes, the application of the term *rhomboidal* to any other solid seems to involve a degree of ambiguity. The term *rhombic* is, besides, more conformable to the practice of our own language.

All minerals which are composed of similar elementary particles combined in equal proportions, and whose molecules are similar in form, are said to belong to one *species*.

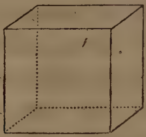
The same species of mineral is frequently observed to crystallize in a great variety of forms.

From among the variety of crystalline forms under which any species of mineral may present itself, some one is selected as the *primary*, and the remainder are termed *secondary* forms.

A *primary form* is that parent or derivative form, from which all the secondary forms of the mineral species to which it belongs, may be conceived to be derived according to certain laws.

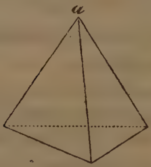
The *primary forms* are at present supposed to consist of only the following classes.

Fig. 10.



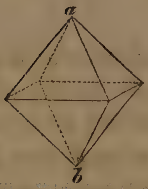
The *cube*, fig. 10, contained within six square planes.

Fig. 11.



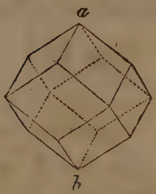
The *regular tetrahedron*, fig. 11, contained within four equilateral triangular planes. The solid angle at *a*, is sometimes called its summit.

Fig. 12.



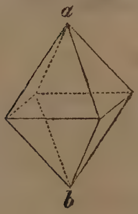
The *regular octahedron*, fig. 12, resembling two four-sided pyramids united base to base. The *planes* are *equilateral triangles*, and the common base of the two pyramids (which will hereafter be denominated *the base of the octahedron*) is a square.

Fig. 13.



The *rhombic dodecahedron*, fig. 13, contained within twelve *equal rhombic planes*, having six solid angles, consisting each of four acute plane angles, two opposite ones as *a*, *b*, being sometimes called the summits, and eight solid angles consisting each of three obtuse plane angles.

Fig. 14.



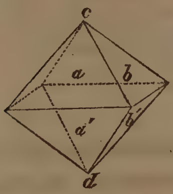
An *octahedron with a square base*, fig. 14, contained within eight *equal isosceles* triangular planes; the bases of the triangles constitute the edges of the base of the octahedron.

When the plane angle at *a* measures less than 60°, the octahedron is called acute.

When the angle at *a* is greater than 60°, the octahedron is called obtuse.

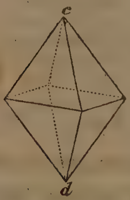
The *square base* serves to distinguish this class from the two which follow it. The *isosceles* triangular planes distinguish it from the regular *octahedron*.

Fig. 15.



An *octahedron with a rectangular base*, fig. 15. The planes of which are generally *isosceles* triangles, *but not equal*. The plane angles at *c* and *d* of the planes *a* and *a′* being more obtuse than those of the planes *b* and *b′*; and the planes *a*, and *a′*, inclining to each other at a different angle from that at which those marked *b*, and *b′*, meet.

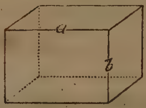
Fig. 16.



An *octahedron with a rhombic base*, fig. 16, contained within eight equal *scalene* triangular planes.

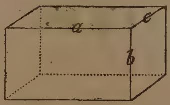
The solid angles at *a*, *b*, fig. 12 and 14, and *c*, *d*, fig. 15 and 16, are sometimes called the summits of the octahedron.

Fig. 17.



A *right prism with a square base**, fig. 17, or *right square prism*, the edge, *a*, being always greater or less than *b*: if *a*, and *b*, were equal, the figure would be a cube.

Fig. 18.



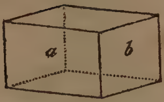
A *right prism with a rectangular base*, fig. 18, or *right rectangular prism*, whose three edges *a*, *b*, *c*, are unequal. For if any two of those were equal, the prism would be square.

---

* A prism is a solid whose *lateral edges* are parallel, and whose *terminal planes* are also parallel.

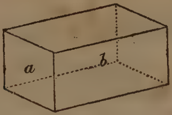
Those prisms which *stand perpendicularly* when resting on their base, are called *right* prisms. Those which *incline from the perpendicular*, are called *oblique* prisms.

Fig. 19.



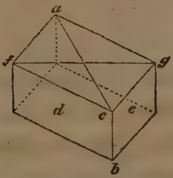
A *right rhombic prism*, or *right prism whose base is a rhomb*, fig. 19, and whose *lateral* planes *a*, *b*, are *equal*. These planes may be either *square* or *rectangular*.

Fig. 20.



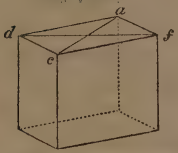
A *right oblique-angled prism*, or *right prism whose base is an oblique-angled parallelogram*, fig. 20, and whose *adjacent* lateral planes *a*, *b*, are unequal. One of these planes must be rectangular, the other may be either a square or a rectangle.

Fig. 21.



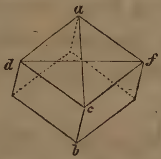
An *oblique rhombic prism*, or oblique prism whose *base is a rhomb*, fig. 21, and whose *lateral planes d*, *e*, are *equal oblique-angled parallelograms*—if they were equal rhombs the solid would be a rhomboid.

Fig. 22.



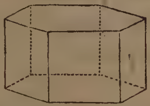
A *doubly oblique prism*, fig. 22, whose *bases and whose lateral planes are generally oblique-angled parallelograms.* The only equality subsisting among these planes, is between each pair of opposite or parallel ones.

Fig. 23.



The *rhomboid*, fig. 23, a solid contained within *six equal rhombic planes*, and having two of its solid angles, and only two, as *a*, *b*, composed each of *three equal plane angles;* these are sometimes called the summits.

Fig. 24.

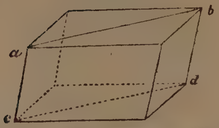


The *regular hexagonal prism*, fig. 24, or right prism whose bases are regular hexagons.

The *secondary forms* of crystals consist of all those varieties belonging to each species of mineral, which differ from the primary form.

These, although extremely numerous, may be reduced to a few principal classes, as will appear in the sequel.

Fig. 25.



A *line*, as *a b*, or *c d*, fig. 25, drawn through two opposite angles of any parallelogram, and dividing the plane into two equal parts, is called a *diagonal* of that plane.

In the oblique rhombic prism, the doubly oblique prism, and the rhomboid, fig. 21, 22, and 23, the line *a c*, which appears to lean from the spectator, will be termed the *oblique diagonal*, and the line *f g* of the oblique rhombic prism, and *d f* of the rhomboid, the *horizontal diagonal.*

The line *d f* of the doubly oblique prism, may also for the sake of distinction be termed its *horizontal diagonal;* although from the nature of the figure, that line must be oblique when the lateral edges are perpendicular.

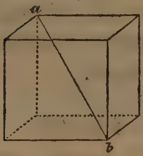
The *diagonal* plane of a *solid*, as *a b c d*, fig. 25, is an imaginary plane passing through the diagonal lines of two exterior *parallel* planes, dividing the solid into two equal parts.

The *axis of a crystal*, generally, is an imaginary line passing *through the solid, and through two opposite solid angles.*

In *prisms*, this may be termed an *oblique axis*, to distinguish it from another line which passes through the centres of their terminal planes, and may be termed a *prismatic axis.*

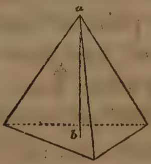
The axis of a pyramid, passes through its terminal point and through the centre of the base.

Fig. 26.



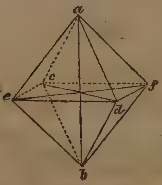
In the *cube*, an axis passes through the centre and through two opposite solid angles, *a b*, fig. 26; from the perfect symmetry of its form, the cube has a similar axis in four directions, or passing through its centre and through each pair of opposite solid angles.

Fig. 27.



The axis of the *regular tetrahedron* passes through the centres of the summit and base as *a b*, fig. 27, and it has a similar axis in four directions in consequence of the symmetrical nature of its form.

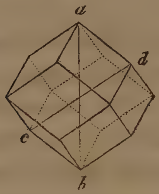
Fig. 28.



In *all octahedrons* the axis passes through the two summits and through the centre of the base, as *a b*, fig. 28; the *regular* octahedron, having all its solid angles similar, may be said to have a similar axis in three directions.

But the lines *c d*, *e f*, joining the opposite lateral solid angles of *irregular octahedrons*, may be called the *diagonals of their base.*

Fig. 29.



The *rhombic dodecahedron* has two *dissimilar sets of axes* passing through its centre; *one set*, as *a b*, fig. 29, passes through the pairs of opposite solid angles, which consist each of *four acute plane angles*, and may be called the greater axes; *another set*, as *c d*, passes through the solid angles which consist of *three obtuse plane angles* each, and may be called the lesser axes of the crystal.

Fig. 30.

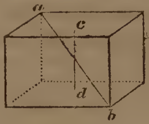
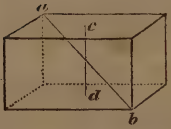


Fig. 31.



The *right square*, and *right rectangular prisms*, have each an axis in four directions similar to *a b*, fig. 30 and 31, but as prisms they have an additional *prismatic* axis, *c d*.

Fig. 32.

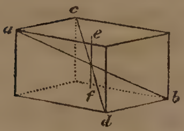
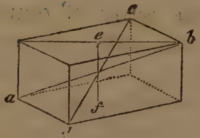
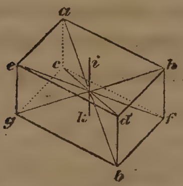


Fig. 33.



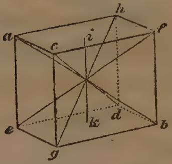
The *right rhombic prism*, fig. 32, and *right oblique angled prism*, fig. 33, have each *two greater* and *two lesser axes*. The greater axis, *a b*, passes through the solid angles which terminate the *acute edges* of the prism, and the lesser, *c d*, through those which terminate the *obtuse edges* of the prism. They have also the prismatic axis, *e f.*

Fig. 34.



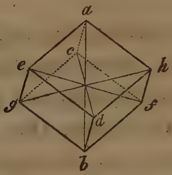
The *oblique rhombic prism* has, besides the prismatic axis, *i k*, fig. 34, a *greater*, a *lesser*, and *two transvere axes*. The greater axis is that which passes through the two *acute solid angles* of the prism *a*, *b*; the lesser that which passes through the two *obtuse solid angles* of the prism *c*, *d*, and the *tranverse*, those which pass through the lateral solid angles, *e*, *f*, *g*, *h*.

Fig. 35.



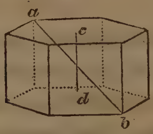
The *doubly oblique prism* has four unequal axes passing through the pairs of opposite solid angles, *a b*, &c. fig. 35; it has also the prismatic axis *i k*.

Fig. 36.



The line *a b*, fig. 36, which passes through the summits of the *rhomboid*, may be called the *perpendicular axis*, and those lines, *c d*, *e f*, *g h*, which pass through the opposite pairs of lateral solid angles may be termed the *transverse axes*. But the lines *a b*, and *c d*, are sometimes called the greater and lesser axes of the rhomboid.

Fig. 37.



The line *a b*, fig. 37, passing through the opposite solid angles of the *hexagonal prism*, may be termed an axis; but the prismatic axis, *c d* of this form, is that which is most generally regarded as its axis.

The diagonals and axes of crystals are imaginary lines, by means of which the secondary planes of crystals may frequently be described with greater precision than could be attained without their assistance; they also facilitate the mathematical investigations into the relations which subsist between the primary and secondary forms.

The diagonal planes are imaginary planes of a similar character.

A crystal is said to be *in position*, when it is so placed, or held, as to permit its being the most easily and precisely observed and described.

For this purpose *tetrahedrons* are made to rest on one of their planes, as in the figure already given.

Octahedrons are supposed to be held with the axis vertical, and in this position the plane angles at *a* and *b*, fig. 28, are called the terminal angles, and the edges *a c*, *a d*, *a e*, *a f*, the terminal edges, or edges of the pyramid.

The edges *e d*, *d f*, &c. may be termed edges *of the base*; and the angles *a e d*, *a d f*, lateral angles.

The *angles of the base* are the angles *c e d*, or *e d f*.

The *cube* stands on one of its planes, and *all prisms* on their respective bases.

Rhombic dodecahedrons are supposed to be held with a greater axis vertical, as in the former figure.

The *rhomboid* is also supposed to be held with its perpendicular axis vertical.

Crystals are supposed to be first formed by the aggregation of a few *homogeneous molecules*, which arrange themselves around a single central molecule in some determinate manner; and they are conceived to increase in magnitude, by the continual additions of similar molecules to their surfaces.

In these additions, the molecules appear to arrange themselves so as to form laminæ, or plates, which successively, either partially, or wholly, cover each other.

These plates are theoretically supposed to be either *single*, that is, of the *thickness of single molecules*, or to be *double*, *treble*, &c. that is of the *thickness of two*, *three*, or *more molecules*.

Fig. 38.

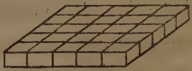


Fig. 38 represents a *single* plate of molecules.

Fig. 39.

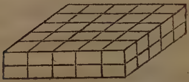
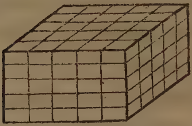


Fig. 39 represents a *double* plate.

Fig. 40.



When such additions envelope the whole of a smaller crystal, its original form is preserved through every increase of size.

Fig. 40 represents a *right rectangular prism* which has increased in magnitude without change of figure.

When the additions do not cover the whole surface of a primary form, but there are rows of molecules omitted on the edges, or angles of the superimposed plates, such omission is called a *decrement*.

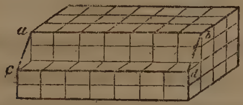
The term *decrement* has been adopted to express these omitted rows of molecules, because, in consequence of such omissions, the primary form on which the diminished plates are successively laid, appears to *decrease* as it were, on the edge or angle on which such omissions take place.

Decrements are said to *begin at*, or to *set out from*, the particular edge or angle at which the omission of molecules first takes place; and to *proceed along* that plane on which the defective plate of molecules is conceived to be superimposed. And they are said to take place either in *breadth* or in *height*.

Decrements in breadth are those which result from the reduction of the *superficial area* of the superimposed plate, by the abstraction of rows of molecules from its edges or angles.

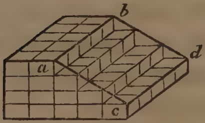
Decrements in height relate to the *thickness* of the plate from which the abstraction of rows of molecules takes place.

Fig. 41.



Let *c d* fig. 41, represent an edge of a primary form, and let *a b* represent an edge of a double plate of molecules, from which one row has been abstracted; the *decrement*, or *omitted portion* of this superimposed plate, would be stated to consist of *one row* in *breadth*, or one row omitted upon the terminal surface of the primary crystal, and *two rows in height*, signifying that the omitted row belonged to a *double plate* of molecules; *a b c d* would be the position of the new plane produced by this decrement.

Fig. 42.



Decrements have been divided by the Abbé Haüy into three principal classes—*simple*, *mixed*, and *intermediary*. The simple and mixed may however, in strictness, be regarded as varieties of the same class.

Simple decrements are those which consist in the abstraction of *any number of rows*, in *breadth* of *single* molecules, or of *single rows*, belonging to plates of *two* or *more* molecules in *thickness*.

Fig. 42 exhibits a *simple* decrement by *one* row in *breadth* on the *edge c d* of the primary form.

Fig. 43.

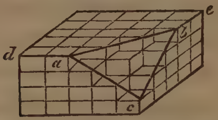
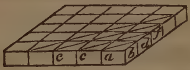


Fig. 43 exhibits a *simple* decrement by one row in *breadth*, on the angle *c* of the primary form.

For the sake of rendering the expression *rows of molecules* generally applicable to decrements both on the angles and edges of a primary form, the term *row* is applied to express the *single* molecule *first* abstracted from the *angle* of any plate.

Fig. 44.



In fig. 44, the single molecule *a*, *b*, is regarded as the *first row* to be abstracted from the angle of the imaginary plate; the two molecules *c*, *d*, as the *second row*; the three molecules *e*, *f*, as the *third row*, and so on.

Fig. 45.

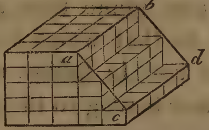


Fig. 45 shews a *simple* decrement by *two rows in height* on the *edge* of the primary form.

Fig. 46.

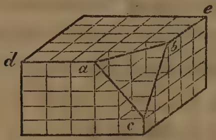


Fig. 46 shews a *simple* decrement by *two rows in height* on the *angle* of the primary form.

It is observable in these figures, that each successive plate is less by one row of molecules than the plate on which it rests. It is by this continual recession of the edges of the added plates, that the crystal appears to *decrease* on its edges or angles, and that new planes are produced. The edges of the new planes which would be produced by the four preceding decrements, are shewn by the lines *a b c d*, fig. 42 and 45, and by the lines *a b c*, fig. 43 and 46.*

A *mixed decrement* is one in which *unequal numbers* of molecules are omitted in height and in breadth, *neither of the numbers being a multiple of the other*, such as three in height and two in breadth, or four in height and three in breadth; for if either number were a multiple of the other, as would be the case if the supposed decrement took place by *two* rows in height and *four* in breadth, or *three* in height and *six* in breadth, the new plane thus produced would be perfectly similar to that which would result from a decrement by *one* row in height and *two* in breadth, and would therefore belong to the planes produced by *simple* decrements.

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* The molecules of crystals are so minute, as to render those inequalities of surface imperceptible which are occasioned by decrements.

Fig. 47.

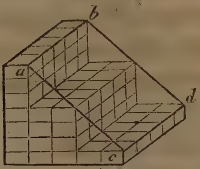
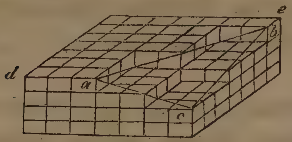


Fig. 47 shews a *mixed decrement* on an *edge* of the primary form by *two rows in breadth and three in height*, and the lines *a b c d* mark the position of the new plane produced by this decrement.

It has been found convenient to express *mixed* decrements by fractions, of which the *numerator, or upper figure*, denotes the *number of molecules in breadth*, and the *denominator, or lower figure*, the *number in height*, abstracted from the edge or angle of the superimposed plates; thus, a decrement by $\frac{3}{4}$ would imply a decrement by three molecules in breadth and four in height.

Intermediary decrements affect only the *solid angles* of crystals, and may be conceived to consist in the abstraction of rows of *compound molecules* from the successively superimposed plates, *each compound molecule containing unequal numbers of single molecules* *in length, breadth, and height.* Thus if we suppose the compound molecule abstracted in an intermediary decrement to belong to a *single* plate, it must consist of some *other* numbers of molecules in the directions *d*, and *e*, fig. 48.*

Fig. 48.



In fig. 48 the compound molecule consists of a *single* molecule in height, *two* on the edge *d*, and *three* on the edge *e*, producing the new plane *a b c*.

Fig. 49.

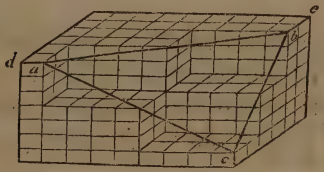


Fig. 49 exhibits an intermediary decrement in which the compound molecule consists of *three* single molecules in height, *four* on the edge *d*, and *two* on the edge *e*, producing the new plane *a b c*.

In the *simple* and *mixed* decrements upon an angle, as shewn in fig. 43 and 46, the number of molecules abstracted in the direction *d*, will always be equal to the number abstracted in the direction *e*. Thus if it be a simple decrement by one row in breadth, one molecule will apparently be omitted on each of the edges *d*, and *e*, as in fig 43. But in an *intermediary* decrement, the numbers are obviously *unequal* in the direction of those edges, and the number in *height* will also differ from *both the numbers in the direction of the edges*, as in fig. 48 and 49.

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* It may be remarked that the planes produced by *simple* and *mixed* decrements, intersect one or more of the primary planes in lines parallel to one of their edges or diagonals. The term *intermediary* has been used to express this third class of decrement, because the line at which the secondary plane produced by it, intersects any primary plane, is never parallel to either an edge or diagonal of that plane, but is an *intermediate* line between the edge and the diagonal, as may be observed by comparing the figures 42, 43, and 48.

The new planes produced by decrements are denominated *secondary* planes, and the primary form, when altered in shape by the interference of secondary planes, is said to be *modified* on the edges or angles on which the secondary planes have been produced. And such edges or angles are sometimes also said to be *replaced* by the secondary planes.

The *law of a decrement* is a term used to express the number of molecules in height, and breadth, abstracted from each of the successively superimposed plates, in the production of a secondary plane.

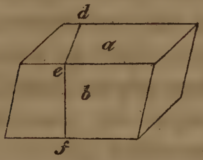
When an *edge*, or *solid angle*, is replaced by one *plane*, it is said to be *truncated*. When an edge is replaced by *two planes*, which respectively incline on the adjacent primary planes at equal angles, it is *bevilled*.

If any secondary plane *replacing an edge, and being parallel to it, incline equally on the two adjacent primary planes*, or if *replacing a solid angle, it incline equally on all the adjacent primary planes*, it is called a *tangent plane*.

OF THE GONIOMETER.

The instruments used for measuring the angles *at which the planes of crystals meet*, or, as it is frequently expressed, *incline to each other*, are called *goniometers*.

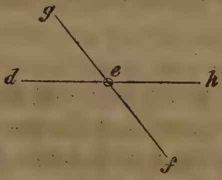
Fig. 50.



Let us suppose the angle required at which the planes *a*, and *b*, of fig. 50, incline to each other.

The inclination of those planes is determined by the portion of a circle which would be intercepted by two lines e d, e f, *drawn upon them from any point* e *of the edge formed by their meeting, and perpendicular to that edge—the point* e *being supposed to stand in the centre of the circle.*

Fig. 51.



Now it is known that if two right lines as *g f*, *d h*, fig. 51, cross each other in any direction, the opposite angles *d e f*, *g e h*, are equal.

If therefore we suppose the lines *g f*, *d h*, to be very thin and narrow plates, and to be attached together by a pin at *e*, serving as an axis to permit the point *f* to be brought nearer either to *d*, or to *h*; and that we were to apply the edges *e d*, *e f*, of those plates, to the planes of the crystal fig. 50, so as *to rest* upon the lines *e d*, *e f*, it is obvious that the angle *g e h*, of the moveable plates fig. 51, would be exactly equal to the angle *d e f* of the crystal fig. 50.

Fig. 52.

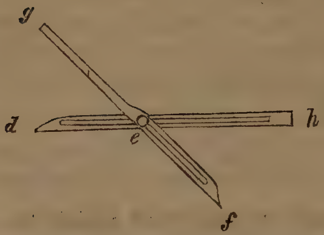
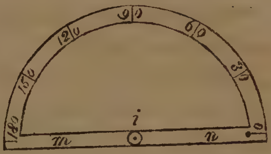


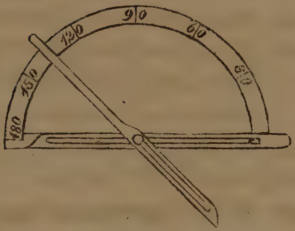
Fig. 53.



The *common* goniometer is a small instrument calculated for measuring this angle *g e h*, of the moveable plates. It consists of a semi-circle, fig. 53, whose edge is divided into 360 equal parts, those parts being half degrees, and a pair of moveable arms *d h*, *g f*, fig. 52. The semicircle having a pin at *i*, which fits into a hole in the moveable arms at *e*.

The method of using this instrument is, to apply the edges *d e*, *e f*, of the moveable arms, fig. 52, to the two adjacent planes of any crystal, *so that they shall accurately touch or rest upon those planes* in directions *perpendicular to the edge at which they meet.* The arm *d h*, is then to be laid on the plate *m n* of the semicircle fig. 53, the hole at *e* being suffered to drop on the pin at *i*, and the edge nearest to *h* of the arm *g e*, will then indicate on the semicircle, as in fig. 54, the number of degrees which the measured angle contains.

Fig. 54.



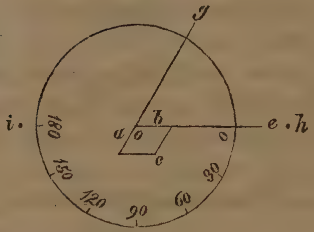
When this instrument is applied to the planes of a crystal, the points *d* and *f*, fig. 52, should be previously brought sufficiently near together for the edges *d e*, *e f*, to form a more acute angle than that about to be measured. The edges being then gently pressed upon the crystal, the points *d*, and *f*, will be gradually separated, until the edges coincide so accurately with the planes, that no light can be perceived between them.

The common goniometer is however incapable of affording very precise results, owing to the occasional imperfection of the planes of crystals, their frequent minuteness, and the difficulty of applying the instrument with the requisite degree of precision.

The more perfect instrument, and one of the highest value to Crystallography, is the *reflective goniometer* invented by Dr. Wollaston, which will give the inclination of planes whose area is less than $\frac{}{100'000}$ of an inch, to a minute of a degree.

This instrument has been less resorted to, than might, from its importance to the science, have been expected, owing perhaps to an opinion of its use being attended with some difficulty. But the observance of a few simple rules will render its application easy.

Fig. 55.



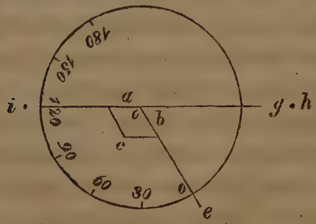
The principle of the instrument may be thus explained.

Let us suppose *a b c*, fig. 55, to be a crystal, of which one plane only is visible in the figure, attached to a circle, graduated on its edge, and moveable on its axis at *o*; and *a* and *b* the two planes whose inclination we require to know.

And let us further suppose the lines *o e*, *o g*, to be imaginary lines resting on those planes in directions *perpendicular to their common edge*, and the dots at *i* and *h*, to be some permanent marks in a line with the centre *o*.

Let us suppose the circle in such a position, that the line *o e* would pass through the dot at *h*, if extended in that direction as in fig. 55.

Fig. 56.

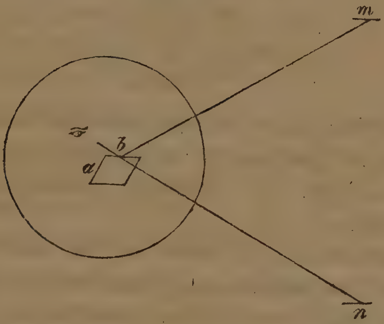


If we now turn round the circle with its attached crystal, as in fig. 56, until the imaginary line *o g*, is brought into the same position as the line *o e* is in fig. 55, we may observe that the No. 120 will stand opposite the dot at *i*.

This is the number of degrees at which the planes *a* and *b* *incline to each other*. For if we suppose the line *o g*, extended in the direction *o i*, as in fig. 56, it is obvious that the lines *o e*, *o i*, which are perpendicular to the common edge of the planes *a* and *b*, would intercept exactly 120° of the circle.

Hence an instrument constructed upon the principle of these diagrams, is capable of giving with accuracy the mutual inclination of any two planes, if the means can be found for placing them successively in the relative positions shewn in the two preceding figures.

Fig. 57.

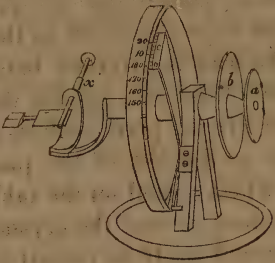


When the planes are sufficiently brilliant, this purpose is effected by causing an object, as the line at *m*, fig. 57, to be reflected from the two planes *a*, and *b*, successively, *at the same angle*.

It is well known that the images of objects are reflected from bright planes at the same angle as that at which their rays fall on those planes; and that when the image of an object reflected from a horizontal plane is observed, that image appears as much below the reflecting surface, as the object itself is above it.

If therefore the planes *a*, and *b*, fig. 57, be successively brought into such positions, as will cause the reflection of the line at *m*, from *each* plane, to appear to coincide with another line at *n*, both planes will be successively placed in the relative positions of the corresponding planes in figs. 55 and 56.

Fig. 58.



To bring the planes of any crystal successively into these relative positions by the assistance of the reflective goniometer, the following directions will be found useful.

The instrument, as shewn in the sketch fig. 58, should be first placed on a pyramidal stand, and the stand on a small steady table, placed about 6 to 10 or 12 feet from a *flat* window.

The graduated circular plate should *stand perpendicularly from the window, the pin* x *being horizontal, with the slit end nearest to the eye.* *

Place the crystal which is to be measured, on the table, *resting on one of the planes whose inclination is*

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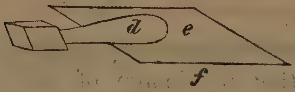
* This goniometer is sometimes drawn with the pin *x in the direction of its axis*, in which position of the pin, the instrument may be regarded as nearly useless.

required, and with the edge at which those planes meet, the farthest from you, and parallel to the window in your front.

Fig 59.



Attach a portion of wax about the size of *d*, to one side of a small brass plate *e*, fig. 59—lay the plate on the table with the edge *f* parallel to the window, the side to which the wax is attached being uppermost, and press the end of the wax against the crystal until it adheres; then lift the plate with its attached crystal, and place it in the slit of the pin *x*, with that side uppermost which rested on the table.

Bring the eye now *so near* the crystal, as, without perceiving the crystal itself, to permit your observing distinctly the images of objects reflected from its planes; and raise or lower that end of the pin *x* which has the small circular plate affixed to it, until one of the horizontal upper bars of the window is seen reflected from the upper or *first* plane of the crystal, which corresponds with plane *a*, fig. 55 and 56, and until the image of the bar is brought nearly to coincide with some line below the window, as the edge of the skirting board where it joins the floor.

Turn the pin *x on its own axis*, if necessary, until the reflected image of the bar of the window *coincides accurately with the observed line below the window.*

Turn now the small circular plate *a* on its axis, and *from you*, until you observe the *same bar* of the window reflected from the *second* plane of the crystal corresponding with plane *b*, fig. 55 & 56, and nearly coincident with the line below; and *having, in adjusting* *the first plane, turned the pin x on its axis* to bring the reflected image of the bar of the window to coincide *accurately* with the line below, *now move the lower end of that pin laterally, either towards or from the instrument*, in order to make the image of the *same bar*, reflected from the *second* plane, coincide with the *same line below.*

Having assured yourself by looking repeatedly at both planes, that the image of the horizontal bar reflected successively from each, coincides with the same line below, the crystal may be considered as adjusted for measurement.

Let the 180° on the graduated circle be now brought opposite the *o* of the vernier at *c*, by turning the middle plate *b*; *and while the circle is retained accurately in this position*, bring the reflected image of the bar from the *first plane* to coincide with the line below, by turning the *small circular plate a*. Now turn the *graduated circle from you*, by means of the *middle plate b*, until the image of the bar reflected from the *second plane* is also observed to coincide with the same line below. In this state of the instrument the vernier at *c* will indicate the degrees and minutes at which the two planes incline to each other.

SECTION I.

GENERAL VIEW.

THE regularity and symmetry observable in the forms of crystallized bodies, must have early attracted the notice of naturalists; but they do not appear to have become objects of scientific research, as a branch of natural history, until the time of Linnæus. He first gave drawings and descriptions of crystals, and attempted to construct a theory concerning them, somewhat analogous to his system of Botany.

We are indebted however to Romé de L'Isle for the first rudiments of crystallography. He classed together those crystals which bore some common resemblance, and selected from each class some simple form as the primary, or fundamental one; and conceiving this to be *truncated* in different directions, he deduced from it all its secondary forms; and it was he who first distinguished the different species of minerals from each other by the measurements of their primary forms.

The enquiries of Bergman were nearly contemporaneous with those of the Abbé Haüy, and both these philosophers appear to have entertained at the same time nearly the same views with regard to the structure of crystals; both having supposed that the production of secondary forms might be explained by the theory of *decrements* on the edges or angles of the primary.

Here, however, Bergman's investigation appears to have terminated, while the Abbé Haüy proceeded to complete this theory, by determining the forms and dimensions of the molecules of which he conceived the primary forms were composed, and by demonstrating mathematically the laws of decrement by which the secondary forms might be produced.

He also established a peculiar nomenclature, to designate *individually* each of the observed secondary forms of crystals; the nomenclature consisting of terms derived from some remarkable character or relation peculiar to each *individual* form. But the disadvantage accruing to the science from encumbering each individual crystal with a separate name, must be immediately apparent, when it is considered that the rhomboid of carbonate of lime alone is capable of producing some millions of secondary crystals by the operation of a few simple laws of decrement.

The number of names requisite to designate all these, if they existed, would form an insuperable obstacle to the cultivation of the science of crystallography, even if it were practicable to devise some sufficiently short and simple terms for the purpose.

To obviate the inconvenience arising from the use of so many individual names, the Comte de Bournon adopted a much simpler method of denoting the secondary forms. He numbered all the individual modifications he had observed, from one onwards, and as the secondary forms are produced either by a single modification, or by the concurrence of two or more single modifications, any secondary form whatever might, according to his method, be expressed by the numbers which designate all the particular modifications which it is found to contain.

Mr. Phillips has adopted this method in his papers on oxide of tin, red oxide of copper, &c. published in the Transactions of the Geological Society, and has thus proved its utility for the purpose of crystallographical description.

The descriptive system of the Comte de Bournon, with some alterations, will be adopted in this volume, as well as the theory of decrements which constitutes the basis of the Abbé Haüy's System of Crystallography.

SECTION II.

MOLECULES.

The homogeneous molecules which are aggregated together in the production of crystals, are supposed to be minute, symmetrical, solid particles, contained within plane surfaces. They are also conceived to be again separable from each other by *mechanical* division, which however stops very short of the separation of single molecules from the mass which has been formed by their union.

For, however minutely we may divide a piece of carbonate of lime, we cannot imagine that we have ever obtained any single portion or molecule containing only one atom or proportion of carbonic acid, and one atom or proportion of lime.

This effect of mechanical division merely implies that the *molecules* are *separated at their surfaces* by cleavage, and are *not divided or broken.* And it thus serves to distinguish them from the *elementary particles* or *atoms* which enter into their composition, and which cannot be separated from each other but by *chemical agency.**

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* Although it is not immediately connected with Crystallography, I am induced to state an observation here which has occurred to me relative to the forms of the *homogeneous molecules* of minerals, when compared with the forms of the *atoms*, or *elementary particles*, of which those molecules are composed.

We certainly know nothing of the forms of the *atoms* of those elementary substances which do not occur crystallized, such as oxygen, hydrogen, and many others. But we infer from analogy that the *atoms* of sulphur, carbon, the metals, and such other elementary substances as

The figures of the solid molecules require to be explained in reference to each of the five following classes of primary forms.

1. The cube and all the other classes of parallelopipeds, or solids contained within six planes.
2. The regular octahedron and all the other classes of octahedrons.
3. The regular tetrahedron.
4. The rhombic dodecahedron.
5. The hexagonal prism.

If we attempt to fracture a piece of *galena*, it will split into rectangular fragments. But we find by observing the secondary forms of galena, that its primary crystal may be a cube, and we know also that by supposing this cube to be composed of cubic molecules, the angles at which the secondary planes incline upon the primary, may be computed and determined with mathematical precision. We are therefore led to infer, *that if the rectangular fragments obtained by cleavage could be reduced to single molecules, those molecules would be cubes.*†

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are found crystallized, are similar in form to the *molecules* of other crystallized substances which present *similar primary forms.*

Now according to two suppositions, the first being that entertained by the Abbé Haüy, the other arising out of a theory which will be presently stated, the *molecule of sulphur* may be an *irregular tetrahedron*, or a *right rhombic prism*, and the *molecule of silver* a *regular tetrahedron*, or a *cube.* But the *compound of sulphur and silver* crystallizes in the form of a *cube.* Hence the molecule of sulphuret of silver, arising out of the *chemical* union of *irregular tetrahedrons* with the *regular tetrahedrons* or *cubes*, according to one supposition, or of *right rhombic prisms and cubes*, according to the other supposition, *performs the function of a cube.* If this subject were pursued it might be shewn that the cubic function is performed by molecules very variously composed.

† Whether these little cubes would consist of one or more *atoms* of lead and of sulphur, or how these elementary particles would be combined in the production of a cubic molecule, are circumstances not immediately relating to Crystallography, however interesting they may be as separate branches of enquiry.

If we reduce a crystal of *carbonate of lime* to fragments, the planes of those fragments will be found to incline to each other at angles which are respectively equal to those of the primary rhomboid. We therefore infer that the *molecule of carbonate of lime is a minute rhomboid similar to the primary form.*

Sulphate of barytes may be split into right rhombic prisms, whose angles are respectively equal to those of the primary crystal. It is therefore supposed *that the primary crystal and the molecules of this substance are similar prisms.*

Having thus found that crystals belonging to several of the classes of parallelopipeds may be split into fragments resembling their respective primary forms; and having assumed that these fragments represent the *molecules* of each of those forms respectively, it has been concluded that the *primary forms* and the *molecules* of *all the classes of parallelopipeds* are respectively similar to each other.

This similarity does not however exist between the *other classes* of primary forms and their respective molecules.

Fig. 60.

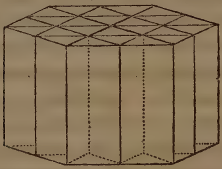
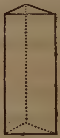


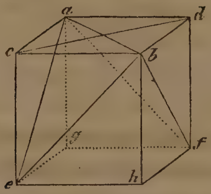
Fig. 61.



If a regular *hexagonal prism* of *phosphate of lime* be split in directions parallel to all its sides, it may be divided into *trihedral prisms whose bases are equilateral triangles*; these may be regarded as the *molecules* of this class of primary forms.

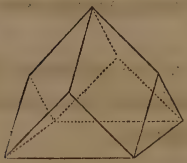
Fig. 60 shews the hexagonal prism composed of trihedral prisms; and fig. 61 shews the trihedral prism separately.

Fig. 62.



If we reduce *a cube of fluate of lime* to fragments, we shall find that it does not split in directions parallel to its planes as galena does, but that it splits obliquely. If we suppose fig. 62 a cube of fluor, and we apply the edge of a knife to the diagonal line *a b*, and strike it in the direction of *c*, we may remove the solid angle *a b e c*.

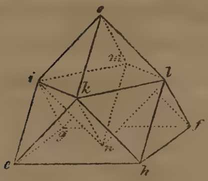
Fig. 63.



If we again apply the edge of the knife to the same line *a b*, and strike it in the direction of *f*, we may remove another solid angle *a b f d*; applying the knife again in the direction of the line *c d*, and striking successively in the directions *g*, and *h*, we may remove two other solid angles.

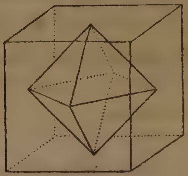
The new solid produced by these cleavages is represented by fig. 63.

Fig. 64.



If we apply our knife again to the line *i k*, *k l*, *l m*, *m i*, fig. 64, and strike in the direction of *n*, we may remove the remaining solid angles of the cube, and we shall then obtain the regular octahedron *i k l m n o*.

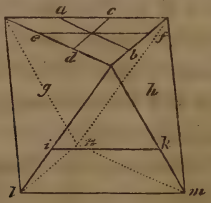
Fig. 65.



The position of this octahedron in the cube is shewn by fig. 65.

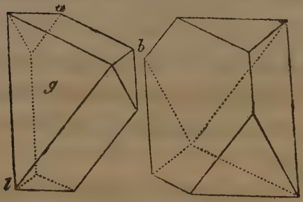
This octahedron is the primary form of fluate of lime, and it may obviously be cleaved in a direction parallel to its own planes.

Fig. 66.



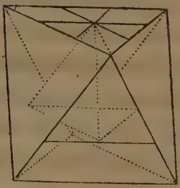
To illustrate more perspicuously the relation we are about to trace between the octahedron and tetrahedron, it will be convenient to place the octahedron of fluor, which we have just obtained, in the position represented in fig. 66, *resting on one of its planes.*

Fig. 67.



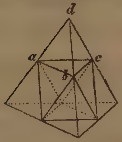
In this position of the crystal, if we suppose the three lines *a b*, *c d*, *e f*, to be drawn through the centre, and parallel to the edges, of the now uppermost plane, and if we apply our knife to the line *a b*, we may cleave the crystal parallel to the plane *g*, and may detach the portion *a b g l*, fig. 67.

Fig. 68.



By cleaving again from the lines *c d*, and *e f*, parallel to the plane *h*, and to the back plane of fig. 66, and by also cleaving parallel to the plane on which the figure rests, beginning at the line *i k*, we shall obtain a regular tetrahedron as seen in fig. 69. In fig. 68 this tetrahedron is exhibited in the position which it occupied in the octahedron.

Fig. 69.



The tetrahedron thus obtained, may be reduced again to an octahedron, as shewn in fig. 69, by removing a smaller tetrahedron, as *a b c d*, from *each* of its solid angles.

And all the fragments separated from the octahedron by the cleavages just described, may also be reduced, by cleaving in the proper directions, to regular octahedrons and tetrahedrons.

In this case *two distinct solids are obtained from the cleavage of an octahedral crystal; and the Abbé Haüy has chosen to assume the tetrahedron as the molecule of the octahedral crystal, upon the supposition that if the cleavage were continued until only single molecules remained to be separated, these molecules would be tetrahedrons; and the octahedron is, according to his theory, conceived to be composed of tetrahedral solids united by their points, and octahedral spaces.* *

From considerations analogous to these, the Abbé Haüy has concluded that the *tetrahedron, when it occurs as a primary form, is constituted also of tetrahedral molecules and octahedral spaces.*

---

* The same imaginary structure has also been supposed by the Abbé Haüy to exist in every class of octahedrons, the molecules peculiar to each being distinct irregular tetrahedrons, varying in their angles and relative dimensions in each particular case.

But it will be attempted to be shewn presently that *this imaginary structure does not belong to the octahedron, and that the tetrahedral solid does not represent the molecule of that form.*

The *regular dodecahedron* may be cleaved into *obtuse rhomboids, obtuse octahedrons*, and *irregular tetrahedrons*, as will be shewn in the section on cleavage. *Of these the Abbé Haüy has chosen the irregular tetrahedron for the molecule of the dodecahedron, and he has supposed that the decrements on this form are produced by the abstraction, not of single molecules, but of masses of single molecules packed into the figure of those obtuse rhomboids which are produced from its cleavage.**

The very complicated system of molecules which the Abbé Haüy has, by this view of the structure of the octahedron and dodecahedron, introduced into his otherwise beautiful theory of crystals, and the apparent improbability that the molecules of the cube, the regular octahedron, tetrahedron and dodecahedron, *among whose primary and secondary forms so perfect an identity subsists*, should really differ from each other, have induced me to propose a new theory of molecules in reference to all the classes of octahedrons, to the tetrahedrons, and the rhombic dodecahedron, which I shall now state.

Fluate of lime, as we have seen, has for its primary form a *regular octahedron*, under which it sometimes occurs in nature; but it is generally found in the form of *a cube*, and sometimes as a *rhombic dodecahedron*, and it has *a cleavage in the direction of its primary planes.*

Galena, whose primary form is *a cube*, is also found under the forms of an *octahedron*, and *rhombic dodecahedron*, with *a cleavage parallel to its cubic planes.*

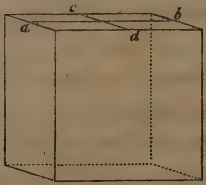
---

* Under the head of cleavage I shall endeavour to explain the nature of the relation which the different solids obtained by cleavage from the tetrahedron, octahedron, and rhombic dodecahedron, respectively bear to those primary forms, and to each other; and to shew *that they do not in either case represent the molecules of these forms.*

Grey copper, whose primary form is a *tetrahedron,* occurs under the forms of the *cube, octahedron,* and *rhombic dodecahedron.*

Blende is found sometimes, though rarely, crystallized in *cubes,* sometimes in *octahedrons, tetrahedrons,* and *rhombic dodecahedrons.*

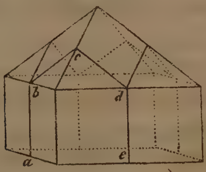
Fig. 70.



If we attempt to fracture a cube of blende, we find it will split in directions parallel only to its *diagonal planes.* These cleavages will truncate the edges of the cube, and if continued until all the edges are removed, and the face of the cube disappear, *a rhombic dodecahedron* will be produced, which has been considered the *primary crystal* of blende.

If a cube of blende, fig. 70, be cleaved in directions parallel to its *diagonal planes,* beginning at the lines *a b, c d,* fig. 71 will be produced.

Fig. 71.



If fig. 71 be further cleaved in directions corresponding to *a b c d e,* so as to remove all the *perpendicular edges,* and to *obliterate the remainder of the perpendicular planes* of the cube, fig. 72 will remain.

Fig. 72.

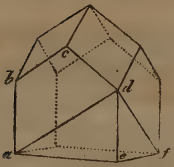


Fig. 73.

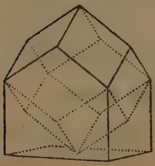


Fig. 73 exhibits the dodecahedron contained in fig. 72; this may be obtained by cleavages in directions corresponding with the lines *a d f*, fig. 72, which will remove the solid angles of the base on which fig. 72 and 73 rest.

Fig. 74.

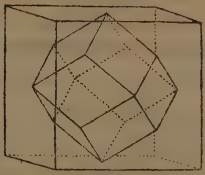


Fig. 74 shews the position of the rhombic dodecahedron in the cube.

Having thus observed that the *cube*, the *regular tetrahedron* and *octahedron*, and the *rhombic dodecahedron are common as primary or secondary forms to different crystallized substances, we may reasonably infer that they are produced in each instance by mole-*

cules of a form which is common to all; and let us suppose this common molecule to be a cube.

Fig. 75, 76, 77, and 78, shew the arrangement of the cubic molecules in each of these forms.

Fig. 75.

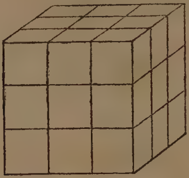


Fig. 75 in the cube.

Fig. 76.

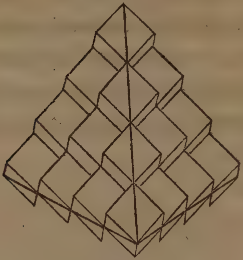


Fig. 76 in the tetrahedron.

Fig. 77.

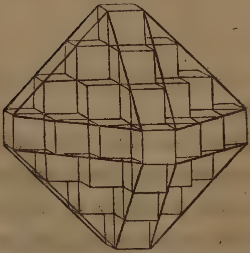


Fig. 77 in the octahedron.

Fig. 78.

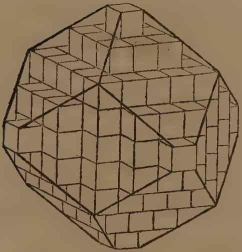


Fig. 78 in the rhombic dodecahedron.

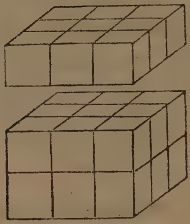
These arrangements of cubic molecules cannot be objected to on account of any supposed imperfection of surface which would be occasioned by the faces of all the primary forms, except the cube, being constituted of the edges, or solid angles, of the molecules. For as we observe that the octahedral and dodecahedral planes of some of the secondary crystals of galena, which are obviously composed of the solid angles, or edges, of the cubic molecules, are capable of reflecting objects with great distinctness, it is evident that the size of the molecules of galena is less than the smallest perceptible inequality of the splendent surface of those planes, and hence we infer generally that *there will be no observable difference in brilliancy between the surfaces of the planes obtained by cleavage parallel to the sides of molecules, and of those which would expose their edges or solid angles.*

This theory may be reconciled with the cleavages which are found to take place parallel to the primary planes of the tetrahedron, the octahedron, and the rhombic dodecahedron, as well as to those of the cube, if we suppose the cubic molecules capable of being held to-

gether with different degrees of attractive force in different directions.[*]

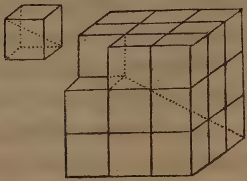
I shall call this force *molecular attraction.*

Fig. 79.



When this attraction is *least* between the *planes of the molecules*, they will be more easily separated by cleavage *in the direction of their planes*, than in any other direction, as shewn in fig. 79, and a cubic solid will be obtained.

Fig. 80.



When the attraction is *least* in the direction of the *axis of the molecules*, they will be the most easily separated in that direction, as in fig. 80, and the *octahedron* or *tetrahedron* will be the result of cleavage.

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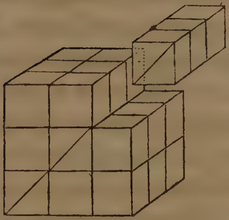
[*] It is possible to conceive that the nature, the number, and the particular forms, of the *elementary particles* which enter, respectively, into the composition of these three species of cubic molecules, may vary so much as to produce the variety of character which I have supposed to exist.

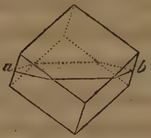
Fig. 81.



And if the attraction be *least* in the direction of its *diagonal planes*, the *edges* will be most easily separated, as in fig. 81, and a *rhombic dodecahedron* will be the solid produced by cleavage.

This supposition of greater or less degree of molecular attraction in one direction of the molecule than in another, is consistent with many well known facts in Crystallography.

Fig. 82.



The primary form both of corundum, and of carbonate of lime, is a rhomboid; and the crystals of these substances may be cleaved parallel to their primary planes, the carbonate of lime cleaving much more readily than the corundum. But the corundum may also be cleaved in a direction *a b*, fig. 82, perpendicular to its axis, which carbonate of lime cannot be.

This cleavage would *either divide the rhombic molecules in half*, or, the *cleavage planes would expose the terminal solid angles of the contiguous molecules.*

But it is contrary to the nature of molecules that they should be thus divided, and we may therefore infer from this transverse cleavage that the *molecular attraction* is *comparatively less* in the direction of the perpendicular axis of the molecules of corundum, than it is in the same direction of those of carbonate of lime. And from the *greater adhesion* of the *planes* of *corundum*, than of *those of carbonate of lime*, we infer that the attraction is *comparatively greater* between the planes of the molecules of the corundum, than between those of carbonate of lime.*

This supposition of the existence of a greater or less degree of molecular attraction in one direction of the molecule than in another, appears to explain the nature of the *two sets* of cleavages which occur in Tungstat of lime: *one of these sets* is parallel to the planes of *an acute octahedron with a square base*, which we will call the primary crystal; the *other set* would produce *tangent planes* upon the terminal edges of that crystal. If we suppose the molecules to consist of *square prisms* whose *molecular attraction* is *greatest* in the direction of their *prismatic axis*, and nearly equal in the direction of their *diagonal planes*, and of their *oblique axes*, the *first set* of cleavages may be conceived to expose the *edges of the molecules*, and the *second set* to expose their *solid angles*.

---

* I am aware of an objection that may be made to this view of the subject, by supposing all the cleavages which are not parallel to the primary planes of a crystal, to be parallel to some secondary plane, and to be occasioned by the slight degree of cohesion which frequently subsists between the secondary planes of crystals and the plates of molecules which successively cover them during the increase of the crystal in size; but although the second set of cleavages may sometimes be connected with the previous existence of a secondary plane, it may also be explained according to the theory I have assumed.

Those cleavage planes which would not expose the planes, edges or solid angles of the molecules, must be considered to belong always to

This theory may, by analogy, be extended to the form of molecules of every class of octahedron.

For we may conceive the *molecules of all the irregular octahedrons to be parallelopipeds, whose least molecular attraction is in the direction of their diagonal planes.*

Thus the *molecules of octahedrons with a square, a rectangular, and a rhombic base, would be square, rectangular, and rhombic prisms respectively; the dimensions of such molecules being proportional respectively to the edges of the base and to the axis of each particular octahedron.*

According to the view here taken, the following table will exhibit the form of the molecules belonging to each of the classes of primary forms.

The cube	molecule, a cube.	
regular tetrahedron		
——— octahedron		
rhombic dodecahedron		
all quadrangular prisms	molecules, similar prisms.	
octahedron with a square base	molecule, a square prism	Proportional in dimensions to the edges of the base, & to the axis of each particular octohedron, respectively.
——————— rectangular base	molecule, a rectangular prism	
——————— rhombic base	molecule, a rhombic prism	
rhomboid	molecule, a similar rhomboid	
hexagonal prism	molecule, an equilateral triangular prism.	

Having thus advanced a new theory of molecules in opposition to one that had been long established, and possibly without a much better claim to general

the class of *planes of composition*, a term which Mr. W. Phillips has applied to those cleavage planes which result from cleavages parallel to secondary planes only.

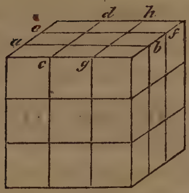
reception than the former theory possessed, I cannot avoid observing that the *whole theory of molecules and decrements*, is to be regarded as little else than *a series of symbolic characters*, by whose assistance we are enabled to investigate and to demonstrate with greater facility the relations between the primary and secondary forms of crystals. And under this view of the subject, we ought to divest our notions of *molecules*, and *decrements*, of that absolute reality, which the manner in which it is necessary to speak of them in order to render our illustrations intelligible, seems generally to imply.

SECTION III.

STRUCTURE.

THE structure of crystals, or the order in which their molecules are arranged, may be inferred from an experiment with common salt. If we dissolve a portion of this salt in water, and then suffer the water to evaporate slowly, crystals of salt will be deposited on the sides and bottom of the vessel. These will at first be very minute, but they will increase in size as the evaporation proceeds; and if the quantity of salt dissolved be sufficient, they will at length attain a considerable bulk. If the forms of the small crystals be examined, they may be found to consist of entire, or modified cubes. If we continue to observe any of these cubic or modified crystals during their increase in bulk, we may find that the forms of some of them undergo a change, by the addition of new planes, or the extinction of some that had previously existed. But we shall also frequently find that both the cube, and the modified crystal, when enlarged, preserve their respective forms. *The increase of a crystal in size appears therefore to be occasioned by the addition of molecules to some, or all, of the planes of the smaller crystal, whether these planes be primary or secondary.*

Fig. 83.



If we apply the edge of a knife to the surface of any one of these cubic crystals of common salt, in a direction parallel to *an edge* of the cube, as at *a b*, fig. 83, the crystal may, by a slight blow, be cleaved parallel to one of its sides.

If we apply a knife in the same manner successively to the other lines *c d*, *e f*, *g h*, and to the other surfaces of the crystal, so that its edge be parallel in each instance to the edge of the cube, we shall find that there are cleavages parallel to all the planes of the cube; and if the crystal be split with perfect accuracy, a cubic solid may be extracted; and the rectangular plates which have been removed by these cleavages, may be also subdivided into smaller cubes.

From these circumstances we infer that *the molecules which have successively covered the planes of the small crystals, are cubes, and that they are so arranged as to constitute a series of plates*, as shewn in p. 18. And we further conclude that the molecular attraction is *least*, in common salt, between the *surfaces* of the molecules.

This regular structure is supposed to belong to all regularly crystallized bodies.

It frequently happens that the regular crystallization of bodies has been prevented by some disturbing cause, in which case the *crystalline mass* will be curved or otherwise irregular, or it may even present a granular character. This granular character would be presented if the solution we have supposed of common salt were rapidly evaporated and suddenly cooled.

SECTION IV.

CLEAVAGE.

The *spliting* a crystal in the manner already described, is, in the language of Mineralogy, termed *cleaving* it.

The direction in which the crystal can be split is called the *direction of the cleavage*, or the *natural joint* of the crystal.

The direction of the *natural joints* may depend, according to the preceding theory, upon the comparative degrees of molecular attraction existing in the different directions of the molecules. This may be so proportioned in different directions, as to occasion other cleavages than those which are parallel to the planes which we may assume as the primary planes, as in the instances already cited of the corundum, and tungstat of lime.

When this occurs the crystal is said to possess *two or more sets of cleavages*. Those which are parallel to the planes of the primary form, are called the *primary* set, and those which are not parallel to those planes are termed *supernumerary* sets.

The oxide of tin, described by Mr. Phillips in the Geological Transactions, has *three* sets of cleavages; one parallel to the planes of an obtuse octahedron with a square base, which is considered the *primary set*, and two others which are *supernumerary*, and are parallel to the edges, and to the diagonals, of the square base, being at the same time perpendicular to the plane of that base.

If all the planes of any primary form be *similar*, as those are of the cube, rhomboid, and some other forms, the *primary cleavages* will generally be effected with equal facility in the direction of each of those planes, and the new planes developed by this cleavage will be similar in lustre and general character. This may be illustrated by cleaving galena and carbonate of lime.

Where the planes of a primary form are not all similar, as in all prisms, and some octahedrons, the primary cleavage is not effected with equal facility in all directions, nor do the new planes all agree in their general characters. Hence the cleavage planes of a mineral will frequently enable us to determine what is *not* its primary form, by their similarity or dissimilarity; but, as will be seen in the section on primary forms, the cleavage is not sufficient to determine what the primary form really is.

Felspar, cyanite, and sulphate of lime, afford instances of the greater facility with which a cleavage takes place in one or two directions than in any other.

The Abbé Haüy has supposed that these unequal cleavages are occasioned by the unequal extension of the different primary planes. The broader planes, presenting more points of contact than the narrower ones, may, he imagines, be held together with greater force than the narrower ones are. This may possibly be the cause of the observed inequality of cleavage, or possibly where the planes are unequal, the degree of attraction between point and point is unequal also.

There are among minerals some substances which yield readily to mechanical division in *one or two directions, but do not admit of distinct cleavage in a third direction*, so as to produce a regular solid.

This circumstance has introduced into mineralogy the terms *single* cleavage, or *double*, *triple*, *fourfold*, &c. cleavage, which are sometimes perplexing to a learner, as they may be confounded with the different *sets* of cleavages before spoken of.

But these terms *single*, *double*, *triple* cleavage, &c. *are intended to refer strictly to the sets of primary cleavage only*.

When a mineral can be split in only *one direction*, the cleavage is said to be *single*; when in *two directions*, which may be conceived to give four sides of a prism, it has a *double* cleavage.

When there is a cleavage in *three directions*, such as to produce either the lateral planes of the hexagonal prism, or a solid bounded by six planes which are parallel when taken two and two, it is termed a *triple cleavage*.

A *four-fold* cleavage, or one in *four directions*, will produce a tetrahedron, an octahedron, or a perfect hexagonal prism; the two latter solids consisting of four pairs of parallel planes, lying in as many different directions.

The *rhombic dodecahedron* possesses *six pairs of parallel planes lying in different directions*, and may be said therefore to have a *sixfold cleavage*.

Sometimes the *natural joints* of a crystal may be perceived by turning it round in a strong light, although it cannot be cleaved in the direction of those joints.

Different specimens of the same substance will also yield to the knife or hammer with unequal degrees of facility; and even carbonate of lime, which splits readily in general, will sometimes present a conchoidal fracture.*

As a *crystalline solid cannot be contained within planes lying in less than three directions*, it is obvious that it cannot be produced by a *single* or *double* cleavage.

The *solids* obtained by cleavage may therefore, according to what has preceded, consist either of *primary* forms produced by *triple*, *fourfold*, or *sixfold primary* cleavages, or of other forms resulting from the *supernumerary* cleavages, either alone or combined with the *primary*.

But *another class* of solids may also result from cleavage when that takes place *parallel to some only* of the *primary planes* of those forms which possess *fourfold* or *sixfold* cleavages.

From a *primary triple* cleavage it is clear that only a *single solid* can be produced, that solid being a parallelopiped. But from either a *fourfold* or *sixfold* primary cleavage, more than one solid may result, according as the cleavage takes place *parallel* to *all*, or only to *some*, of the primary planes.

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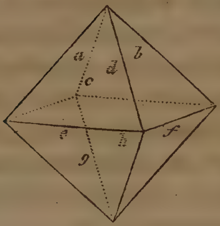
* Some practice is necessary in order to cleave minerals neatly, and some experience in the choice of the instruments to be used for this purpose.

In many instances, the mineral being placed on a small anvil of iron or lead, a blow with a hammer will be sufficient for dividing it in the direction of its natural joints; and sometimes a knife or small chissel may be applied in the direction of those joints, and pressed with the hand, or struck with a hammer; or the crystal may be held in the hand and split with a small knife; or it may be split by means of a pair of small cutting pincers *whose edges are parallel.*

A small short chissel, fixed with its edge outward in a block of wood, is a convenient instrument for resting a mineral upon which we are desirous of cleaving.

Fig. 84.



If we cleave an octahedron parallel to *six only* of its planes, omitting any two opposite ones, as *b* and *e*, fig. 84, and if we continue the cleavage until only the *central points* of the planes *b* and *e* remain, a figure of *six sides* will evidently be produced.

This figure is a rhomboid whose plane angles are 60° and 120°.

Fig. 85.

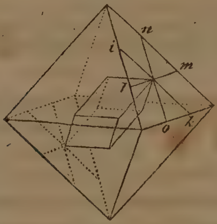


Fig. 85 shews the position of this rhomboid in the octahedron, from which it is evident that the cleavage would be continued as far as the lines *i k*, *l m*, *n o*, and those which are parallel to them on the opposite plane.

Fig. 86.

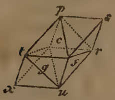
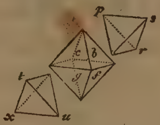


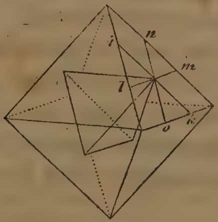
Fig. 86 exhibits the same rhomboid separately, the planes being marked with the same letters as are placed on such planes of the octahedron as are parallel to those of the included rhomboid.

Fig. 87.



Our *rhomboid* may thus be regarded as an *imperfect octahedron, two of its planes being concealed, or covered by small tetrahedrons p r s*, and *t u x*, as in figure 86. These tetrahedrons consist of masses of cubic molecules, and by their removal, as in fig. 87, we shall obviously reproduce the *perfect* octahedron.

Fig. 88.



If we now cleave the octahedron *parallel to any four alternate planes*, as *c d*, *e f*, fig. 84, and continue the cleavage as far as the lines *i k*, *l m*, *n o*, fig. 88, and until only the central points of the four planes *a*, *b*, *g*, *h*, remain, we shall produce a regular tetrahedron, as shewn by the interior lines in the figure.

Fig. 89.

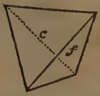
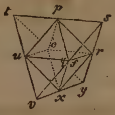


Fig. 89 exhibits this tetrahedron separately, its planes being marked with the same letters as appear on the planes of the octahedron, fig. 84, which are parallel to those of the included tetrahedron.

Fig. 90.



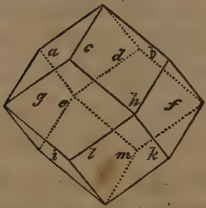
The tetrahedron thus obtained may be regarded as an *imperfect octahedron*, four of its planes being concealed, or covered by smaller tetrahedrons, *p q r s*, *p q u t*, *u q x v*, *q x r y*, as in fig. 90, and it is capable of being reduced again to the *perfect* octahedron by the removal of those masses of cubic molecules which constitute the tetrahedrons by which the concealed planes are covered.

The tetrahedron and octahedron have thus obviously *the same set of cleavages*, and if the tetrahedron be the primary form, the octahedron may be regarded as an imperfect tetrahedron, requiring certain additions to complete that form.*

---

* The student is advised to trace the relation of the octahedron to the acute rhomboid and tetrahedron, by means of an octahedron of fluor produced by cleavage or otherwise. Let him place this on a table, and by the assistance of a small hammer and a knife, he may procure from it, by well observing the figures as he proceeds, the acute rhomboid, and tetrahedron, and from them he may re-produce the octahedron.

Fig. 91.



If the rhombic dodecahedron, fig. 91, be cleaved parallel to the planes, *a*, *b*, *c*, *d*, and to the four planes opposite to these, *until the four remaining planes of the dodecahedron disappear, an obtuse octahedron will be produced.*

Fig. 92.

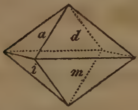
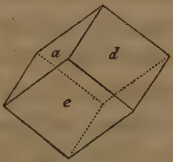


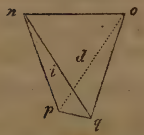
Fig. 92 exhibits this octahedron separately, the planes being marked by the same letters as appear on the corresponding planes of the dodecahedron.

Fig. 93.



If the cleavage be effected parallel only to the planes *a*, *d*, *e*, and *h*, *b*, *k*, until the other primary planes disappear, an obtuse rhomboid will result, as seen in fig. 93; this rhomboid measures 120° over the edges at which the planes *a*, and *e*, meet.

Fig. 94.



If the cleavage take place parallel only to the planes *c d* and *i k*, and be continued until only the four cleavage planes remain, an irregular tetrahedron, fig. 94, will be produced, whose planes meet at an angle of 90° at the edges *n o*, *p q*, and at an angle of 60° at the other edges.

Thus an *obtuse octahedron*, *d* on *m*, fig. 92, measuring 60°, an *obtuse rhomboid of* 120°, and an *irregular tetrahedron*, obtained by *partial cleavages* from the *rhombic dodecahedron*, may be regarded as *imperfect dodecahedrons, to which figure they may be reduced, by detaching from each solid the portions of cubic molecules which respectively cover the obscured dodecahedral planes.*

But it is very obvious that these imperfect forms may be obtained as well *by cleaving at once through the interior of the crystal, in directions parallel to* 8 *to* 6 *or to* 4 *only of the primary planes, as by beginning to cleave from the outside, and arriving by degrees at the new figure in the manner already described.*

Hence it appears that a *dissection of the octahedral and dodecahedral crystals by cleavages, parallel to some only, of the primary planes, will yield only the imperfect solids above described, not any of which will represent the molecules of which the crystals are composed.*[*]

---

[*] Blende will afford the student an opportunity of producing, by cleavage, the solids represented by fig. 91 to 94.

The relation of the tetrahedron to the octahedron in reference to the theory of cubic molecules, may be explained in the following manner.

The Abbé Haüy's theory, it will be recollected, supposes that if the tetrahedron obtained by cleavage from the octahedron, were to be successively reduced to an octahedron and four still smaller tetrahedrons, we should at length arrive at a tetrahedron consisting of four single tetrahedral molecules *enclosing only an octahedral space, instead of an octahedral solid.*

But according to the structure assigned to the octahedron by the theory of cubic molecules, that figure is an entire solid; and the smallest tetrahedron that can be imagined to exist, will contain an octahedral solid, and would be reduced to an octahedron by the removal of four *cubic* molecules from its four solid angles, and not of four *tetrahedrons.*

Fig. 95.

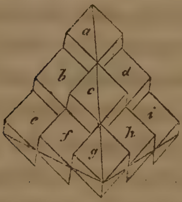
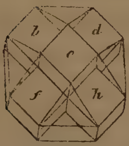


Fig. 96.



Let fig. 96 be supposed to represent the smallest octahedron that can be imagined to exist, formed of seven cubic molecules, and let fig. 95 represent a tetrahedron containing this minute octahedron. The tetrahedron would obviously be reducible to the octahedron, as other tetrahedrons are, by the removal of all its solid angles.

But it is apparent that the solid angles to be removed in this instance, are the small cubes *a e g i*, and by their removal the octahedral solid shewn in fig. 96 will remain.

This octahedron is supposed to rest on one of its planes, and the molecules *b c*, *c h*, *f c*, *c d*, may be conceived to constitute four of its edges.

Thus the necessity of adopting the tetrahedron as the molecule of the octahedron is removed, and in consequence a more simple theory of the structure of the octahedron, may be substituted for that which has been established upon the adoption of tetrahedral molecules.

By a similar mode of reasoning, the compatibility of the cubic molecule with the solids obtained by cleavage from the rhombic dodecahedron, might be shewn; and by adopting the cubic molecule, a more simple theory of decrement, in relation to the rhombic dodecahedron, may be substituted for that which has been established upon the assumption of the irregular tetrahedron as the integrant molecule, and the obtuse rhomboid as the subtractive molecule.

SECTION V.

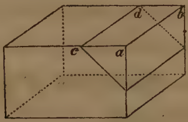
DECREMENTS.

Decrements have been already defined to be either *simple*, *mixed*, or *intermediary*; and the *simple* decrements have been divided into two classes, according as they take place in *breadth*, or in *height*: see Definitions, page 19.

The manner in which all the classes of decrements operate in the production of new planes, has also been explained.

Simple and *mixed decrements* take place either on the *edges* of crystals, or on the *angles*, and produce new planes *which intersect one at least of the primary planes, in lines parallel to one of its edges or diagonals.*

Fig. 97.



If either a *simple* or *mixed* decrement take place on the edge *a b*, of any primary form whatever, a new plane is produced, *whose intersection c d, with the primary plane along which the decrement may be conceived to proceed, is parallel to the edge a b, from which it may be said to begin or set out.*

Fig. 97 shews the character of the secondary plane produced by a *simple* or *mixed* decrement on the edge of a rectangular prism.

Fig. 98.

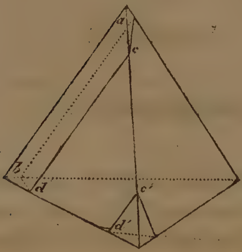


Fig. 98 shews the character of a similar plane on the edge of a tetrahedron.

Fig. 99.

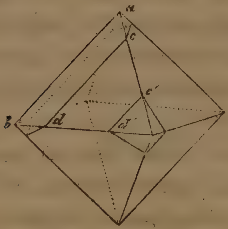


Fig. 99 shews the character of a similar plane on the edge of an octahedron.

If a *simple* or *mixed decrement* take place on the *angle* of a *tetrahedron* or *octahedron*, one new *edge* $c' d'$, of the secondary plane, will be always parallel to an edge $a\ b$, of the primary form, as shewn in figures 98 and 99.

If either a *simple* or *mixed* decrement take place on the *angle* of any primary form, *except the tetrahedron and octahedron*, a new plane is produced, whose intersection with the primary plane, along which the decrement may be conceived to proceed, *is parallel to the diagonal of that plane.*

Fig. 100.

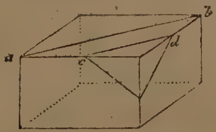
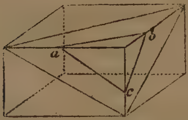


Fig. 100 exhibits a plane produced on the angle of a rectangular prism, by a simple or mixed decrement; the edge *c d*, of the secondary plane, being parallel to the diagonal *a b*, of the primary form.

Fig. 101.



Intermediary decrements may be said to take place *only on the solid angles* of crystals, by the omission of *unequal numbers* of molecules in the direction of the edges which meet at such solid angle. And a plane is thus produced, *none of whose lines of intersection*, *a b*, *c b*, *a c*, with the *primary planes*, *are parallel* to *any edge*, or *diagonal of those planes.*

Fig. 101 shews the position of a plane produced by an *intermediary decrement* on the angle of a rectangular prism.

Fig. 102.

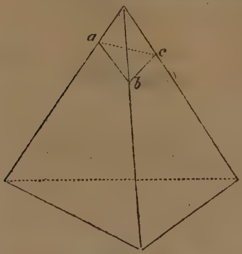


Fig. 102 contains a similar decrement on the angle of a tetrahedron.

Fig. 103.

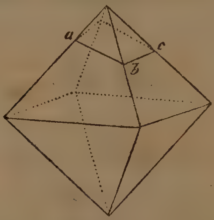
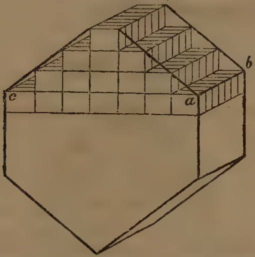


Fig. 103 shews the effect of a similar decrement on the angle of an octahedron.

In the illustrations hitherto given of the nature of decrements, and of the characters of the secondary planes produced by them, we have considered the effect produced upon only *a single edge or angle of the primary form.* But as decrements generally take place equally on *all* the *similar* edges and angles of any primary form, it will be necessary now to enquire into the manner in which these similar edges and angles are affected, when they are *all* operated upon at the same time, by any given decrement.

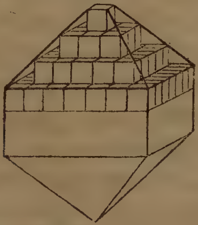
From the definition already given in page 3, of the nature of *similar edges* and *angles*, it will appear that in the *rectangular prism, those edges only are similar* *which are parallel to each other.* And if we refer to the tables of modifications of that form, which will be found in a subsequent section, we shall observe it is only the parallel edges which are affected by the modifications *b*, *c* and *d*.

Fig. 104.



Let us now suppose a modification belonging to the class *c* to have taken place on a *right rectangular prism*, and let us suppose the secondary crystal produced by this modification represented by fig. 104. The upper part of this figure shews the manner in which the new planes are conceived to be produced, by the continual abstraction of single rows of molecules on both the edges *a b*, and *c*, of each of the superimposed plates, until the last plate consists of only one row, forming the new edge of the secondary crystal.

Fig. 105.



The *square prism* has all its *terminal edges similar*, and *all its terminal angles also similar*, and consequently when *one* of those edges or angles is affected by any decrement, they will generally *all* be so.

Fig. 105 exhibits modification *c*, of the square prism, the upper part of the figure shewing the manner in which the secondary planes are conceived to be generated, by the continual abstraction of single rows of molecules from *each* edge of *each* of the successively superimposed plates.

Fig. 106.

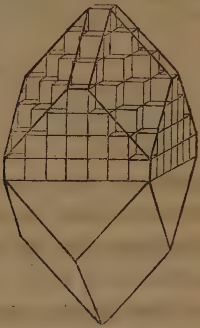
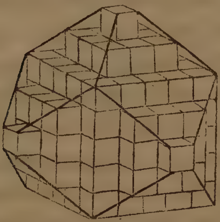


Fig. 106 exhibits the effect of a decrement by one row of molecules on the angle of a square prism, producing a secondary form belonging to the class *a* of the modifications of that figure. See tables.

Fig. 107.



The cube has its *three adjacent edges similar*, and consequently they are *all affected equally* by decrements upon the edge of that form, except in some particular cases which will be referred to in a future section.

Fig. 107 shews the effect of a decrement by one row of molecules on the edges of a cube; producing the planes of the rhombic dodecahedron.

Fig. 108.

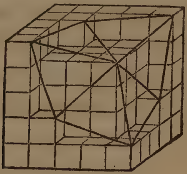


Fig. 108 shews the manner in which an intermediary decrement, taking place at the same time upon the three adjacent angles of the cube, is conceived to produce six planes on the solid angle.

The causes which occasion decrements do not appear at present to be understood: crystals so minute as to be seen only by the aid of a microscope, are found variously modified; hence the circumstance, whatever it may be, which occasions the modification, begins to operate very soon after the crystal has been formed.

Perhaps it may influence the arrangement of the first few molecules which combine to produce the crystal in its nascent state; and as we find that crystals during their increase in magnitude, sometimes undergo a change of form, by the extinction of some modifying planes, or the production of others, it is evident that the cause which occasions a decrement, may be suspended, or may be first brought into operation, at any period during the increase of a crystal in size.

For the purpose, however, of affording a clearer illustration of the theory of decrements, it has been found convenient to imagine that the primary form of any modified crystal had attained such a magnitude, before the law of decrement had begun to act upon it, as to require for the completion of the modified crystal, the addition of only those *defective* plates of molecules by which the modifying planes were produced. A primary form of this magnitude, is evidently the greatest that could be inscribed in the given secondary form. And a primary form so related to the secondary, is in theory termed the *nucleus* of the secondary form.

This *nucleus* may frequently be extracted from the secondary crystals by cleavage.

If we take a crystal of carbonate of lime of the variety called dog-tooth spar (the metastatique of the Abbé Haüy,) and begin to cleave it at its summit, we shall first remove those molecules which were last added in the production of the crystal; and by continuing to detach successive portions, thus proceeding in an order the inverse of that by which the modified crystal has been formed, we may remove the whole of the laminæ, which enclose or cover the theoretical primary nucleus.

As far as we have proceeded with the theory of decrements, we have supposed the diminished plates of molecules to be laid constantly upon the primary form, in order to produce the modifications which are found to exist in nature. But from a comparison of the angles at which some secondary planes incline on the primary, and on each other, it is probable that the decrements sometimes take place on secondary crystals. Thus, for example, we may conceive decrements to take place on any *secondary* rhomboid of carbonate of lime by the abstraction of *secondary* molecules similar in form to that *secondary* rhomboid.

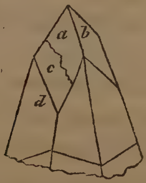
These *secondary* molecules would consist of certain numbers of primary ones arranged in the same order as they would be in the production of the entire secondary crystals, and they would in fact be minute secondary crystals.

There is an interesting paper on this subject by the Abbé Haüy, in the 14th vol. of the Annales du Museum, p. 290, where, in order to express the laws of decrement in as low numbers as possible, he has in several instances conceived the decrements to take place on secondary forms.

Another circumstance apparently influenced by a cause in some degree similar to that which produces decrements, is the colour which occasionally covers *some* of the modifying planes of a crystal while the other planes remain uncoloured.

A specimen of carbonate of lime, from St. Vincent's rocks near Bristol, which now lies before me, affords an instance of this.

Fig. 109.



In this specimen the planes *a*, *b*, of some of the crystals *and those planes only*, are covered with particles of, I believe, oxide of iron, upon which no molecules of carbonate of lime appear to have been subsequently deposited; but a thin plate of that substance is observed on some crystals as at *c*, to cover part of the coloured plane, having apparently begun to form at the edge *d*, and to have proceeded from that edge over part of the coloured plane *a*, but scarcely touching the colouring matter.

Mr. Beudant has collected together many facts relative to the influence which particular circumstances are supposed to exert upon the formation of crystals in the laboratory; but these facts are insufficient to explain the causes which determine the particular crystalline form of a mineral, or to account for its modifications. His memoirs, however, are interesting, and may be found in the Annals of Philosophy, vol. xi. p. 262; and in the Royal Institution Journal.

Decrements appear to be sometimes influenced, as will appear in the next section on the symmetry of crystals, by the capacity of a body to become electric by heat, but I am not aware that any explanation has been given of the manner in which this influence is exerted.

SECTION VI.

SYMMETRY.

AMONG the definitions will be found an explanation of what is meant by *similar edges, angles and planes of a crystal.*

It has been discovered, by the observation of a great number of secondary forms of crystals, that when a modification takes place on any one edge or angle of any primary form, a similar modification *generally* takes place on all the *similar* edges or angles. And this has been observed to occur so frequently, as to induce the conclusion of its being the effect of a general law, which the Abbé Haüy has called the *law of Symmetry.* This law however does not act universally with regard to all such similar edges or angles as are included under the definitions already given. The tourmaline will present an instance of deviation from this law. The primary form of the tourmaline is a rhomboid, and the three edges terminating in *a*, fig. 36, are *similar* to the three terminating in *b*; the *six lateral edges*, as well as the *six lateral solid* angles, are also respectively *similar.*

Yet it is found that the three edges terminating in *a*, are sometimes truncated, while those terminating in *b*, are not. It is also observed that sometimes only the *alternate three* of the lateral solid angles are modified, the three others remaining entire; but the tourmaline is pyro-electric, that is, capable of becoming electric by heat, and many other substances which are endued with a similar property, appear equally subject to a similar interference with the general law of symmetry.

But other substances which are not pyro-electric, as, for example, iron-pyrites, afford instances of deviation from that kind of symmetrical modification by which the cube of galena is affected. Pyro-electricity is not therefore the only disturbing cause which influences the deviation from the general law of symmetry.

With all the known exceptions however to this law, there are still so many substances influenced by it, that the character it confers on crystals is generally serviceable for determining the class of primary form to which any secondary crystal belongs.

SECTION VII.

PRIMARY FORMS.

THE *derivative* or *parent* form, from which the *secondary* forms of any crystallized mineral may be conceived to be derived by the operation of certain laws of decrement, has been denominated the *primary form* of such mineral.

It may be added that the *primary form of a mineral should not be inconsistent with its known cleavages, and it should generally be such also as would produce the secondary forms of the species to which it belongs by the fewest and simplest laws of decrement.**

It is for the sake of rendering our notions of a primary form more precise, that we give this limiting, and in some degree arbitrary, definition of the term. Our purpose throughout this treatise is, to find the shortest and most direct road from the *secondary crystal* to the *mineral species* to which it belongs.

But as we must travel first from the *secondary* to the *primary* form, it is essential that our ideas of that figure which we agree to call the primary form, should be as precise as possible.

The *primary forms* of crystals may sometimes be developed by cleavage.

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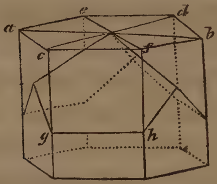
* The term *primary*, so defined, is merely relative, being used in contradistinction to *secondary*. It appears therefore preferable to the term *primitive*, which has been generally used to designate this original or parent form, and which seems to imply something more intrinsic, and absolute, than is required by the science into which it is introduced.

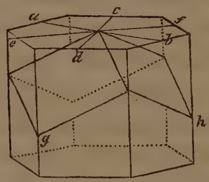
Thus a hexagonal prism of *carbonate of lime*, which is one of the *secondary forms of that mineral*, may be cleaved in a direction parallel to the planes of a *rhomboid*, which is its *primary* form.

Fig. 110.



When the prism results from a decrement on the inferior angle of the rhomboid, as shewn in the table of modifications of the rhomboid class *e*, this cleavage will take place parallel to the three alternate terminal edges of the prism, as shewn in fig. 110.

Fig. 111.



But when the prism results from a modification on the inferior edges of the rhomboid, corresponding with modification class *n*, the cleavage will take place on the alternate solid angles of the prism as in fig. 111.

A similar prism of phosphate of lime, *which is the primary form of that substance*, cannot be cleaved in any other direction than parallel to its own planes.

But cleavage alone cannot be relied on for determining the primary form of a mineral. For if, as it frequently happens, two or more solids can be extracted from a crystallized mineral by cleavage, we must refer to its secondary forms, in order to determine which of those solids ought to be adopted as its primary form. And if the secondary forms can be derived from one of such solids by the operation of single decrements, while it would require two or more decrements to operate simultaneously on the other, in order to produce the same secondary forms, that solid will be adopted as the primary, from which the secondary forms belonging to the species might be derived by the single decrement.

A case however may occur in which two different solids may be produced by cleavage, from *either* of which, the secondary forms of the particular species of mineral in which it occurs, may be derived by laws of decrement equally simple. Whenever this happens we shall be at liberty to adopt either as the primary, and we should probably adopt that which predominates most among the secondary forms.

In the section on cleavage we have seen that the same *set* of cleavages will produce either a regular *tetrahedron*, or a regular *octahedron*, or a *particular rhomboid*, one only of which is to be regarded as the primary form of the species in which such cleavages occur.

In this case the secondary forms of the mineral we are supposed to be examining, can alone enable us to determine which to adopt.

The secondary forms of a rhomboid differ so much from those of the octahedron and tetrahedron, as to admit no doubt in the instance of fluor, that the primary form of this substance is not a rhomboid.

According to the law of symmetry, the modifications of a regular octahedron should *equally* affect *all* its *edges* or *angles*, and tend *equally* to the extinction of *all* its planes, while the modifications of the tetrahedron, regulated by the same law, would affect only *some of the edges or angles of a crystal containing all the octahedral planes.*

It may be observed by examining the crystals of spinelle and red oxide of copper, that the solid angles are sometimes replaced by four planes resting on the primary planes.

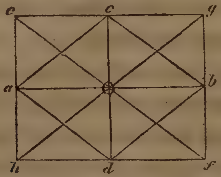
This change of figure results from a *single modification of the octahedron* belonging to class *b*. See tables.

But in order to produce four similar planes on each of the solid angles of the *octahedron*, regarded as a *secondary form of the tetrahedron, two modifications* of the tetrahedron must concur. We must suppose *each of the solid angles of the tetrahedron to be replaced by three planes resting on its edges, at the same time that its edges are bevilled.*

The octahedron will therefore, under the definition I have given of a primary form, be adopted as the primary form of spinelle and red oxide of copper, and of such other minerals as present secondary forms of a similar character.

I shall cite only one other instance of the insufficiency of cleavage alone to determine the primary form of a mineral, although many more might be adduced.

Fig. 112.



The petalite has *two sets* of cleavages, one in the direction of *a b*, *c d*, fig. 112, at right angles to each other, and another *e f*, *g h*, such that the angle *e o g*, measures about 100° and the angle *g o f*, about 80°.

Now these cleavages are respectively parallel to the sides of either a rectangular, or a rhombic prism, and either of these may therefore be the primary form of petalite. But there is no cleavage which indicates with certainty whether the prism, whichever of the two we may adopt, be *right* or *oblique*.

We cannot therefore determine the primary form of petalite, without a reference to crystals possessing their natural terminal planes, or such modifications of those planes as will shew whether the primary crystal be right or oblique.

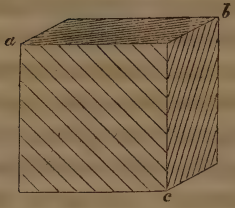
If we discover from the crystal of petalite that the primary form is a *right* prism, we should still be at liberty to take either the *right rectangular* or *right rhombic* prism.

But as the angles at which the planes of the rhombic prism incline to each other, are those by which the particular species of mineral would in this case be distinguished, we should at once adopt the *rhombic prism* as the primary form.

It has been observed in the section on cleavage, that the *character of the planes developed by cleavage* sometimes afforded indications of the primary form.

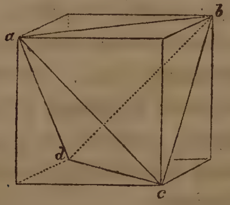
The striæ on the natural surfaces of the secondary planes of crystals have been also considered to afford indications of primary form. As those which are parallel to the shorter diagonal of the dodecahedral planes of the aplome, have been regarded as indications of the primary form being a cube, as it appears to be from its cleavage.

Fig. 113.



But this circumstance cannot be relied upon in all instances for determining the primary form of a mineral. For the cubic planes of iron pyrites, which are found by cleavage to be the primary planes, are striated in such directions, as might lead us to suppose them secondary planes, and its primary form to be a pentagonal dodecahedron. And the cubic crystals of blende are frequently striated parallel to the *alternate diagonals*, as shewn in fig. 113, which would indicate a *tetrahedral* primary form. But the cleavage is parallel, as we have seen, to *all* the *diagonal planes* of the cube, producing a rhombic dodecahedron, and not parallel to the planes *a b i*, as it should be if the tetrahedron were the primary form and consequently were to be produced by cleavage.

Fig. 114.



To illustrate the relation of these striæ to the supposed tetrahedron, we shall derive that figure from a cube by cleavage, and to do this we may again have recourse to a cube of fluor. Let fig. 114 represent this cube. If we apply a knife to the diagonal *a b*, and detach successively the two solid angles *a b c*, *a b d*, and then place the crystal with the edge *a b*, downwards, and remove the solid angles *c d a*, *c d b*, the figure we shall obtain will be the regular tetrahedron.

Each of the *classes of primary forms contained in page* 6 *to* 11, *except the cube, the regular tetrahedron, octahedron, and rhombic dodecahedron, comprehends many individual forms belonging to as many species of minerals; which individual forms differ from each other in some of their angles, or in the relative lengths of some of their adjacent edges.*

Section VIII.

SECONDARY FORMS.

The secondary forms of crystals are either *simple* or *compound*. The *simple* consist of modifications of the primary forms, produced by *single decrements*. The *compound* consist of *several modifications occuring together on one crystal*, produced by as many decrements operating simultaneously upon it. The cube with the solid angles truncated or replaced by three or six planes, is an instance of a simple secondary form produced by a single modification; but if the edges be also truncated or bevilled, or the solid angle be both truncated and replaced by three or six planes, it will afford an example of a compound secondary form.

The secondary planes frequently obliterate entirely the primary ones, and produce a new form apparently belonging to another class of primary forms, as in the instance of the rhomboid being converted into a six-sided prism by the truncation of all its solid angles, or of its terminal solid angles and its lateral edges.

Particular secondary forms sometimes predominate in particular species of minerals, as the cube in fluate of lime, whose primary form is an octahedron. Particular modifications of primary forms are also found to affect particular districts of country.

Thus the dodecahedral variety of carbonate of lime, commonly called dog-tooth spar, occurs the most frequently in Derbyshire.

In Cumberland, the most common variety is a six-sided prism terminated by the planes of an obtuse secondary rhomboid.

In the Hartz, the entire six-sided prism occurs more frequently than in other places.

Particular secondary forms are found to occur constantly among some species of minerals, and rarely among other species belonging to the same class of primary forms.

Thus the regular hexagonal pyramids, which occur constantly among the secondary forms of quartz, rarely occur in carbonate of lime.

The causes of these peculiar habitudes of minerals have not I believe been investigated, nor do I apprehend that the investigation would lead to any satisfactory result. They appear to belong to that class of facts, which our limited knowledge of the operations of nature does not enable us at present to comprehend.

SECTION IX.

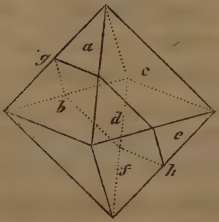
HEMITROPE AND INTERSECTED CRYSTALS.

BESIDES the secondary forms referred to in the preceding section, there is another class of crystals which were denominated *macles* by Romé de l'Isle, but which the Abbé Haüy has called *hemitrope crystals*, having assigned the term *macle* to a species of mineral more generally known by the name of chiastolite.

The term *hemitrope* has been derived from the resemblance of this class of forms to crystals which might be conceived to have been slit in a particular direction, and then to have had one half partly turned round on an imaginary axis, passing through the centre of, and perpendicular to, the planes of section.

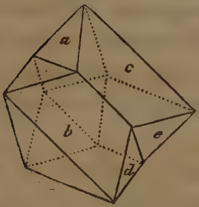
This kind of structure may be readily understood from one or two examples.

Fig. 115.



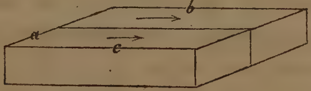
If we conceive an octahedron *a b c d e f*, fig. 115, to be cut through in the direction *g h*,

Fig. 116.



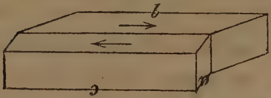
and if we suppose the half *b d f*, partly turned round as in fig. 116, until *b* is opposite to *c*, and *d* opposite to *e*, a hemitrope crystal would be produced, resembling one of the varieties of the common spinelle.

Fig. 117.



Again, if we suppose a right prism, whose base is an oblique angled parallelogram, fig. 117, to be cut through its centre and parallel to its lateral planes;

Fig. 118.



and if we then conceive the portion marked *a c*, turned round until the edges *b* and *c*, again become parallel as in fig. 118, we shall have a form of hemitrope crystal not of unfrequent occurrence in sulphate of lime.

These examples are sufficient to illustrate the manner in which hemitrope crystals may be conceived to be produced. But we cannot for a moment imagine that they are so produced. The arrangement of the molecules in so apparently capricious a manner, is doubtless the consequence of some law operating on the structure of the crystal from the commencement of its formation, and analogous to those laws by which other secondary forms are produced.

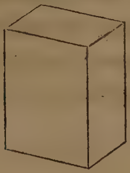
Hemitrope crystals consist of portions of either unmodified primary forms, or of secondary forms; and the plane of the imaginary section is found to be parallel either to the primary planes, or to some secondary plane which would result from some regular decrement.

Oxide of tin, as shewn in Mr. Phillips's paper on that substance in the 2nd vol. of the Geological Transactions, exhibits a considerable variety of this species of secondary form. It is very common also in felspar, in rutile, and sphene, and it occurs in many other substances.

One character by which hemitrope crystals may generally be known, is the *re-entering angle* produced by the meeting of some of their planes. This is very obvious in the figures 114 and 116. But even where this re-entering angle does not appear, there is generally some line, or some other character, which indicates the nature of the crystal.

Crystals are frequently found *intersecting each other* with greater regularity than can be ascribed to accident, and forming a class very analagous to hemitropes, and probably governed in their structure by the same general laws to which those forms owe their existence.

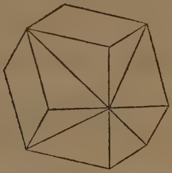
Fig. 119.



The staurotide affords a good example of this variety of form.

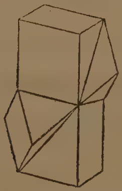
The primary form of the staurotide is a right rhombic prism, fig. 119.

Fig. 120.



Two of these prisms frequently cross each other at right angles as in fig. 120.

Fig. 121.



And sometimes at an oblique angle as in fig. 121.

In other minerals it is sometimes observed that three, four, or more crystals, intersect each other in this manner, and produce figures apparently remote in character from the primary forms to which they belong, and from which they could not be deduced by any ordinary law of decrement. This class of combined or intersecting crystals generally occurs in arragonite: such forms also frequently occur in carbonate of lead, in sulphuret of copper, and in other species of minerals.

SECTION X.

EPIGENE AND PSEUDOMORPHOUS CRYSTALS.

In addition to the variety of crystals already described, there are others whose forms are not natural to the substances in which they occur.

To one class of these the Abbé Haüy has applied the name of *Epigene*, where a *chemical alteration has taken place in the substance of the crystal subsequently to its formation.*

Thus crystals of blue carbonate, and of red oxide of copper, are frequently found converted into green carbonate. Sulphuret, and carbonate, of iron, are changed into oxides, without losing their peculiar crystalline forms; and the same alteration takes place in other substances.

Another class of crystals, not belonging to the substances in which they occur, have been denominated *Pseudomorphous.* These have been formed either *within cavities from which crystals of some other substance have been previously removed by some natural cause, probably by solution*, or *upon crystals* of some other substance which have *subsequently disappeared*; the space they occupied either remaining a void, or having been afterwards filled with some other matter.

Both these classes of crystals, particularly the first, may present some little difficulty to the young mineralogist, but this will be overcome by an improved acquaintance with the minerals in which they occur.

SECTION XI.

ON THE TABLES OF MODIFICATIONS.

THE preceding sections have explained the theory of crystals in reference to

The forms of their molecules.
The manner in which those molecules are aggregated in the production of crystals, and to
The nature of decrements, or the manner in which the secondary forms of crystals may be conceived to be produced.

The most important practical purpose of this theory is, to enable the mineralogist to determine the species to which any crystallized mineral belongs, from an examination of any of its crystalline forms.

Minerals which *differ in species*, may belong either to *different classes*, or to the *same class*, of the primary forms. *If several species belong to the same class*, they will be found, with the exception of such as crystallize in cubes, regular tetrahedrons, regular octahedrons, and rhombic dodecahedrons, *to differ from each other, sometimes in the angles at which their primary planes incline to each other, and sometimes in the comparative lengths of some of their primary edges.*

Thus the general class of square prisms may consist of any number of particular prisms, belonging to as many different species of minerals; and these individual or particular prisms may be conceived to differ from each other in the relation of their respective heights to the length of the edge of their square base. The class of rectangular prisms may be supposed to contain many particular prisms, which vary from each other in the relative dimensions of their planes.

The individuals of the class of rhombic prisms may vary from each other in their relative heights, and in the angle at which the lateral planes incline to each other. Those of the class of rhomboids will differ in the angles at which their planes incline respectively to each other; constituting a series of particular rhomboids, of which the most acute, and the most obtuse, will be the two extremes; and similar differences may be imagined to exist among the individuals belonging to such of the other classes of primary forms as admit of analagous variations.

But crystals rarely present themselves under their respective primary forms; they are usually modified by new planes, producing secondary crystals, from which the primary forms are to be inferred.

And although, as we have already seen, we may, to a certain extent, be guided by cleavage in our attempts to discover the primary forms of minerals, those forms cannot in general be determined without a reference to the secondary crystals.

Hence the relations between the various secondary forms of crystals, and their respective primary forms, constitute a highly important feature in the science of which we are treating.

The secondary forms of crystals have been explained to consist of modifications of the primary, occasioned by decrements on some of their edges or angles.

The character of the modifying planes, and their geometrical relations to the primary form, as connected with the theory of decrements, will be explained in the Appendix, together with a system of notation connected with the same theory, and capable of expressing the figure of any secondary form by means of certain symbols.

But there are many who form collections of minerals as an amusement, who are not in the habit of mathematical investigations, and who cannot avail themselves of the theory of decrements, for the purpose of tracing the relations between the secondary and primary forms of crystals; and I am not aware of the existence of any published work unconnected with that theory, which attempts to point out these relations so as to enable the mineralogist to trace in a secondary crystal the characters of the primary to which it belongs.*

I have attempted to supply this desideratum by the following tables of modifications of the primary forms, and by the explanations which follow them. And although these may not furnish the young enquirer with all the assistance he desires, but may leave him still to encounter some difficulty in his pursuit, he will certainly derive advantages from the opportunity the tables will afford him, of comparing all the classes of simple secondary forms, belonging to the several classes of primary ones, with each other, and with their respective primary forms; as he will thus obtain a general view of the entire series of simple secondary forms belonging to all the known classes of the primary.

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* A systematic method of *describing* crystals was taught by Werner; but that method has been found *inconvenient* even for the purpose of *description*, and it supplies no rules for deducing the primary form of a crystal from any of its secondary forms. The system of Mohs has not yet been sufficiently developed to the English reader, to enable him to judge fairly of its merits; but from what has been published here, it appears that the purpose I have attempted to effect, may also be effected by his system, although by a less direct course.

The consideration of infinite lines which he has introduced into his system, and his notation founded on this character, are parts of his theory which will probably render its public reception less general than it might have been from its merits in other respects.

In these tables, *not merely the observed modifications of crystals, but all the numerous modifications of which each class of primary form is susceptible, while influenced by the law of symmetry, are reduced into classes, and arranged in an orderly series*; and I have added some of the *observed* instances of departure from this law, in the production of peculiar and anomalous secondary forms.

From a general view of the tables, it will be seen that the first classes of modifications are those on the *solid angles* of the primary form; these being succeeded by the modifications upon the *edges*, beginning in all cases with the simplest change of form.

It has been already stated, that each of the *classes of primary forms, may comprise many individual forms belonging to as many different species of minerals*; which individual forms, with the exception of such as belong to the cube, the regular tetrahedron and octahedron, and rhombic dodecahedron, will be found to differ from each other in the measurement of some of their angles, or in the ratios of some of their edges.

Each of the *classes of modifications*, excepting those which produce *tangent planes*, and which consequently admit of no variation, *may also comprise a series of individual modifications*; which individual modifications will be found to differ from each other, in the angles at which the modifying planes incline on the adjacent primary planes.

Thus the series of modifying planes which would be comprehended under class *b*, in the table of modifications of the cube, may be conceived to be comprised within two natural limits; the one being the primary plane of the cube, and the other the plane *a*. For it is obvious that the inclination of *b* on P may approach nearly to 180°, but it can never reach that limit. And it may pass from such an obtuse angle, through an almost unlimited series of planes inclining less and less on P, until at length the inclination would be nearly the same as that of *a* on P; but it is apparent that it can never attain this limit, there being only the one plane *a*, which can incline to the primary planes at that angle.

The series of planes belonging to class *c* of the cube, are limited within the planes *a* and *e*. The angle at which the planes marked *c* incline to each other, may be conceived to increase, until it approaches very near to 180°; the three planes would then very nearly become one, and similar to *a*, but they can never reach that limit. And they may also be conceived to incline more and more upon the edge, until at length they would very nearly coincide with the plane *e*, but they could never exactly coincide with that plane.

This may be readily comprehended if we refer to the tables, and remark that each of the series of new solids which would result from the series of planes belonging to class *c*, would be contained within 24 *planes*, while only one single solid contained within 8 *planes* could be produced from class *a*, and only one other by class *b*, contained within 12 *planes*; neither of which therefore could fall within the series resulting from class *c*.

The series of individual modifications belonging to class *d* of the cube, may be conceived to lie within several limits, according to the direction in which the change of position of the modifying planes may be supposed to take place. The angle at which the two planes *d*, resting on P′, meet, may be supposed to increase until those planes would nearly approach to one plane, corresponding with class *b*; or the angle at which the two planes *d*, which meet at the edge of the cube, incline to each other, may approach nearly to 180°, when those two planes would nearly become one analogous to class *c*. Or if the two planes *d*, which meet at the edge of the cube, be conceived to incline more and more on that edge, they will at length approach nearly to the position of the planes of class *f*, but they can never coincide with those planes.

These remarks on the nature of the differences which may be conceived to exist among the *individual modifications* belonging to some of the classes contained in the following tables, might be easily extended to all. But it will not be difficult for the reader to apply them himself to all the other variable classes contained in the tables. The nature of the variation, however, in many of the classes, will be pointed out in the tables.

When the sets of new planes, resulting from the *individual modifications* belonging to any one of the classes, are so much extended as entirely to efface the primary planes, a *series* of entirely new solids will result. The planes of these new solids will *generally* possess one common character throughout the series; that is, their planes will *generally* be all triangular, or all quadrilateral, or all pentagonal, or be of some other polygonal form; but the planes belonging to each individual of the series, will differ from those belonging to every other individual, in their angles, or the relative lengths of some of their edges.

But although it is *generally* true that the planes of the entire series of secondary forms, resulting from any given class of modifications, are similar throughout the series, it is not invariably so. Among the series of secondary forms resulting from particular classes of modifications of the rhomboid, there are some contained within scalene triangular planes, and others, belonging to the same class, contained within isosceles triangular planes. These differences will generally be pointed out in the tables.

The *series* of *new solids*, or complete simple secondary forms, resulting from each of the classes of modifications, will be described in the tables under the appellation of *new figures*.

It is to be observed that very few of the *individual* modifications belonging to the several classes, have yet been found to exist among crystals. And it is doubtful whether all the classes even have been hitherto noticed.

The figures of the primary and secondary forms, given in the following tables, are not to be regarded as representations of crystalline forms of any particular minerals, but as exhibiting a type, or general character, of each of the classes of primary forms, and of the modifications belonging to each of those classes.

A position is chosen for the figure of each of the primary forms, which is supposed to be constant throughout the series of modifications belonging to each. The position of the greater number will be readily comprehended from the figures themselves, but there are a few which require a short explanation.

The figure of the *right rectangular prism* has its *greater edge horizontal*, and its *largest plane* is supposed to be a *terminal one*.*

---

* The horizontal lines here referred to, are those which are parallel to the upper or lower edges of the page.

The figure of the *octahedron with a rectangular base*, has the *greater edge of its base horizontal.*

The figures of the *right rhombic prism*, and of the *octahedron with a rhombic base*, have their *greater diagonals horizontal.*

The figure of the *right oblique-angled prism*, has its *greater diagonal horizontal*, and the *greater lateral plane of the prism standing opposite the right hand of the spectator.*

Thus where the *angles of the base of a primary form are right angles*, four of its *terminal edges*, as seen in the figures, are *horizontal.* But where the *angles of the base are not right angles*, the figure is drawn with its *greater diagonals horizontal, and all its terminal edges are oblique lines.*

I conceive that an advantage will attend the placing the figures of crystals of different minerals, belonging to the same class of primary forms, always in the same position. The crystals of different substances may be then more easily compared with each other, and their peculiar characters be more readily observed. Euclase, and sulphate of lime, are right oblique-angled prisms; but the figures of euclase which accompany the Abbé Haüy's description of that substance, are made to rest on one of the lateral planes of the primary form, as it is exhibited in p. 10, while his primary form of sulphate of lime is made to rest on its base as that figure does.

On each of the primary forms, it will be observed, that certain letters are placed, which are intended to designate the angles, edges, and planes, of crystals, and to denote their similarity or dissimilarity.

We are indebted to the Abbé Haüy for the adaptation of letters to these purposes; and in a future section the manner will be explained in which they are applicable in describing the secondary forms of crystals.

Some, or all, of the vowels A E I O, are used to designate the *solid angles*; some of the consonants, B C D F G H, to designate the *primary edges*; and P M T to designate the *primary planes* of crystals.

The same letter is repeated where the angles, edges, or planes are similar; and different letters are used where those angles, edges, or planes are dissimilar, according to the definitions of similar angles, &c. given in p. 3.

Thus the letter A is repeated on all the angles of the cube, these being all similar; while A and E are placed on the alternate angles of the right rhombic prism, to shew that there, the opposite angles only are similar.

So in the *cube*, the letter P is repeated on all the planes because they are all similar.

In *the right rhombic prism*, the letter P stands only on the terminal plane, the lateral planes having the letter M placed upon them. This implies that the lateral planes are not similar to the terminal plane; but the letter M being repeated on *both* the lateral planes, denotes that *these are similar to each other.*

In the *right oblique-angled prism*, the lateral planes are distinguished from each other by the letters M, and T, implying *that they are dissimilar to each other*, as both are to the terminal plane which is designated by P.

The ′ and ″ added to some of these letters, serve merely to distinguish two or more *similar planes* from each other.

Thus, by carefully observing the position of the edges of the base of any figure, as it is explained in p. 101 and 102, and the letters used to designate the planes, as explained above, we may immediately discover the class of primary form to which the figure belongs.

The front planes exhibited in the drawing of any crystal are generally one half the number belonging to it; hence it is obvious that if these be described, the parallel planes of crystals being always similar to each other, the planes which are parallel to those shewn in the drawing, may be conceived to be described also.

It may be remarked that the modifying planes of the series of forms contained in the tables, are produced by cutting off portions of the figure of the primary crystal and thus reducing its bulk. It is almost unnecessary, after what has been already said on the formation of crystals, to observe, that nature proceeds by building up the secondary forms instead of thus truncating the primary. And we may, if we please, imagine, that the secondary figures given in these tables, have been produced by additions to primary crystals of smaller relative dimensions than those placed at the head of the several tables.

The *inclination* of any two planes to each other, as *a* and P, modification *a* of the cube, or *the angle at which they meet*, is commonly expressed in this manner, *a on* P *so many degrees and minutes.*

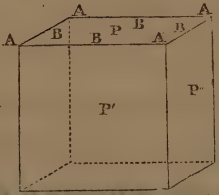
TABLES OF PRIMARY FORMS

AND

THEIR MODIFICATIONS.

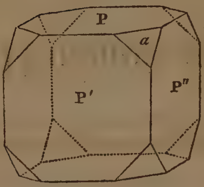
THE CUBE, AND

Primary form.

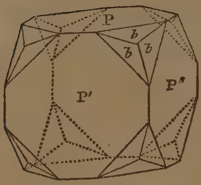


Modifications.

Class *a*.



Class *b*.



ITS MODIFICATIONS.

Primary form. The cube.

Modifications.

Class *a*. Solid angles replaced by *tangent* planes.

The modifying planes are equilateral triangles.

When the modifying planes are so extended as to efface the primary planes, a regular octahedron will be produced.

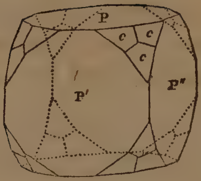
Plane *a* on P, P′ or P″, 120° 15′ 52″.

Class *b*. Solid angles replaced by three planes resting on the primary planes.

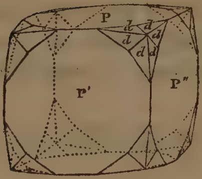
Each of the series of new figures produced by this class, would be contained within 24 equal trapezoidal planes. But the trapezoidal planes belonging to each figure of the series would differ in their angles from those belonging to every other figure of the same series.

Obs. A *trapezoid* is a four-sided figure whose *opposite edges are unequal*, and in which, if lines be drawn through the angles *a*, and *b*, *c*, and *d*, they

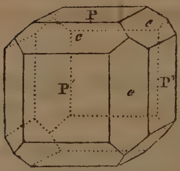
Class *c*.



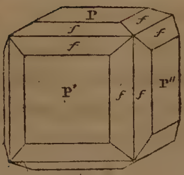
Class *d*.



Class *e*.

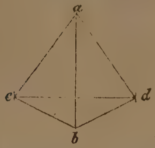


Class *f*.



will intersect each other at right angles, as in the following figure.

Fig. 133.



Class *c*. Solid angles replaced by three planes resting on the edges of the cube.

The series of new figures resulting from this class, would be contained within 24 isosceles triangular planes.

Class *d*. Solid angles replaced by six planes.

The new figures would be contained within 48 triangular planes.

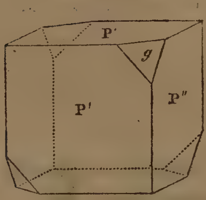
Class *e*. Edges replaced by *tangent* planes.

The new figure would be the rhombic dodecahedron. Plane *e* on P or P′, 135°.

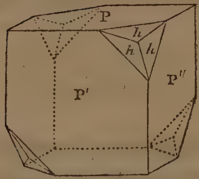
Class *f*. Edges replaced by two planes.

This class would produce a series of four-sided pyramids on the planes of the cube. The planes of the rhombic dodecahedron, and those of the cube, will evidently be the limits of this series.

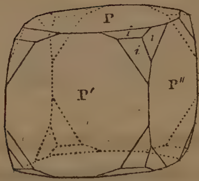
Class *g*.



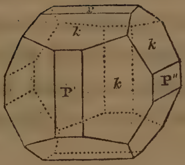
Class *h*.



Class *i*.



Class *k*.



The following modifications do not accord with the law of symmetry as defined in p. 77.

Class *g*. *Alternate* solid angles replaced by tangent planes.

A regular tetrahedron would result from this modification.

Class *h*. *Alternate* solid angles replaced by three planes resting on the primary planes.

The series of new figures resulting from this class, would be similar to those produced by class *c* of the tetrahedron.

Class *i*. Solid angles replaced by three planes, *each of which inclines unequally on the three adjacent primary planes*.

The *unequal* inclination of each of the modifying planes on the *three adjacent primary planes*, distinguishes this class from class *b*, each plane of which inclines *equally* on *two* of the adjacent planes, and *unequally* on the *third* plane.

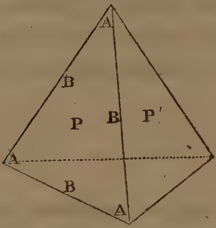
The character of this class is similar to that of class *d*, if we suppose the *alternate three* only of the modifying planes to be produced on each solid angle.

Class *k*. Edges replaced by single planes inclining at unequal angles on the adjacent primary planes, and bearing the same analogy to class *f*, that class *i* does to class *d*.

A series of dodecahedrons with pentagonal planes would result from this class of modifications, but

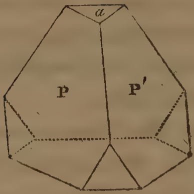
THE REGULAR TETRAHEDRON, AND

Primary form.

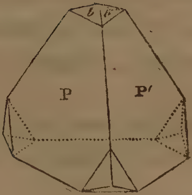


Modifications.

Class *a*.



Class *b*.



there is only one of the series known to exist among crystals.

It appears that in all these irregular modifications, only one half the number of modifying planes is produced, which would be required by the law of symmetry.

ITS MODIFICATIONS.

Primary form. The regular tetrahedron.
Plane P on P′, 70° 31′ 43″.

Modifications.

Class *a*. Solid angles replaced by tangent planes.

When the modifying planes *first touch each other on the edges of the tetrahedron*, a regular octahedron is produced.

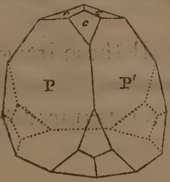
When the primary planes are entirely effaced, the new figure is a tetrahedron similar to the primary.

Plane *a* on P or P′, 109° 28′ 16″.

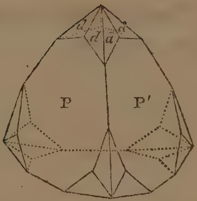
Class *b*. Solid angles replaced by three planes resting on the primary planes.

The new figures would be dodecahedrons, generally with trapezoidal planes; but one individual modification belonging to this class produces the rhombic dodecahedron.

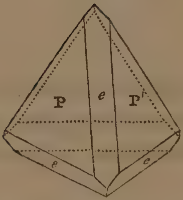
Class *c.*



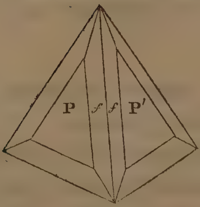
Class *d.*



Class *e.*



Class *f.*



Class *c*. Solid angles replaced by three planes resting on the primary edges.

The new figures would be dodecahedrons with triangular planes.

The planes resulting from classes *b*, and *c*, would produce a great variety of dodecahedral solids; not any of which, except the rhombic dodecahedron, can be derived from the octahedron according to the law of symmetry.

Class *d*. Solid angles replaced by six planes.

The new figures would be contained within 24 triangular planes.

Class *e*. Edges replaced by tangent planes.

Produces the cube.

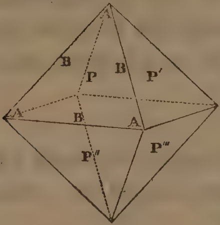
Plane *e* on P or P′, 120° 15′ 52″.

Class *f*. Edges replaced by two planes.

The new figures would be dodecahedrons with triangular planes, appearing as three-sided pyramids on the planes of the tetrahedron.

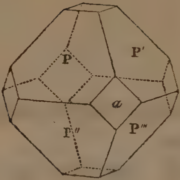
THE REGULAR OCTAHEDRON, AND

Primary form.

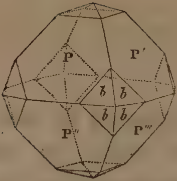


Modifications.

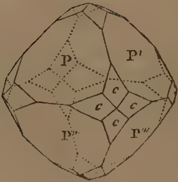
Class *a*.



Class *b*.



Class *c*.



ITS MODIFICATIONS.

Primary form. The regular octahedron.
Plane P on P′, or P″, 109° 28′ 16″.

Modifications.

Class *a*. Solid angles replaced by tangent planes.

The new figure resulting from this modification is the cube.

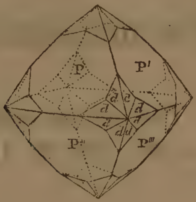
Class *b*. Solid angles replaced by four planes resting on the primary planes.

The new figures would be contained within 24 trapezoidal planes.

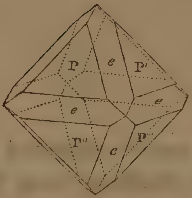
Class *c*. Solid angles replaced by four planes resting on the primary edges.

The new figures would be contained within 24 isosceles triangular planes.

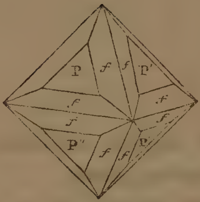
Class *d*.



Class *e*.



Class *f*.



Class *d*. Solid angles replaced by eight planes.

The new figures would be contained within 48 triangular planes.

Class *e*. Edges replaced by tangent planes.

The new figure would be the rhombic dodecahedron.

Plane *e* on P or P′, 144° 44′ 8″.

Class *f*. Edges replaced by two planes.

The new figures would present a trihedral pyramid on each primary plane.

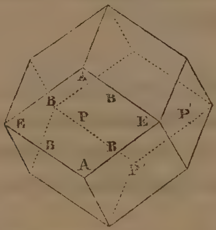
It has been already stated that the cleavages of the octahedron and the tetrahedron are perfectly similar, and that it is only by means of the secondary planes of each, that we can discriminate these primary forms from each other.

The octahedron may be distinguished as a primary form from the tetrahedron, according to the rules laid down in a former section, by such of its *single* modifications as could be produced on the tetrahedron only by the *simultaneous operation of two or more separate modifications*.

But it is obvious that *all* the modifications of the octahedron, except mod. *a* and *d*, would require a *double decrement* for their production, supposing the tetrahedron were the primary form.

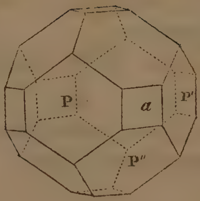
THE RHOMBIC DODECAHEDRON, AND

Primary form.



Modifications.

Class *a*.



This may be easily conceived, if we recollect that the octahedron itself is a figure resulting from the modification *a* of the tetrahedron; and that the edges of the modifying plane *a*, of the tetrahedron, are not affected by modification *e*, or *f*, which replace the primary edges of the tetrahedron; and that the primary edges of the tetrahedron are not affected by the modifications *b*, and *c*, which would replace the edges of the modifying plane *a*. *Three* classes of modification must therefore concur upon the tetrahedron, to produce most of the secondary forms of the octahedron.

ITS MODIFICATIONS.

Primary form. The rhombic dodecahedron.
Plane P on P′, 90°.
P on P″, 120°.

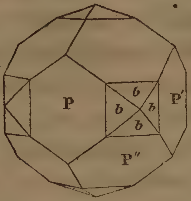
For the sake of avoiding a frequent repetition of the description of the two kinds of solid angles of this figure, those contained within *four acute plane angles* will be called the *acute solid angles*, and those contained within *three obtuse plane angles*, will be called the *obtuse solid angles*.

Modifications.

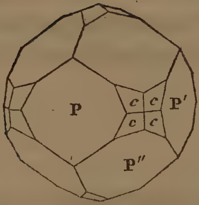
Class *a*. Acute solid angles replaced by tangent planes.

The new figure would be the cube.
Plane *a* on P, 135°.

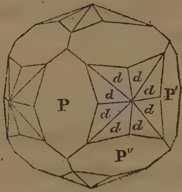
Class *b.*



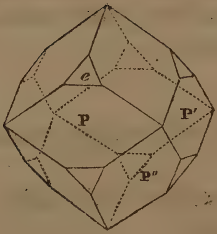
Class *c.*



Class *d.*



Class *e.*



Class *b*. Acute solid angles replaced by four planes resting on the primary planes.

The new figures would be contained within 24 isosceles triangular planes.

Class *c*. Acute solid angles replaced by four planes resting on the primary edges.

The new figures would be contained within 24 trapezoidal planes.

Class *d*. Acute solid angles replaced by eight planes.

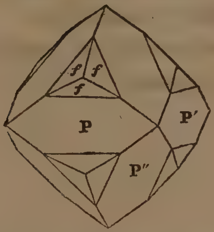
The new figures would be contained within 48 triangular planes.

Class *e*. Obtuse solid angles replaced by tangent planes.

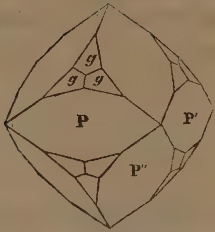
The new figure would be the regular octahedron.

Plane *e* on P 144° 44′ 8″.

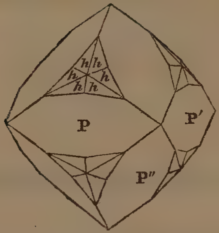
Class *f*.



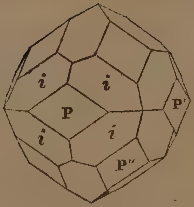
Class *g*.



Class *h*.



Class *i*.



Class *f*. Obtuse solid angles replaced by three planes resting on the primary planes.

The new figures would be contained within 24 isosceles triangular planes.

Class *g*. Obtuse solid angles replaced by three planes resting on the primary edges.

The new figures would be contained within 24 trapezoidal planes.

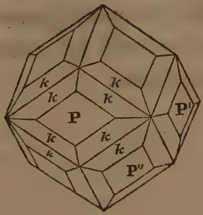
Class *h*. Obtuse solid angles replaced by six planes.

The new figures would be contained within 48 triangular planes.

Class *i*. Edges replaced by tangent planes.

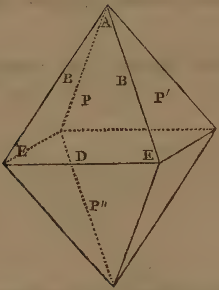
The new figures would be contained within 24 trapezoidal planes.

Class *k*.

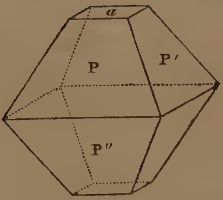


THE OCTAHEDRON WITH A SQUARE

Primary form.



Modifications.
Class *a*.



Class *k*. Edges replaced by two planes.

The new figures would be contained within 48 triangular planes.

It may be remarked that the secondary forms belonging to the four preceding classes of primary forms, are nearly similar to each other. And, with the exception of the tetrahedron, each primary form is found to be also a secondary form to each of the other primary.

The table of secondary forms, which will be found at the end of the tables of modifications, exhibits the relation to each other of each of the preceding classes of secondary forms.

BASE, AND ITS MODIFICATIONS.

Primary form. The octahedron with a square base.

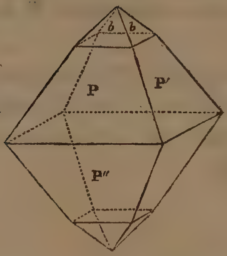
The individuals belonging to this class will differ from each other in the inclination of P on P″, and consequently of P on P′.

Modifications.

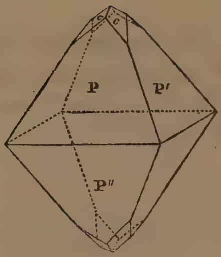
Class *a*. Terminal solid angles replaced by tangent planes.

As this modification contains only *two* parallel planes, those planes can never efface the primary planes and produce a solid. The same observation

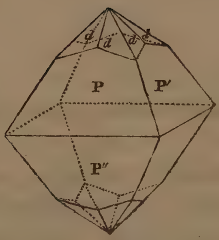
Class *b*.



Class *c*.



Class *d*.



will apply to all those classes which produce *only two, or four, parallel planes* upon the primary form. In these cases, as no entire secondary form can result from the *secondary planes alone*, no new figure is described.

Class *b*. Terminal solid angles replaced by four planes resting on the primary planes.

The new figures would be more obtuse octahedrons.

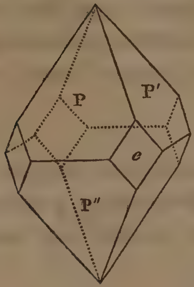
Class *c*. Terminal solid angles replaced by four planes resting on the primary edges.

Another series of octahedrons more obtuse than the primary would result from this class.

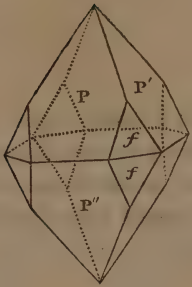
Class *d*. Terminal solid angles replaced by eight planes.

The new figures would be double eight-sided pyramids united at a common base, and measuring unequally over the two pyramidal edges of each of the planes. The surfaces of the new planes would generally be scalene triangles.

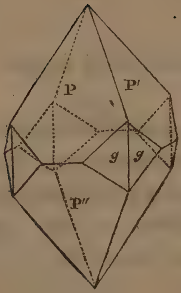
Class *e*.



Class *f*.



Class *g*.



Class *e*. Solid angles of the base replaced by tangent planes.

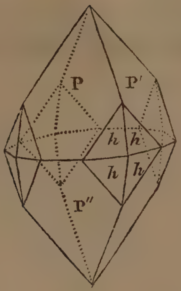
This modification would produce only four sides of a prism.

Class *f*. Solid angles of the base replaced by two planes resting on the edges of the summits.

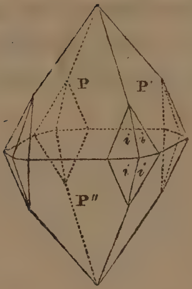
This modification would produce a series of octahedrons more acute than the primary.

Class *g*. Solid angles of the base replaced by two planes resting on the edges of the base.

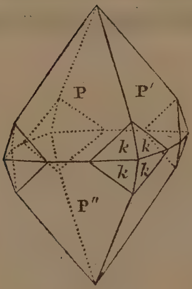
Class *h*.



Class *i*.



Class *k*.



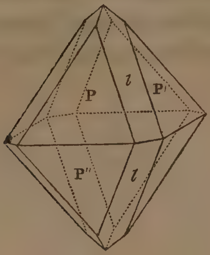
Class *h.* Solid angles of the base replaced by four planes resting on the primary planes, and having their edges parallel to the edges of the pyramids.

Class *i.* Solid angles of the base replaced by four planes inclining more on the terminal edges than modification *h.*

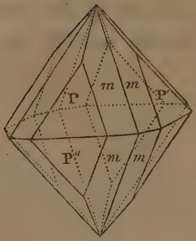
Class *k.* Solid angles of the base replaced by four planes inclining more on the edges of the base than modification *h.*

A series of double eight-sided pyramids might result from class *h*, *i*, and *k*, analogous to those resulting from class *d*, but more acute.

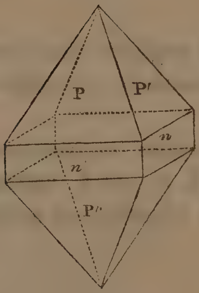
Class *l*.



Class *m*.



Class *n*.



Class *l*. Edges of the pyramids replaced by tangent planes.

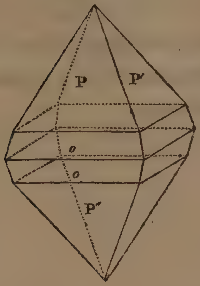
The new figure resulting from this modification would be an octahedron with a square base, but more obtuse than the primary.

Class *m*. Edges of the pyramids replaced by two planes.

Class *m* would produce eight-sided pyramids similar in character to those resulting from class *d*.

Class *n*. Edges of the base replaced by tangent planes.

Class *o*.



Class *o*. Edges of the base replaced by two planes. The new figures would be octahedrons, more acute than the primary.

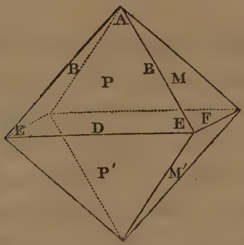
The character, and number, of the modifications of this, and some other of the following classes of primary forms, arise from the dissimilarity between the edges and angles of the summits, and those of the base, in the octahedrons; and between the angles of the terminal planes, or the terminal and the lateral edges of the prisms, and sometimes between the lateral edges themselves.

The primary crystals belonging to this class of primary forms, are distinguishable from *regular* octahedrons by the unequal inclinations of the plane P on P′, and P on P″; and the secondary forms may be distinguished by the modifications taking place on some only of the edges or angles, and not on all, as they do on the regular octahedron. If two of its edges measure over the summit more than 90°, the octahedron of this class is called obtuse; if less than 90°, it is called acute. In the regular octahedron the edges measure exactly 90° over the summit.

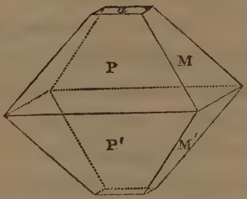
The secondary octahedrons belonging to this class, have, like the primary, square bases.

THE OCTAHEDRON WITH A RECTANGULAR

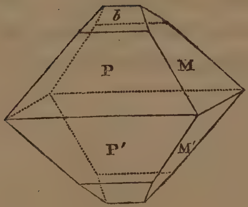
Primary form.



Modifications.
Class *a*.



Class *b*.



BASE, AND ITS MODIFICATIONS.

Primary form. An octahedron with a rectangular base.

In this figure the broad planes P P′, meet at the edge of the base at a more obtuse angle than the narrow ones M M′. The edge D may therefore be denominated the *obtuse edge* of the base, and the edge F the *acute edge*; or they may be termed the *greater* and *lesser* edges of the base.

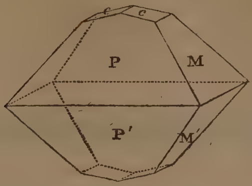
The individuals belonging to this class of primary forms will differ from each other in the inclination of P on P′, or of M on M′.

Modifications.

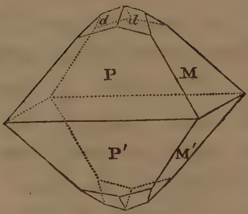
Class *a*. Terminal solid angles replaced by single planes.

Class *b*. Terminal solid angles replaced by two planes, resting on the broad planes.

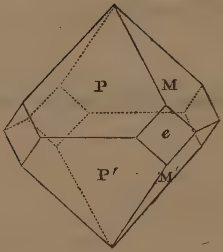
Class *c*.



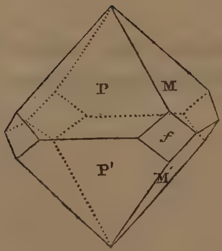
Class *d*.



Class *e*.



Class *f*.



Class *c*. Terminal solid angles replaced by two planes, resting on the narrow planes.

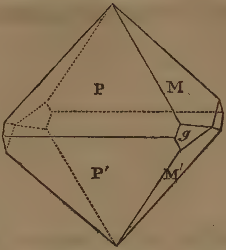
Class *d*. Terminal solid angles replaced by four oblique planes.

The new figures would be octahedrons with rhombic bases.

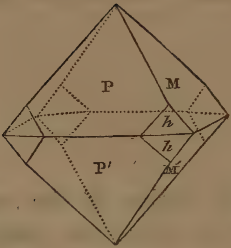
Class *e*. Solid angles of the base replaced by single planes, whose edges are parallel to the edges of the pyramids.

Class *f*. Solid angles of the base replaced by single planes, inclining on the greater edges of the base more than those of modification *e*.

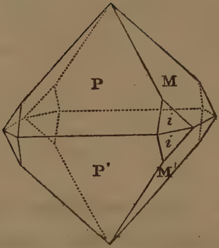
Class *g*.



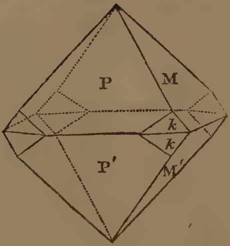
Class *h*.



Class *i*.



Class *k*.



Class *g*. Solid angles of the base replaced by single planes inclining more on the lesser edges of the base than those of modification *e*.

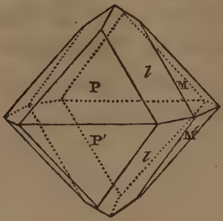
Classes *e f* and *g*, would give the lateral planes of a series of right rhombic prisms.

Class *h*. Solid angles of the base replaced by two planes, one edge of each new plane being parallel to a pyramidal edge of the broad primary planes.

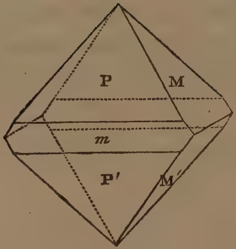
Class *i*. Solid angles of the base replaced by two planes, one edge of each new plane being parallel to a pyramidal edge of the narrow primary planes.

Class *k*. Solid angles of the base replaced by two planes, neither of whose edges are parallel to any edge of the primary form.

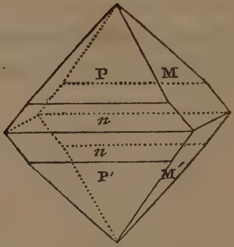
Class *l*.



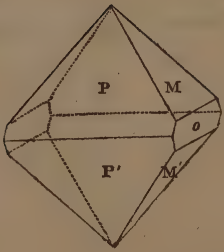
Class *m*.



Class *n*.



Class *o*.



Class *l*. Edges of the pyramids replaced by single planes.

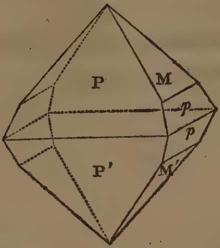
A series of octahedrons with rhombic bases would result from Classes *h*, *i*, *k*, and *l*.

Class *m*. Greater edges of the base replaced by single planes.

Class *n*. Greater edges of the base replaced by two planes.

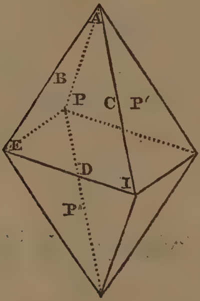
Class *o*. Lesser edges of the base replaced by single planes.

Class *p*.



THE OCTAHEDRON WITH A RHOMBIC

Primary form.



Class *p*. Lesser edges of the base replaced by two planes.

This species of octahedron has been adopted here as a primary form, in conformity with the opinions entertained by the Abbé Haüy of its belonging to certain species of minerals. But it is probable that the right rhombic prism is really the primary form, of most, if not of all, those species, and that there is not any mineral whose crystals are strictly referable to this class of octahedrons.

BASE, AND ITS MODIFICATIONS.

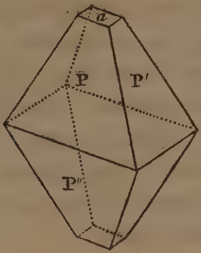
Primary form. An octahedron with a rhombic base.

It has been already observed that this figure is drawn with the *greater diagonal of the base horizontal.* Hence the primary planes meet at the edge B, at a more acute angle than at the edge C. The edge B will therefore be denominated an *acute edge* of the pyramid, and the edge C, an *obtuse edge* of the pyramid. The solid angle at E will be termed the *acute lateral solid angle*, and that at I the *obtuse lateral solid angle.*

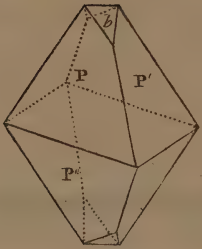
The individuals belonging to this class will differ from each other in the inclinations of P on P′ and on P″.

Modifications.

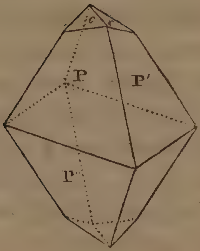
Class *a*.



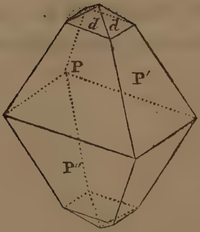
Class *b*.



Class *c*.



Class *d*.



Modifications.

Class *a*. Terminal solid angles replaced by tangent planes.

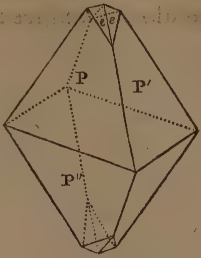
Class *b*. Terminal solid angles replaced by two planes, resting on the obtuse edges of the pyramids.

Class *c*. Terminal solid angles replaced by two planes, resting on the acute edges of the pyramids.

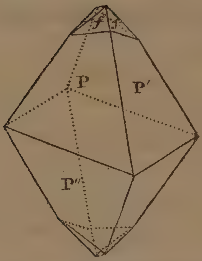
Class *d*. Terminal solid angles replaced by four planes, resting on the primary planes.

The edges of the planes *d*, which intersect the primary planes, are parallel to the edges of the base.

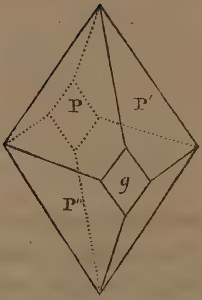
Class *e*.



Class *f*.



Class *g*.



Class *e*. Terminal solid angles replaced by four oblique planes, inclining on the obtuse edges of the pyramids.

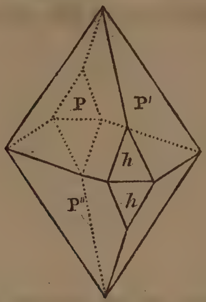
Class *f*. Terminal solid angles replaced by four oblique planes, inclining on the acute edges of the pyramids.

Modifications *e*, and *f*, may be distinguished from modification *d*, by the edges of the modifying planes which intersect the primary planes, not being parallel to the edges of the base.

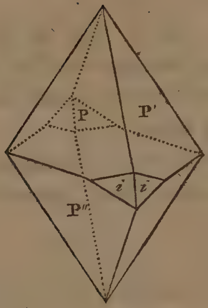
Classes *d*, *e*, and *f*, would produce a series of octahedrons with rhombic bases, more obtuse than the primary.

Class *g*. Obtuse lateral solid angles replaced by tangent planes.

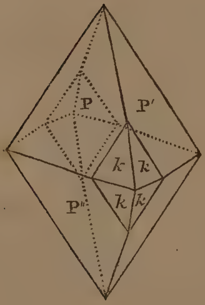
Class *h*.



Class *i*.



Class *k*.



Class *h*. Obtuse lateral solid angles replaced by two planes, resting on the edges of the pyramids.

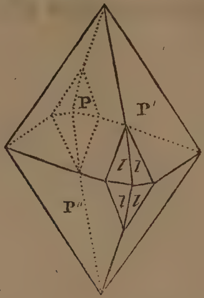
Class *i*. Obtuse lateral solid angles replaced by two planes, resting on the edges of the base.

Planes *i* might be the lateral planes of a right rhombic prism.

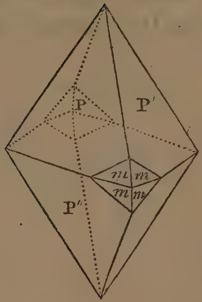
Class *k*. Obtuse lateral solid angles replaced by four planes, resting on the primary planes.

The edges produced by the intersection of the planes of this modification with the primary planes, are parallel to the primary edges of the pyramids.

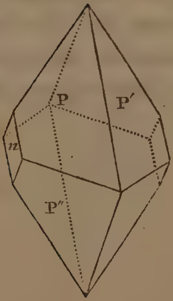
Class *l*.



Class *m*.



Class *n*.



Class *l*. Obtuse lateral solid angles replaced by four oblique planes, inclining on the obtuse edges of the pyramids.

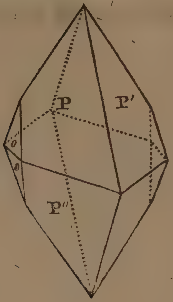
Class *m*. Obtuse lateral solid angles replaced by four oblique planes inclining on the edges of the base.

The edges of planes *l*, and *m*, intersect the primary planes in lines which are not parallel to the primary edges of the pyramids.

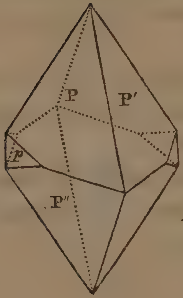
The new figures resulting from classes *k*, *l* and *m*, would be octahedrons, more acute than the primary, with rhombic bases.

Class *n*. Acute lateral solid angles replaced by tangent planes.

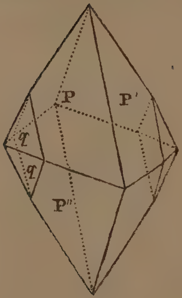
Class *o*.



Class *p*.



Class *q*.



Class *o*. Acute lateral solid angles replaced by two planes, resting on the edges of the pyramids.

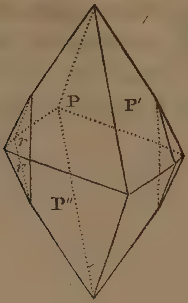
If we suppose the octahedron to be placed with its axis horizontally, the planes of classes *b*, *c*, *h*, or *o*, might be the lateral planes of right rhombic prisms.

Class *p*. Acute lateral solid angles replaced by two planes, resting on the edges of the base.

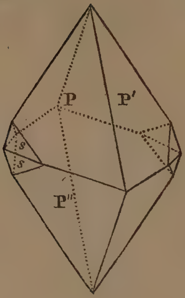
Planes *p* might be the lateral planes of a right rhombic prism.

Class *q*. Acute lateral solid angles replaced by four planes, resting on the primary planes, and intersecting those planes in lines parallel to the edges of the pyramids.

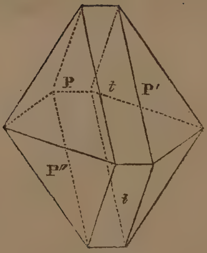
Class *r*.



Class *s*.



Class *t*.



Class *r*. Acute lateral solid angles replaced by four oblique planes, inclining on the edges of the pyramids.

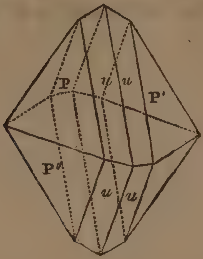
Class *s*. Acute lateral solid angles replaced by four oblique planes, inclining on the edges of the base.

The planes *r*, and *s*, intersect the primary planes in lines which are not parallel to the edges of the pyramids.

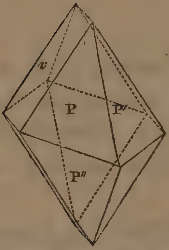
The new figures resulting from class *q*, *r*, and *s*, would be octahedrons, more acute than the primary, with rhombic bases.

Class *t*. Obtuse edges of the pyramids replaced by tangent planes.

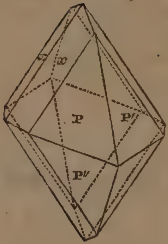
Class *u*.



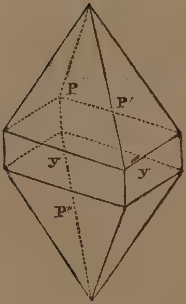
Class *v*.



Class *x*.



Class *y*.



Class *u*. Obtuse edges of the pyramid replaced by two planes.

Class *v*. Acute edges of the pyramid replaced by tangent planes.

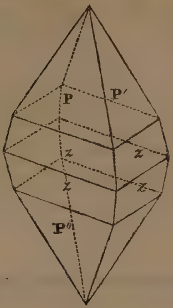
Class *x*. Acute edges of the pyramid replaced by two planes.

The new figures produced by class *u*, and *x*, would be octahedrons, more obtuse than the primary, with rhombic bases.

Class *y*. Edges of the base replaced by tangent planes.

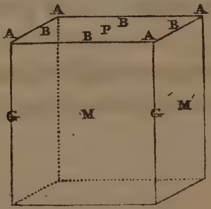
Planes *y* might be the lateral planes of a right rhombic prism.

Class *z*.



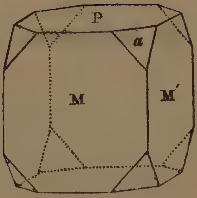
THE RIGHT SQUARE PRISM, AND

Primary form.



Modifications.

Class *a*.



Class z. Edges of the base replaced by two planes.

The new figures would be more acute octahedrons, with rhombic bases.

The unequal angles at which the primary planes incline to each other at the edges B and C, sufficiently distinguish this class of octahedrons from the preceding classes.

ITS MODIFICATIONS.

Primary form. A right square prism.

The individuals of this class will differ from each other in the comparative length of the edges G and B.

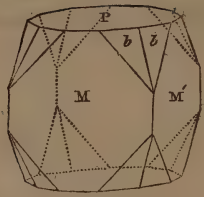
Modifications.

Class *a*. Solid angles replaced by single planes whose edges are *generally isosceles triangles*.

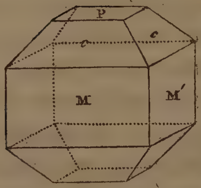
But they are not necessarily always so.

For if the decrements in height be to those in breadth in exactly the same ratio that the terminal edge of the prism bears to its height, the truncating planes on the angles will be equilateral triangles, and those on the edges of the summit would, under similar circumstances, be tangent planes.

Class *b*.



Class *c*.



It is however extremely improbable that this precise relation between the dimensions of the prism and the law of decrement should ever exist. The character of the modifying planes, as given above, may therefore be considered to exist in all the prisms belonging to this class.

Modification *a* would produce a series of four-sided pyramids on each summit, resting on the lateral edges of the prism. And if the modifying planes were so enlarged as to efface the primary planes, a series of octahedrons with square bases would result.

The planes produced by this modification incline equally on M and M′, but at a different angle from that at which they incline on P. This character of the plane *a*, will distinguish it from plane *a* of the cube; and its equal inclination on M and M′, will distinguish it from plane *a* of the right rectangular prism.

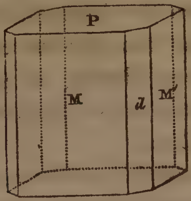
Class *b*. Solid angles replaced by two planes.

The new figures produced by this class would be eight-sided pyramids, similar to those produced by *d*, *h*, *i*, *k* and *m* of the octahedron with a square base.

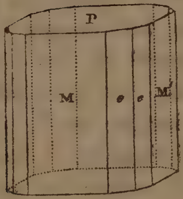
Class *c*. Edges of summit replaced by single planes not forming equal angles generally with the adjacent terminal and lateral planes.

Produces a series of four-sided pyramids resting on the lateral planes of the prism; and by an extension of the modifying planes, a series of octahedrons with square bases would result.

Class *d.*

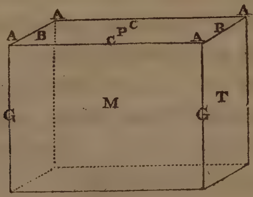


Class *e.*



THE RIGHT RECTANGULAR PRISM, AND

Primary form.



Class *d*. Edges of prism replaced by tangent planes.

The *equal* inclination of the planes *d*, upon the adjacent primary lateral planes, distinguishes these secondary forms from those of the right rectangular prism.

Class *e*. Edges of prism replaced by two planes.

The modifications of the terminal edges alone, or the lateral edges alone, will tend to distinguish the secondary crystals belonging to this class of prisms, from those derived from the cube.

A close resemblance may be remarked between the primary and secondary forms of this class of prisms, and those of the octahedron with a square base; and it is only from the cleavage that we are enabled to decide which of these forms is to be regarded as the primary, in reference to such secondary forms as are common to both.

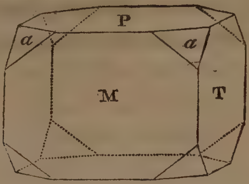
ITS MODIFICATIONS.

Primary form. A right rectangular prism.

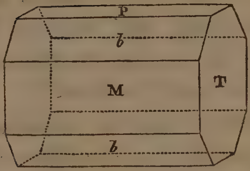
The individuals belonging to this class will differ from each other in the comparative length of the three adjacent edges C, B, and G.

Modifications.

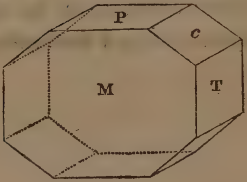
Class *a*.



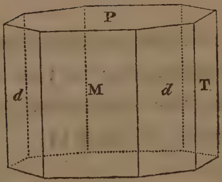
Class *b*.



Class *c*.



Class *d*.



Modifications.

Class *a*. Solid angles replaced by single *scalene* triangular planes, which incline on the three adjacent primary planes at unequal angles.

The new figures which would be ultimately produced by this class of modifications, would be a series of octahedrons with rhombic bases.

Class *b*. Greater terminal edges replaced by single planes.

Class *c*. Lesser terminal edges replaced by single planes.

The new figures which would be produced by a combination of modifications *b* and *c*, would be a series of octahedrons with rectangular bases.

Class *d*. Lateral edges of the prism replaced by single planes.

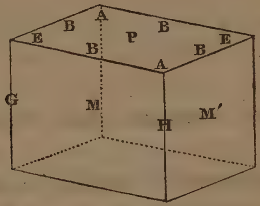
A series of right rhombic prisms would result from this class of modifications.

If more planes than one be found modifying any one of the edges or angles of this form, they are supposed to result from as many individual modifications, as there are planes upon the particular edge or angle on which they occur.

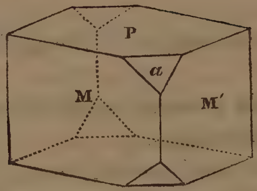
The planes produced by modifications class *b*, *c*,

THE RIGHT RHOMBIC PRISM, AND

Primary form.



Modifications.
Class *a*.



and *d*, will generally incline on the adjacent primary planes at unequal angles.

The occurrence of modification *b*, *alone*, or modification *c*, *alone*, or the *unequal inclination* of plane *d*, or M, and T, will tend to distinguish the crystals belonging to this class of prisms from those of the square prism or cube.

ITS MODIFICATIONS.

Primary form. A right rhombic prism.

The *solid angles at A* will be termed the *obtuse*, and those *at E* the *acute solid angles*, of this class of prisms. The *edge G* will be called the *acute*, and the *edge H* the *obtuse*, *lateral edges* of the prisms.

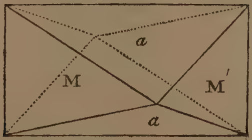
The individuals belonging to this class will differ from each other in the inclination of M on M′, or in the ratio of the edge H to the edge B.

Modifications.

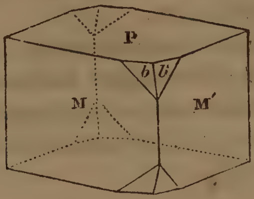
Class *a*. Obtuse solid angles replaced by single planes which intersect the terminal plane parallel to its greater diagonal.

When the planes *a*, increase so much as to meet above the terminal plane, the resulting figure may be an octahedron with a rectangular base, as shewn in fig. 228.

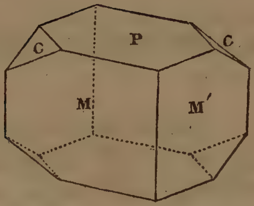
Fig. 228.



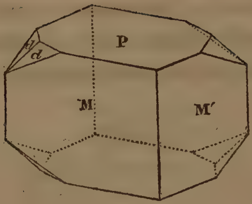
Class *b*.



Class *c*.



Class *d*.



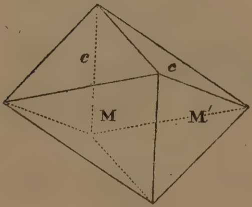
Class *b*. Obtuse solid angles replaced by two planes.

This modification would produce a series of four-sided pyramids, replacing the terminal plane, and the new figures would be octahedrons with rhombic bases.

Class *c*. Acute solid angles replaced by single planes, which intersect the terminal plane parallel to its short diagonal.

An octahedron with a rectangular base, as shewn in fig. 231, may result from this class of modifications also, but reversed in its position when compared with that produced by modification *a*.

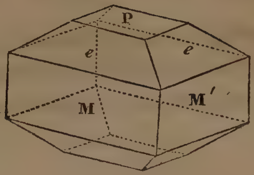
Fig. 231.



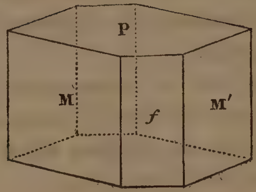
Class *d*. Acute solid angles replaced by two planes.

This class produces octahedrons with rhombic bases, differing from those which might result from class *b*. only in the relative inclination of the secondary planes to each other.

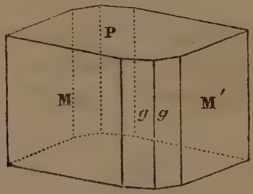
Class *e*.



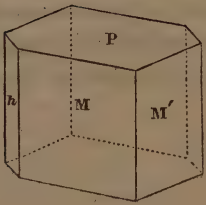
Class *f*.



Class *g*.



Class *h*.



Class *e*. Terminal edges replaced by single planes.

This modification would produce a series of four-sided pyramids replacing the terminal planes, and the new figures would be octahedrons with rhombic bases.

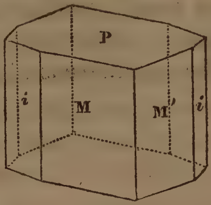
Class *f*. Obtuse lateral edges replaced by tangent planes.

Class *g*. Obtuse lateral edges replaced by two planes.

When the primary lateral planes are effaced by the planes *g*, a series of secondary right rhombic prisms would be produced.

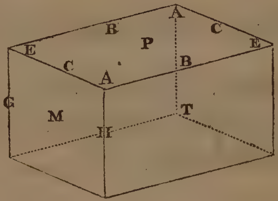
Class *h*. Acute lateral edges replaced by tangent planes.

Class *i*.



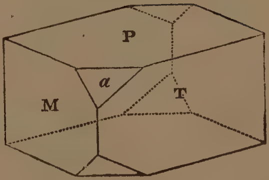
THE RIGHT OBLIQUE-ANGLED PRISM,

Primary form.



Modifications.

Class *a*.



Class *i*. Acute lateral edges replaced by two planes.

This modification would produce another series of right rhombic prisms.

A general analogy may be observed to prevail between these secondary forms, and those of the octahedron and a rhombic base, and it is only from cleavage that we are enabled to refer the secondary forms to either of these primary ones.

AND ITS MODIFICATIONS.

Primary form. A right oblique-angled prism.

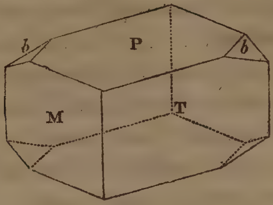
The angles and edges of this class of prisms may be designated as those of the right rhombic prism have been, calling the solid angles at A the obtuse, those at E the acute solid angles; the lateral edges at H the obtuse, those at G the acute lateral edges; the edges B may be called the greater, and C the lesser terminal edges.

The individuals belonging to this class of prisms will differ from each other in the inclination of M on T, and in the relative lengths of the edges C, B, and H.

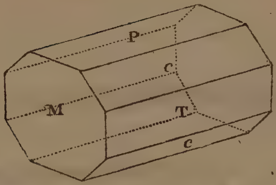
Modifications.

Class *a*. Obtuse solid angles replaced by single planes.

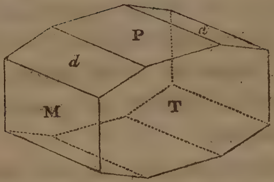
Class *b.*



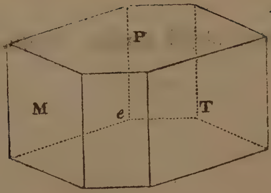
Class *c.*



Class *d.*



Class *e.*



Class *b*. Acute solid angles replaced by single planes.

The planes of both these modifications are scalene triangles.

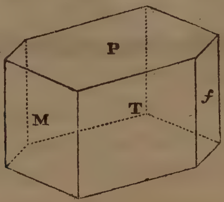
Class *c*. Greater terminal edges replaced by single planes.

Class *d*. Lesser terminal edges replaced by single planes.

The relative dimensions of the terminal edges can be ascertained only from some modification which is supposed to indicate the direction of one of the diagonals of the terminal plane; from this the angle which the diagonal makes with an edge of the same plane may be deduced, and thence the ratio of the terminal edges may be known.

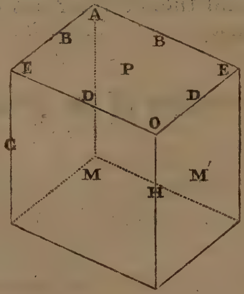
Class *e*. Obtuse edges of the prism replaced by single planes.

Class *f*.



THE OBLIQUE RHOMBIC PRISM, AND

Primary form.



Class *f*. Acute edges of the prism replaced by single planes.

Modifications *e*, and *f*, incline unequally on the adjacent lateral planes.

The characters which distinguish this class of prisms from right rhombic prisms, are, first, the unequal inclinations of any secondary lateral plane on the two adjacent lateral primary planes; and secondly, the occurrence of similar modifying planes upon *two only* of the terminal edges.

There is a remarkable general character of obliquity in the planes resulting from modifications *c*, and *d*, which tends to distinguish the secondary forms belonging to this class of primary forms from those belonging to the right rectangular prism.

Epidote affords a good illustration of this character.

ITS MODIFICATIONS.

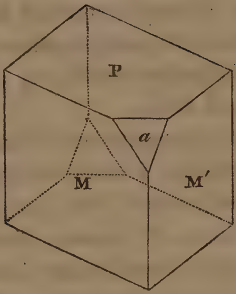
Primary form. An oblique rhombic prism.

The figure is supposed to be oblique in the direction O A, so that the terminal plane forms an obtuse angle with the edge H. The planes M M′, may meet at an acute, or an obtuse angle. For the convenience of description, the *solid angle* at A, will, in either case, be called the *acute solid angle*; that at O, the *obtuse solid angle*; and those at F, the *lateral solid angles*.

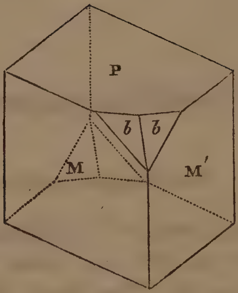
The edges B will be called the *acute terminal* edges, and those at D the *obtuse terminal* edges. The edge H, and its opposite, are the *oblique edges of the prism*, and G and G′ *the lateral edges of the prism.*

Modifications.

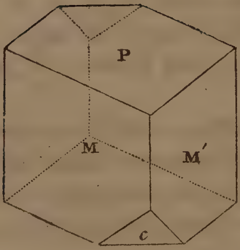
Class *a*.



Class *b*.



Class *c*.



The individuals of this class will differ from each other in the inclination of M on M′, and in the ratio of the edge H to the edge D.

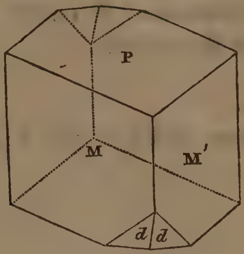
Modifications.

Class *a*. Obtuse solid angles replaced by single planes.

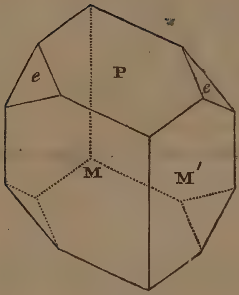
Class *b*. Obtuse solid angles replaced by two planes.

Class *c*. Acute solid angles replaced by single planes.

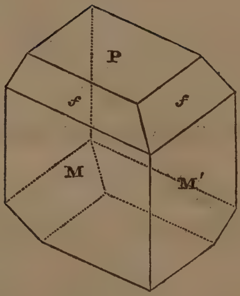
Class *d*.



Class *e*.



Class *f*.



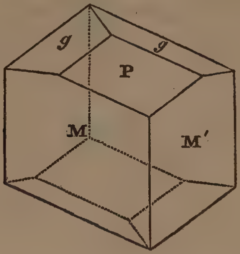
Class *d.* Acute solid angles replaced by two planes.

Class *e.* Lateral solid angles replaced by single planes.

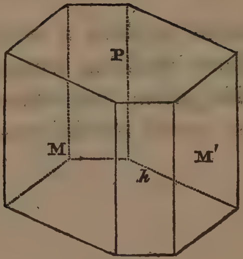
This class would produce a series of octahedrons analogous to those resulting from modification *c* of the right rhombic prism, but whose bases instead of being rectangles, would be oblique-angled parallelograms.

Class *f.* Obtuse terminal edges replaced by single planes.

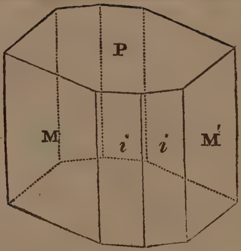
Class g.



Class *h*.



Class *i*.

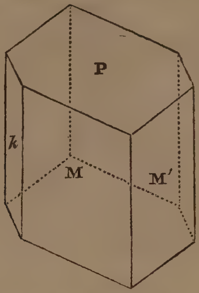


Class *g.* Acute terminal edges replaced by single planes.

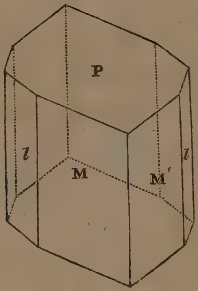
Class *h.* Oblique edges of the prism replaced by tangent planes.

Class *i.* Oblique edges of the prism replaced by two planes.

Class *k*.



Class *l*.



Class *k*. Lateral edges of the prism replaced by tangent planes.

Class *l*. Lateral edges of the prism replaced by two planes.

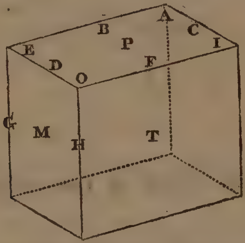
Classes *i*, and *l*, would produce other oblique rhombic prisms, varying from the primary, and from each other, in the angles at which their lateral planes would meet.

In this class of primary forms the cleavage planes, parallel to *one* of the lateral planes, are sometimes brighter than those parallel to the other planes, which is not the general character of symmetric cleavage.

This class of prisms may generally be distinguished from rhomboids, by the unequal angles at which the adjacent planes incline to each other at a terminal edge, and at an adjacent lateral oblique edge; but if these planes should respectively meet at equal angles, as it is possible they may do, the distinction then would arise from the lateral edge being greater or less than the terminal one. For it is possible that the inclination of P on M, or M′, should be equal to that of M on M′, but in this case the edges D would be greater or less than the edges H, and this prism would then bear the same analogy to the rhomboid, that the square prism does to the cube.

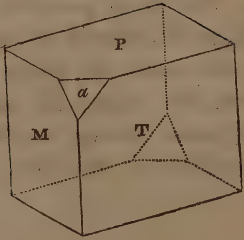
THE DOUBLY OBLIQUE PRISM, AND

Primary form.



Modifications.

Class *a*.



ITS MODIFICATIONS.

Primary form. A doubly oblique prism.

This class of prisms may be supposed to stand in the same relation to the *right oblique-angled* prisms, that the *oblique rhombic prism* does to the *right rhombic prism*; and the following modifications will be better understood, by supposing a *right oblique-angled prism* to become oblique from an acute or an obtuse edge of the prism. This form will then be readily perceived to vary from the oblique rhombic prism, in the dissimilarity of its plane angles A, E, I, and O, of its acute terminal edges B C, and of its obtuse terminal edges D and F.

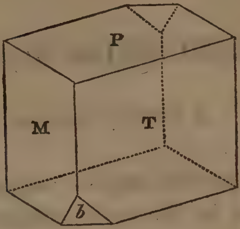
The edges and angles of this class of prisms may be designated by the same terms as have been used for the corresponding ones of the oblique rhombic prism.

The individuals belonging to this class will differ from each other in the inclination of P on M, P on T, and M on T, and in the ratios of the edges D, H, and F.

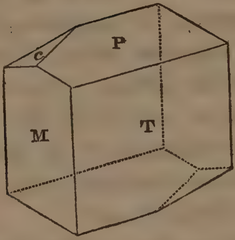
Modifications.

Class *a*. Obtuse solid angles O, replaced by single planes.

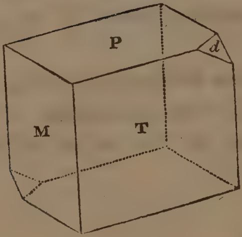
Class *b.*



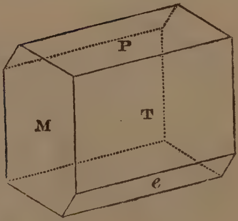
Class *c.*



Class *d.*



Class *e.*



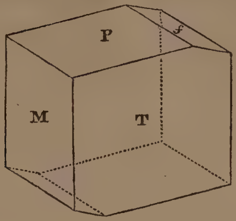
Class *b*. Acute solid angles A, replaced by single planes.

Class *c*. Lateral solid angles E, replaced by single planes.

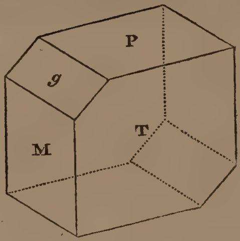
Class *d*. Lateral solid angles I, replaced by single planes.

Class *e*. Acute terminal edges B, replaced by single planes.

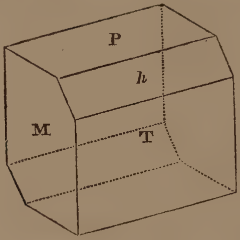
Class *f.*



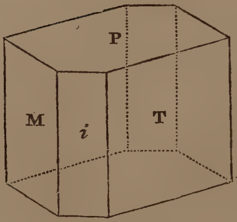
Class *g.*



Class *h.*



Class *i.*



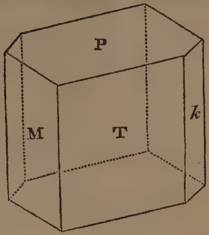
Class *f*. Acute terminal edges C, replaced by single planes.

Class *g*. Obtuse terminal edges D, replaced by single planes.

Class *h*. Obtuse terminal edges F, replaced by single planes.

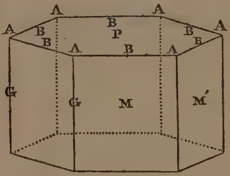
Class *i*. Oblique edges of prism replaced by single planes.

Class *k*.



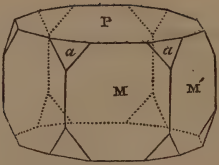
THE REGULAR HEXAGONAL PRISM, AND

Primary form.



Modifications.

Class *a*.



Class *k*. Lateral edges of prism replaced by single planes.

From the dissimilarity of any two adjacent edges or angles of this class of primary forms, the modifications, it will be remarked, are all single planes; some of the compound secondary forms belonging to this class, are among the most difficult crystals to be understood.

ITS MODIFICATIONS.

Primary form. A regular hexagonal prism.

The planes M on M′, measure 120°. M on *d*, 150°.

The individuals belonging to this class will differ from each other in the ratio of the edge G to the edge B.

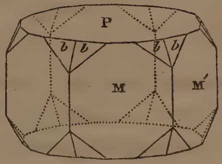
Modifications.

Class *a*. Solid angles replaced by single planes.

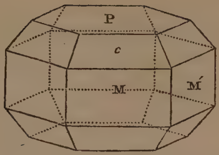
Produces a six-sided pyramid on each summit, resting on the edges of the prism.

The new figures would be a series of dodecahedrons with isosceles triangular planes.

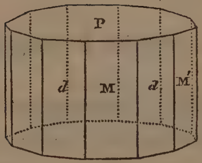
Class *b*.



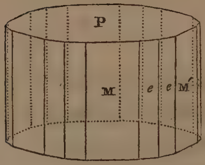
Class *c*.



Class *d*.



Class *e*.



Class *b*. Solid angles replaced by two planes.

The series of new figures would be contained within 24 triangular planes.

Class *c*. Terminal edges replaced by single planes.

Produces a regular hexagonal pyramid on each summit, resting on the planes of the prism.

The new figures would be dodecahedrons with isosceles triangular planes.

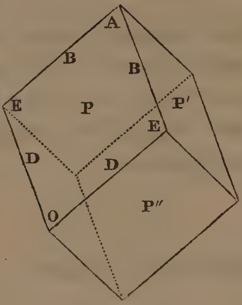
Class *d*. Lateral edges replaced by tangent planes.

Class *e*. Lateral edges replaced by two planes.

The primary and secondary forms of this class, are similar in character to some of the secondary forms of the rhomboid; and it is only by their respective cleavages that they can be distinguished from each other.

THE RHOMBOID, AND

Primary form.



ITS MODIFICATIONS.

Primary form. A rhomboid.

It is found convenient to designate the edges and angles of this figure as follows.

The angle at A is the *superior* angle of the plane P; that at O is the *inferior* angle; those at F are the *lateral* angles; the edges B are the *superior* edges; those at D the *inferior* edges.

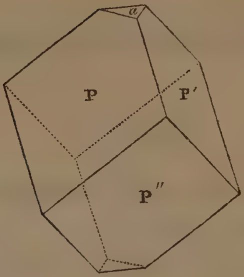
The solid angle at A may also be called the terminal solid angle, or solid angle of the summit. The edges B the terminal edges, or edges of the summit. The solid angles at E, the lateral solid angles, and the edges D, the lateral edges.

The individuals belonging to this class are usually distinguished from each other by the inclination of P on P′. When P on P′ measures more than 90°, the rhomboid is called obtuse; when less, it is called acute.

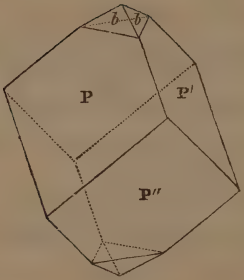
The angle P on P′, is limited between 180° and 60°, but it is obvious it can never reach either of those limits; for the axis must vanish, before the planes P and P′ would reach 180°, or become one plane, and it must be infinite, before these planes could incline to each other at an angle of 60°; in either of which cases the figure would cease to be a rhomboid.

Modifications.

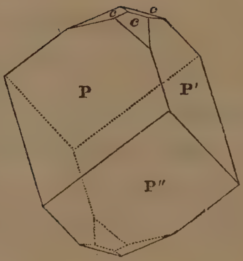
Class *a*.



Class *b*.



Class *c*.



Modifications.

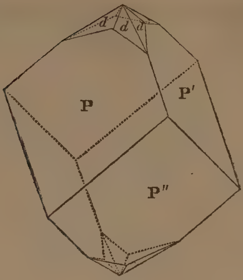
Class *a*. Terminal solid angles replaced by tangent planes.

Class *b*. Terminal solid angles replaced by three planes resting on the primary planes.

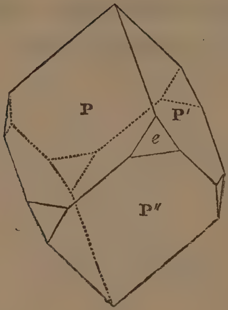
Class *c*. Terminal solid angles replaced by three planes resting on the primary edges.

Modifications *b*, and *c*, would produce a series of rhomboids more obtuse than the primary.

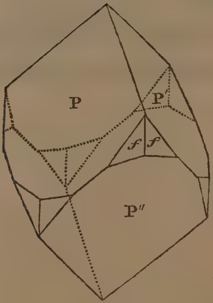
Class *d.*



Class *e.*



Class *f.*



Class *d*. Terminal solid angles replaced by six planes, producing, ultimately, a series of dodecahedrons whose planes are generally scalene triangles.

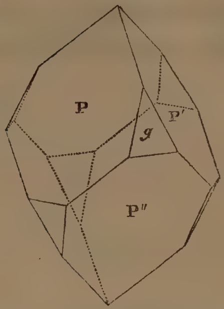
Class *e*. Lateral solid angles replaced by single planes parallel to the perpendicular axis of the rhomboid.

Planes *e*, are the lateral planes of a regular hexagonal prism.

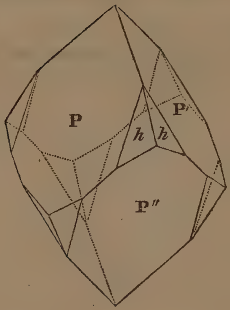
Class *f*. Lateral solid angles replaced by two planes, meeting at an edge which is parallel to the perpendicular axis of the rhomboid.

Planes *f* represent the lateral planes of a series of dodecahedral prisms.

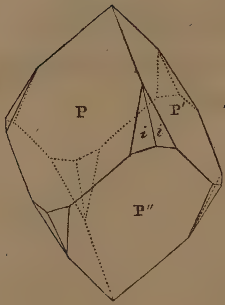
Class *g*.



Class *h*.



Class *i*.

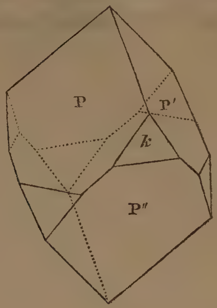


Class *g*. Lateral solid angles replaced by single planes inclining on the superior edges.

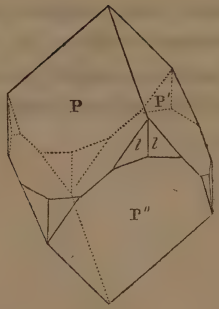
Class *h*. Lateral solid angles replaced by two planes, which intersect each other at an edge that inclines on the superior edges; and which also intersect the adjacent primary planes parallel to their oblique diagonals.

Class *i*. Lateral solid angles replaced by two planes, which like those of class *h*, intersect each other at an edge that inclines on the superior edges, but do *not* intersect the adjacent primary planes parallel to their oblique diagonals.

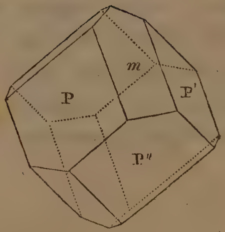
Class *k*.



Class *l*.



Class *m*.



Class *k*. Lateral solid angles replaced by single planes inclining on the primary planes.

Modifications *g*, and *k*, will produce a series of rhomboids more acute than the primary, except one variety of class *g*, which produces a rhomboid similar to the primary.

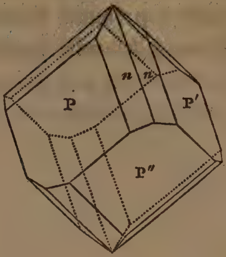
Class *l*. Lateral solid angles replaced by two planes, which intersect each other at an edge that inclines on the primary planes.

Modifications *h*, *i*, and *k*, lead to a series of dodecahedrons whose planes are generally scalene triangles.

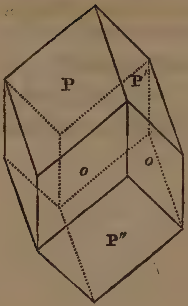
Class *m*. Superior edges replaced by tangent planes.

Produces a rhomboid more obtuse than the primary.

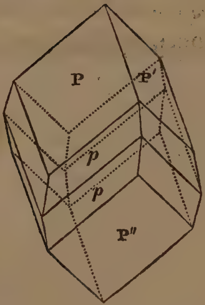
Class *n*.



Class *o*.



Class *p*.



Class *n*. Superior edges replaced by two planes.

A series of dodecahedrons may result from this modification, whose planes are generally isosceles triangles.

Class *o*. Lateral edges replaced by tangent planes.

Planes *o* are the lateral planes of a regular hexagonal prism.

Class *p*. Lateral edges replaced by two planes.

From this modification there results a series of dodecahedrons, whose planes are generally scalene triangles.

The modifications of the rhomboid, and the secondary figures to which they lead, are generally distinguishable from those of the oblique rhombic prism. But those which mark the distinction with the greatest certainty, are modifications *a*, *e*, *g*, *k*, and *m*, of the rhomboid. In the oblique rhombic prism, modification *c*, corresponding with *a* on the rhomboid, is not a tangent plane; and modification *e* of the prism, corresponding in position to some of the planes of modification *e*, *g*, *k*, and *m*, of the rhomboid, affects only four solid angles of the prism instead of the six which are simultaneously modified on the rhomboid.

The three edges of the rhombic prism which meet at the solid angle A, are not generally all modified at the same time, as those of the rhomboid are; nor are the edges G and D modified together as the corresponding edges are in class *m* of the rhomboid.

When the primary planes of the rhomboid are effaced, it is frequently only by observing the direction of the natural joints, or cleavage planes, that we are enabled to determine the classes of modification to which its secondary forms belong.

Several of the preceding classes of primary forms stand in certain relations to each other, which it has not fallen within the scope of the tables to point out.

If we imagine the lateral edges of the *cube* to be lengthened or shortened, a *square prism* would be produced. If while the lateral edges are lengthened, or shortened, we conceive four parallel terminal edges to be lengthened or shortened also, but in a different ratio to the remaining edges of the cube from that in which the lateral edges have been varied, we shall then have the *right rectangular prism*.

It will facilitate our description of the relation of some of the primary forms to certain others, if we conceive the edges to represent wires, united at the solid angles by universal hinges or joints, and capable of being moved in every direction; and, together with the axes, capable also of being lengthened or shortened.

If we conceive one of the axes of the *cube* to be lengthened, the resulting figure would be an *acute rhomboid.* If we suppose the axis shortened by pressing at the same time upon the two opposite solid angles through which the axis to be shortened passes, an *obtuse rhomboid* would be produced.

If two opposite lateral edges of the *square prism* be supposed to be gradually pressed together, so as to shorten one of the diagonals of the terminal plane, and to lengthen the other, the resulting figure would be a *right rhombic prism.*

If we now suppose pressure applied to an acute or an obtuse solid angle of this *rhombic prism*, and the prism to be forced from its perpendicular in the direction of one or other of the diagonals of its terminal plane, an *oblique rhombic prism* would be produced.

If two opposite lateral edges of the *right rectangular prism* be pressed more or less towards each other, a *right oblique-angled prism* would be produced.

And if this *right oblique-angled prism* were slightly forced from its perpendicular position, in the direction of either of its diagonals, a *doubly oblique prism* would result.

So if the vertical axis of the *regular octahedron* were to be lengthened or shortened, an *octahedron with a square base* would be produced. And if two opposite angles of that square base were pressed together so as to shorten one of its diagonals, and to lengthen the other, the resulting figure would be the *octahedron with a rhombic base.*

TABLE OF SECONDARY FORMS.

It may be observed in the preceding tables of modifications, that many of the secondary forms of crystals, are similar to some of the classes of the primary. And it may also be remarked, in many instances, that the secondary forms when complete, or the new figures, as they are termed, are different from all the primary forms.

The following table exhibits the relations of both these descriptions of secondary forms to the several classes of the primary from which they might be produced; and it may thus be regarded as a kind of index to the tables of modifications.

The first column contains a list of the secondary forms, several of which are exhibited in their complete state, or as they would appear if they were contained *within the modifying planes only.*

The second column contains the references to the classes of primary forms, and of modifications, from which the figures in the first column might respectively result.

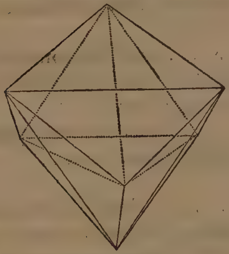
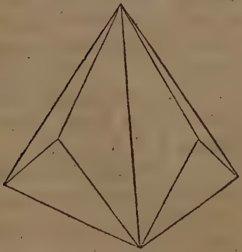
A single example will sufficiently illustrate the use of this table.

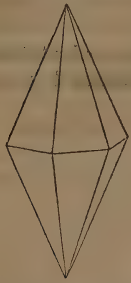
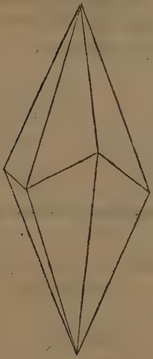
If we desire to know from what primary form a right square prism may be derived, we find that it may result from its own modification *d*; or from an octahedron with a square base, by the concurrence, on one crystal of that form, of the modifications *a* and *e*, or *a* and *n*, when those modifications efface the primary planes. And if we turn to those modifications of the octahedron with a square base, we shall observe that modification *a* would produce the terminal planes, and *e*, or *n*, the lateral planes of a right square prism.

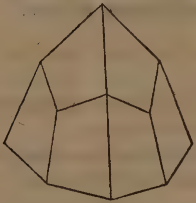
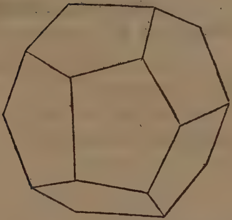
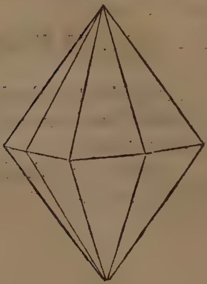
Secondary forms.	*How they may be derived.*
1. CONTAINED WITHIN FOUR PLANES.	
The regular tetrahedron .	From the cube. Mod. *g*.
2. CONTAINED WITHIN SIX PLANES.	
The cube	From the regular tetrahedron. Modification *e*. .... regular octahedron. Modification *a*. .... rhombic dodecahedron. Modification *a*.
The right square prism ..	.... right square prism. Modification *d*. The octahedron with a square base, by a concurrence of Modifications *a* and *e*, or *a* and *n*, on the same crystal.
The right rectangular prism	.... octahedron with a rectangular base, by the concurrence of Modifications *a*, *m*, and *o*, on the same crystal. .... octahedron with a rhombic base, by the concurrence of Modifications *a*, *g*, and *n*, on the same crystal. .... right rhombic prism, if Modifications *f* and *h* occur on the same crystal, and efface the lateral primary planes.

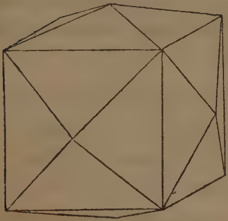
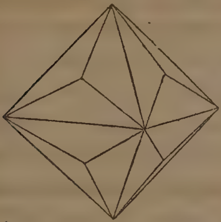
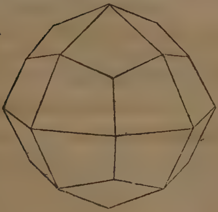
Secondary forms.	*How they may be derived.*
The right rhombic prism .	From the octahedron with a rectangular base, by the concurrence of Modifications a and e, or a and f, or a and g, on the same crystal; or of b and o, or n and o, in which case o would appear a terminal plane; or of c and m, or p and m, appearing then as the terminal plane.
	.... octahedron with a rhombic base, by the concurrence of Modifications a and i, or a and p, or a and y, on the same crystal; or of b and n, or h and n, or t and n; or of c and g, or o and g, or v and g; but in these latter combinations, n and g would appear as terminal planes.
	.... right rectangular prism, Modification d; or Modification b, or c, if we suppose the planes b, or c, to have effaced four of the primary planes, and the figure then rest on the plane M or T.
	.... right rhombic prism, Modification g, or i.
The right oblique-angled prism	.... oblique rhombic prism, by the concurrence of Modifications h and k, on the same crystal, and by supposing the secondary form to rest upon the plane k. In this position the planes k would appear as the terminal planes of the new figure, and plane P as one of the lateral planes.

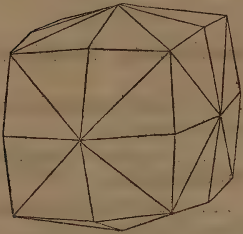
Secondary forms.	*How they may be derived.*
The rhomboid	From the rhomboid, Modification, *b*, *c*, *g*, *k*, or *m*.
3. CONTAINED WITHIN EIGHT PLANES.	
The regular octahedron.	From the cube, Mod. *a*.
	.... regular tetrahedron, Modification *a*, when the secondary planes first touch each other on the edges of the tetrahedron. In this state of the figure, four of the octahedral planes are obviously the primary planes of the tetrahedron.
	.... rhombic dodecahedron, Modification *e*.
The octahedron with a square base	.... octahedron with a square base, Modification *b*, *c*, *f*, *l*, or *o*.
	.... right square prism, Modification *a* or *c*.
The octahedron with a rectangular base	.... right rectangular prism, by the concurrence of Modifications *b* and *c*.
	.... the right rhombic prism, when Modification *a*, or *c*, is combined with four of the primary planes.
The octahedron with a rhombic base	.... octahedron with a rectangular base, Modification *h*, *i*, *k*, or *l*.
	.... octahedron with a rhombic base, Modification *d*, *e*, *f*, *k*, *l*, *m*, *q*, *r*, *s*, *u*, *x*, or *z*.
	.... right rectangular prism, Modification *a*.
	.... right rhombic prism, Modification *b*, *d*, or *e*.

Secondary forms.	*How they may be derived.*
The hexagonal prism	From the rhomboid, by a combination of Modifications *a* and *e*, or *a* and *m*. .... hexagonal prism, Modification *d*. .... right rhombic prism of 120°, Modification *h*.
4. CONTAINED WITHIN TWELVE PLANES.	
a. The planes being isosceles triangles.	
Fig. 289. 	From the regular tetrahedron, Modification *c*. .... cube, Modification *h*.
Fig. 290. 	From the regular tetrahedron, Modification *f*.

Secondary forms.	*How they may be derived.*
Fig. 291. 	From the rhomboid, particular planes of Modification *d, h, i, l*, or *n*. .... hexagonal prism, Modification *a*, or *c*.
b. The planes being scalene triangles.	
Fig. 292. 	.... rhomboid, Modification *d, h, i, l, n*, or *p*.
c. *The planes being rhombs.*	
The rhombic dodecahedron	.... cube, Modification *e*. .... regular tetrahedron, particular Modification belonging to class *b*. .... regular octahedron, Modification *e*.

Secondary forms.	*How they may be derived.*
d. *The planes being trapezoids.*	
Fig. 293. 	From the regular tetrahedron, Modification *b* generally.
e. *The planes being pentagons.*	
Fig. 294. 	.... cube, Modification *k*. .... regular tetrahedron, particular Modification belonging to class *d*.
5. CONTAINED WITHIN SIXTEEN TRIANGULAR PLANES, which are generally scalene, but may be isosceles triangles.	
Fig. 295. 	From the octahedron with a square base, Modification *d, h, i, k,* or *m*. .... right square prism, Modification *b*.

Secondary forms.	*How they may be derived.*
6. CONTAINED WITHIN TWENTY-FOUR PLANES.	
a. The planes being isosceles triangles.	
Fig. 296. 	From the cube, Modification *f*. ....regular tetrahedron, Modification *d*. .... regular octahedron, Modification *c*. .... rhombic dodecahedron, Modification *b*.
Fig. 297. 	.... cube, Modification *c*. .... regular octahedron, Modification *f*. .... rhombic dodecahedron, Modification *f*.
b. The planes being equal trapezoids.	
Fig. 298. 	.... cube, Modification *b*. ....regular octahedron, Modification *b*. .... rhombic dodecahedron, Modification *c*, *g*, or *i*.

Secondary forms.	*How they may be derived.*
7. CONTAINED WITHIN FORTY-EIGHT TRIANGULAR PLANES.	
Fig. 299. 	From the cube, Modification *d*. .... regular octahedron, Modification *d*. .... rhombic dodecahedron, Modification *d*, *h*, or *k*.

SECTION XII.

ON THE APPLICATION OF THE TABLES OF MODIFICATIONS.

THE preceding tables of modifications are adapted principally to two purposes. The first is, by the remarks they contain upon the comparative characters of the secondary forms belonging to the different classes of the primary, to assist the mineralogist in determining the primary form of any mineral from an examination of its secondary forms. And the second is to enable him to describe any secondary crystal, whose primary form is known. An attempt is thus made to supply a language, by means of which the secondary forms of crystals may be described independently of the theory of decrements, and without the assistance of mathematical calculation.

The remarks upon the comparative characters of the secondary forms, may not however be sufficient to lead the observer from the secondary to the primary form of a mineral, without the assistance of a few general rules.

And although I cannot flatter myself that the slight outline I am about to trace of *the method of reading crystals*, will enable a person at once to refer any given secondary crystal to its primary form, it will nevertheless afford him a useful clue to the discovery of that form.

The first step in the process of determining the mineral species to which any given secondary crystal belongs, is, as we have already seen, to determine the *class of primary forms* to which it belongs. And if the individuals of that class differ from each other in the mutual inclination of some of the primary planes, the goniometer must be resorted to for determining that inclination in the crystal we are examining.

It happens, however, not unfrequently, that all the primary planes of a crystal are obliterated, and that the secondary form consists of an entirely new figure.

In this case the observer will encounter a difficulty in his attempt to deduce the characters of the primary form from the secondary crystal, unless the secondary crystal can be referred to one of those *entire secondary forms* described in the table of secondary forms; but in any case this difficulty will be in some degree overcome, by a habit of examining and comparing crystals with each other, although it probably cannot be entirely removed. For if any rules could be given for determining a primary form from the inspection of a secondary form, on which no trace of the primary planes remain, and where no assistance is afforded by cleavage, they must be too numerous and complicated to be serviceable to the young mineralogist, for whose use these pages are principally designed.

In the few rules, therefore, which I propose to give, I shall suppose, *generally*, that the secondary crystal which is to be examined retains some portion of the natural primary planes, or of cleavage planes which are parallel to these.*

---

* The planes of the regular tetrahedron, and of all the octahedrons, and of the rhombic dodecahedron, may be easily recognised from the figures in the preceding tables, and they will not therefore be particularly noticed here. And the different varieties of octahedrons are distinguishable from each other by the angles at which their several planes respectively meet.

To ascertain the *class of primary forms* to which such secondary crystal belongs, we should first observe whether there be on the crystal any series of *planes whose edges are parallel to each other*. If we observe such a series of planes, we should then hold the crystal in such a manner, that the series of *parallel edges* may be *vertical*, or upright. And *while it is in this position*, we should observe whether there be any plane at *right angles* to the series of *vertical* planes we have noticed.*

But on examining the secondary forms of crystals, we may sometimes find that there are *two sets* of parallel edges, either of which being held upright, the crystal would present a series of vertical planes. We should in this case endeavour to ascertain whether the planes belonging to one set, are not so symmetrically arranged with respect to those of the other, as to possess the character of modifications of the *terminal* edges of a primary form; if we find them so, we should not make that the vertical series.

If there be a series of vertical planes, and a horizontal plane, we should observe whether any of the vertical planes are at right angles to each other, and whether there be any oblique planes lying between some of the vertical planes, and the horizontal plane.

We should remark the equality, or inequality, of the angle at which any of the vertical or oblique planes incline on the several adjacent planes.

We should notice whether there be any such symmetrical arrangement of the vertical planes, or of the oblique planes, if there be any, as would induce us to refer our crystal to any particular class of primary forms; and by comparing the characters we thus observe with those described in the tables, we shall probably discover the class of primary forms to which our crystal belongs.

---

* When the series of planes with parallel edges are held *vertically*, the plane at right angles to them will of course be *horizontal*. These may therefore be called *the vertical* and *horizontal planes*; *all other planes will be termed oblique*; and the edges of the horizontal and vertical planes, will be termed *horizontal* and *vertical edges*.

Let us now suppose our crystal to be contained within any series of vertical planes, and to be terminated, not by a horizontal plane, but by a single oblique plane, the crystal may then belong to the class of oblique rhombic prisms, doubly oblique prisms, or rhomboids.

If there be *four oblique planes*, inclining to each other at *equal angles*, the crystal may belong to the class of square prisms, or of octahedrons with square bases.

If there be *four oblique planes*, each of which inclines on two adjacent planes at unequal angles, the crystal will probably belong to the class of right rectangular prisms, right rhombic prisms, or octahedrons with rectangular or rhombic bases.

If the series of vertical planes consist of 6, 9, 12, *or some other multiple of* 3, *and if there be a single horizontal plane*, the crystal may belong to one of the classes of right prisms, rhomboids, or hexagonal prisms.

If there should be *three oblique planes*, the primary form is a rhomboid.

But if the termination consists of *six* oblique and equal planes, the crystal may belong to the class of rhomboids or hexagonal prisms.

Crystals not falling within any of the preceding descriptions, may yet be found to resemble some of the secondary forms given in the tables.

Those which belong to the class of doubly oblique prisms, are sometimes very difficult to be understood; and the relation between the primary and secondary forms of this class, can be learned only by a comparison of the crystals themselves with each other, assisted by the tables of modifications already given.

A circumstance, which has not yet been alluded to, will also frequently render it very difficult to *read a crystal.* This is the *unequal* extension of some of its *parallel* planes. A very remarkable instance of this character prevails in copper pyrites, and has been the occasion of the erroneous opinions entertained until very lately, respecting the primary form of that substance.

In all the works on mineralogy, except that by Professor Mohs, its primary form is stated to be a regular tetrahedron. Mohs, however, discovered that its form was an octahedron with a square base. The two following figures, for the drawings of which, made from crystals in his own possession, I am obliged to Mr. W. Phillips, exhibit crystals containing *equal numbers of similar planes*; fig. 300 having these planes *regularly placed on the primary form*, and fig. 301 representing the same crystal *as it frequently occurs in nature, with some of its planes considerably enlarged.*

The same letters are placed on the corresponding planes of each.

Fig. 300.

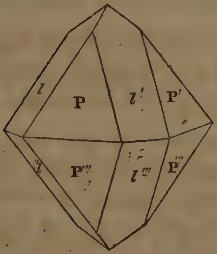
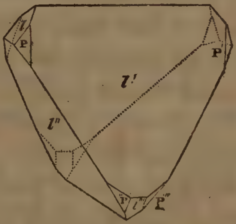


Fig. 301.



Mr. Phillips had discovered from the cleavage of this substance, before the publication of Professor Mohs's book, that its primary form was not a tetrahedron, as appears by a paper in the Annals of Philosophy, new series, vol. 3. p. 297.

When crystals of this irregular character occur, it is generally, only by cleavage, and by using the goniometer, that we can be led to an accurate determination of their true forms.

Having ascertained the *class* of *primary forms* to which our supposed crystal belongs, our next step would be to measure the angles of its primary or secondary planes, in order to determine the *species* to which the mineral itself belongs.

If the crystal belongs to one of the *four regular solids*, whose angles, when the forms are similar, are always equal, its hardness, or specific gravity, or some other character, will the most readily lead to a determination of its mineral species.

But it may happen that the secondary crystal we are examining, may be referred with equal propriety to either of the two, or more, classes of primary forms. If we turn to the modifications of the octahedron with a square base, and to those of the square prism, and imagine the modifying planes of the square prism much enlarged, we shall observe such a resemblance between them, that either form may be taken as the primary, in reference to the secondary forms of both. The same remark will apply to the octahedron with a rectangular or a rhombic base, and the right rectangular, or rhombic prism.

In these cases it has been usual to adopt that as the primary form which is developed by cleavage. But if there be no practicable cleavage, or if there be two sets of cleavages, parallel to the planes of two primary forms, we are then at liberty, as it has been already stated, to adopt either of these, and our choice would probably fix on that which was most predominant among the secondary forms.

If a crystal is to be *described* by the assistance of the preceding tables, we must suppose the primary form to be known; this must be first described according to its class, and if necessary by its angles also.

Its modifications, if they are single, may then be denoted by the letters under which they are arranged in the tables. But as *each of the classes of modifications*, except those which consist of *tangent planes*, *comprehends an almost unlimited number of individual modifying planes, differing from each other in the angles at which they respectively incline on the primary planes*, it becomes necessary to add to the tabular letter which expresses the modification, the value of the angles at which the plane we have observed inclines on the adjacent primary planes.

We have already seen that modification *a* of the right rectangular prism, comprehends a considerable number of planes varying in their relative inclinations on P, M, and T. Let us suppose the crystal we are examining, to belong to the class of right rectangular prisms, and to be modified by a plane *a*, and let the inclination of the plane we have observed, on P, M, and T, be called $m°$, $n°$, and $o°$, these letters signifying any number of degrees and minutes whatsoever.

A crystal containing the primary planes of the right rectangular prism, and a set of planes belonging to modification *a* might then be thus described.

Right rectangular prism, Modification *a*, $m°$ on P.
$n°$ — M.
$o°$ — T.

The character of the plane being thus established, we may in future, in order to avoid the repetition of the measurements; describe the plane as Modification *a*, plane 1, and it may be marked in the figure of the crystal as *a* 1.

Let us now suppose we find on another crystal, another plane modifying the same solid angle; and inclining on P, M, and T, at p°, q°, and r°; and a third plane; also modifying the same solid angle; and inclining on P, M, and T, at s°, t°; v°, we should describe these planes as we did the first, by

$$\text{Modification } a,\ \left.\begin{array}{l} p^\circ \text{ on P} \\ q^\circ - \text{M} \\ r^\circ - \text{T} \end{array}\right\}\text{plane 2.}$$

$$\text{Modification } a,\ \left.\begin{array}{l} s^\circ - \text{P} \\ t^\circ - \text{M} \\ v^\circ - \text{T} \end{array}\right\}\text{plane 3.}$$

And having thus recorded the character of the planes, we may in future describe them as

Modification *a* 2,
Modification *a* 3.

This method of description may be applied, whether the three planes have occurred on the same crystal, or on different crystals.

The inclination of the modifying plane on *two* of the primary planes, is generally sufficient, when a solid angle is modified, for determining the law of decrement; but the third inclination serves as a check upon the accuracy of the other two.

If the edge of any prism be modified by one or more planes, it will be sufficient to give the inclination of *each* plane, on *either* of the primary planes, where the inclination of the primary planes to each other is known, as the inclination on the other primary plane may be readily ascertained. But when an edge is modified, of any crystal whose adjacent primary planes do not meet at a right angle, and when their mutual inclination is unknown, the inclination of the modifying plane on *both* the primary should be given.

This method of description may be readily extended to all the classes of primary forms; and although it may sometimes be rather tedious in its application, it will convey an accurate description of the planes to which it is applied.

It may frequently happen that we are examining a crystal whose primary form is unknown to us, and whose secondary planes do not enable us to determine that form; we can in such case describe the crystal only by giving a drawing of it, accompanied by the inclinations of its several planes to each other.

It will perhaps be found convenient, where it can be done, to number the observed planes, belonging to each class of modifications, in some certain order; when there is a series of secondary planes whose edges are parallel, that plane may be denoted by No. 1, which forms the *most obtuse angle* with the primary plane to which the series may be referred.

SECTION XIV.

ON THE USE OF SYMBOLS FOR DESCRIBING THE SECONDARY FORMS OF CRYSTALS.

IN Section 11, p, 102, it was stated that certain letters had been appropriated by the Abbé Haüy, as symbols, to designate the similar and dissimilar edges, angles, and planes of each of the classes of primary forms. And in the tables of modifications, these letters are placed on the figures of the primary forms, to denote in each its similar, and dissimilar, edges, angles, and planes.

The order in which they are placed on the figures is obviously that of the alphabet; and they are arranged also according to the ordinary method of writing, beginning at the upper part of the figure, and then proceeding from left to right until the several parts of the crystal are marked by the appropriate letters.

This will be very apparent, if we refer to the primary form of the doubly oblique prism.

The letters P M T are retained to designate the primary planes of crystals, although the term primitive, from which those letters were derived, is not used in this treatise.

The letter A is, by the Abbé Haüy, placed, generally, on an obtuse angle of the primary form; but according to the positions in which the primary forms are drawn in the preceding tables, the letter A will not necessarily stand on an obtuse angle, excepting on the rhombic dodecahedron, the right rhombic, the right oblique-angled, and the hexagonal prisms.

The edges and angles of that terminal plane of the prism, on which the figures appear to rest, and the edges and planes which constitute the back of the figures, are supposed to be denoted by a series of small letters, corresponding with the capitals, by which the diametrically opposite edges, angles, and planes, exhibited in the front of the figure, are designated.

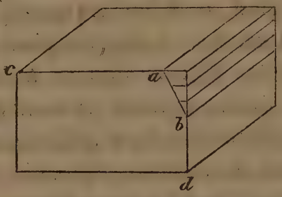
The representation of the secondary forms of crystals by means of these symbols, is effected by annexing numbers, expressive of the particular laws of decrement by which the secondary planes are conceived to have been respectively produced, to the letters which denote the edges or angles on which the decrements have taken place.

These numbers will be termed the indices of the secondary planes, and will generally be represented by the letters p q r s.

Before we proceed, however, to explain the manner in which these symbols may be applied to the representation of the secondary forms of crystals, we shall for a moment consider the theory of decrements more particularly in reference to the descriptive character it affords.

As this character is to be regarded as little else than a symbol, indicating the change of figure which the primary form has undergone, if there be two laws of decrement which will equally well express this change of figure, we are obviously at liberty to adopt either law, as the generator of the new plane by which the figure of the primary form is altered; but it will be found convenient to be guided by some rule in our choice.

Fig. 302.



If, for example, we find a secondary plane, such as *a b*, fig. 302, on the terminal edge of any prism, produced by the abstraction of three rows of molecules in the direction of the lateral edges, and of one row in the direction of the terminal edges, such a plane might be conceived to be produced by a decrement proceeding along the plane *a c*, consisting of three molecules in height and one in breadth, or of three molecules in breadth, if we suppose it to proceed along the planes *b d*; and the symbol denoting either of these decrements, might therefore with equal propriety be used to describe the new plane; and it would be indifferent, as far as the descriptive character of the symbol is regarded, which of the two we should adopt.

The rule which it will be more convenient to follow, is, to suppose all planes on the *terminal* edges of prisms, to be produced by decrements proceeding along the *terminal* planes; and the planes replacing the lateral edges of prisms, and the edges of all the other classes of primary forms, may be conceived to result from decrements proceeding along those planes in the direction of which the greatest number of molecules appear to have been abstracted. And any intermediary decrement may be conceived to proceed along that plane in the directions of whose edges the greatest number of molecules have been abstracted.

It may not be useless to remark, that when two or more planes replace the solid angle of a crystal, if an edge at which the secondary planes intersect each other be parallel to the edge at which one of them intersects the primary plane, they will generally both result from simple or mixed decrements.

Intermediary decrements however sometimes produce a series of planes whose intersecting edges are parallel to each other, and when this happens, the symbols of those planes will have two of their corresponding indices in the same ratio to each other.

The edges of such secondary planes as replace the edges of crystals, and which result from simple or mixed decrements, are always parallel.

From what has been already stated it will appear, that if we are about to describe a secondary crystal, belonging to any species of mineral whose primary form is known, and upon several of whose edges, or angles, *similar* decrements have produced *similar* planes, it will be sufficient, *generally*, to describe one only of the new planes, produced upon one of those edges or angles.

And if two or more laws of decrement have concurred in the production of any secondary crystal, we should be required, *generally*, to describe only one of the planes produced by each particular law. For the change of figure which any primary form has undergone would be, generally, thus indicated. And in drawing the crystal we might construct planes, similar to those which are described, upon all its similar edges or angles.

As it sometimes, however, occurs that all the similar edges or angles of crystals are not similarly modified, it will not be sufficient in all cases to indicate the decrement which has taken place on one edge or angle, but our representative symbol should also indicate the absence of the modifying plane from some other edge or angle, where according to the law of symmetry, it might be expected to appear.

This necessity of distinguishing the symmetrical modifications of crystals, from those which are not so, will render the symbols rather more complicated than they would be otherwise.

The new theory of molecules which has been introduced into this treatise, will render it necessary to vary the character of some of the symbols employed by the Abbé Haüy in reference to the tetrahedron, to all the classes of octahedrons, and to the rhombic dodecahedron; and as these changes will occasion some other slight deviations from his system of notation, it will conduce to perspicuity if we consider the application of the symbols to each of the classes of the primary forms separately. This will be done in a table subjoined to this section, where the order of the primary forms will correspond with that adopted in the tables of modifications.

The general nature of this system of notation will be best illustrated by its application to one of the least regular of the primary forms.

Let us suppose that we are about to represent a secondary crystal belonging to the class of doubly oblique prisms, according to the theory of decrements, and by means of the symbolic letters already alluded to, the primary form being known, and the law of decrement by which the secondary plane has been produced, having been also ascertained.

The crystal is supposed to be held with the plane marked P, horizontal, and with that edge or angle nearest to the eye on which the decrement we are about to describe has taken place.

Let us suppose this crystal to be modified by an individual plane, belonging to the series of modifications of that figure comprehended under class *b*.

The planes belonging to this class of modifications, may incline more or less on either of the adjacent primary planes, and may result from a decrement on either of the adjacent plane angles which constitute the solid angle on which O is placed.

If the modifying plane be produced by a simple or mixed decrement, beginning at the angle O, and proceeding along the terminal plane, consisting of one row of single molecules, it should be expressed thus, $\overset{1}{O}$, and be read, one over O, signifying that the abstraction of molecules from the superimposed plates took place above, or receding from O, in the direction O A. If the decrement be simple, and by two rows of molecules in breadth, it would be expressed by $\overset{2}{O}$, and if it be a mixed decrement by three rows in breath and two in height, it would be denoted by $\overset{\frac{3}{2}}{O}$, and so of any other decrement acting in that direction.

If the modifying plane be occasioned by a simple or mixed decrement, beginning at the angle of the plane M adjacent to O, and proceeding along the plane M, by p rows of molecules, *p signifying any whole number or fraction*, it would be denoted by ${}^{p}O$, and be read p on the left of O.

If the new plane were produced by a simple or mixed decrement by p rows on the angle of the plane T, adjacent to O, and proceeding along that plane, it would be denoted by O^p, and be read *p* on the right of O.

In either of the preceding cases, the intersection of the new plane with the primary plane along which the decrement is conceived to proceed, will, as we have already seen, be parallel to the diagonal of that plane.

Let us now suppose an intermediary decrement to have taken place on the angle O, of such a nature, that the mass of molecules abstracted should belong to a double plate, or be two molecules in height, or as it might be otherwise expressed, 2 molecules in the direction of the edge H, 3 in the direction of the edge D, and 4 in the direction of the edge F.

The appropriate symbol to denote such a decrement, ought obviously to represent this threefold character; which it does by combining the indices expressive of the particular law of decrement, with the letters which represent the edges and angles affected by it, in this manner, (D 3 H 2 F 4). This symbol is placed in a parenthesis to distinguish it from a combination of three simple or mixed decrements, and it would be read thus, 3 on the edge D, 2 on the edge H, 4 on the edge F.*

If instead of the angle marked by O, we now imagine the solid angle on which A is placed to be modified simlarly to that denoted by O; before we describe the modifications of A, the crystal is conceived to be turned round, until the angle on which

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* This mode of representing intermediary decrements differs from that adopted by the Abbé Haüy, in referring the decrement to the adjacent edges; whereas he refers them to two edges and the angle they include. But the form of the symbol here given will best accord with the results obtained by the methods of calculating the laws of decrement, which will be given in the Appendix.

A is placed is nearest to the eye; or we may be supposed to pass round the crystal, until we place ourselves opposite the angle at A; and while the eye and that angle are in these relative positions, we should proceed to describe the new planes, as we did those on the solid angle at O.

If two or more planes, resulting from simple or mixed decrements, are found modifying the same solid angle of any crystal, the symbols representing them are to be placed immediately following each other. Thus if the three planes we have supposed on the angle O, should occur on the same crystal, its change of figure would be thus represented,

$$ {}^{p}O\ \overset{p}{O}\ O^{p}. $$

These symbols not being placed in a parenthesis, are understood to represent three separate planes.

If three intermediary decrements should occur on the same solid angle, their symbols would also be placed following each other, thus,

$$ (D3\ H2\ F4)\ (D1\ H3\ F2)\ (D4\ H1\ F3). $$

Here, each of the three sets of characters being included within a separate parenthesis, three varieties of intermediary decrement are implied; and as they stand singly, it is implied that they are independent of each other; and they are evidently produced by different laws of decrement.

Let us now suppose we are about to describe a decrement on a terminal edge of a doubly oblique prism. The prism is again supposed to be placed or held with that edge nearest to us, the plane P continuing horizontal.

And first let us suppose a terminal edge F to be replaced by a plane resulting from a decrement by *p* rows of molecules proceeding along the plane P; p, *meaning, as before, any whole number or fraction.* The symbol to denote this decrement, would be $\overset{p}{F}$, and be read as before, p over F.

If the decrement be supposed to have proceeded along the plane T by three rows of molecules, as in figure 303, the *general* symbol used to represent the new plane would still be $\overset{p}{F}$, but p would in this case represent the fraction $\frac{1}{3}$, and the *particular* symbol would be $\overset{\frac{1}{3}}{F}$.

If we suppose p to be a fraction, it is evident from what has been already stated, that the numerator of that fraction may be either greater or less than the denominator, according as the decrement in breadth exceeds or falls short of that in height.

If either of the other terminal edges be modified, the modified edge is supposed to be the nearest to the eye, when the modifying plane is described.

This change of position must be understood to take place in every instance where the position of the modified edge or angle requires it.

If two dissimilar planes occur on the same terminal edge of a crystal, the symbol is repeated thus $\overset{p}{F}\ \overset{p}{F}$, which expresses the coexistence of the two planes on one edge.

If the lateral edge H of a doubly oblique prism be modified, and if it has been found that the decrement producing it has proceeded along the plane M, by p rows of molecules, its characteristic symbol would be ${}^{p}H$, and it would be read p on the left of H.

If the decrement appears to have proceeded along the plane T by p rows of molecules, its symbol would be H^{p}, or p on the right of H; p being either a whole number or fraction, expressive of the particular law of decrement, in reference to each plane respectively, as it is supposed to have been ascertained by calculation.

If the two planes occur on the same crystal, they would be denoted by the two symbols being used together, thus, ^{p}H H^{p}.

If it should be required to describe any decrement acting upon an edge or angle of the lower plane of the crystal, upon which the small letters are supposed to be placed, the crystal is imagined to be turned with that plane upwards, the edge or angle on which the decrement has taken place is to be brought the nearest to the eye, and we are then to describe the plane or planes in the manner already directed, only using the small letter, instead of the capital, to indicate the edge or angle which is modified. And if it should be necessary to describe a decrement upon the back planes of the crystal, we are supposed to pass round it, and to substitute small letters in the symbol for the capitals which designate the corresponding front planes.

The preceding explanations will render sufficiently intelligible the *general* method of representing the secondary planes of crystals by means of symbolic characters. Before we proceed, however, to apply this method to the different classes of primary forms, it will be necessary to separate the secondary forms of the crystals to be represented into three principal classes.

1. Those which are strictly *symmetrical*, as modification *a*, *b*, *c*, *d*, *e*, or *f*, of the cube, where similar decrements take place on similar edges or angles, and proceed along similar planes.

2. Those which are partially modified, or on which the same modification does not occur on all the similar edges or angles; as in modification *g*, *h*, *i*, *k*, of the cube.

   These may be termed *defective modifications*, and they may be again subdivided into two portions.

   a. Those in which an edge or angle is replaced by only half the number of planes which the law of symmetry would require.

   b. Those in which only one of two similar edges or angles is modified, while the other remains entire.

3. Those in which two or more similar edges or angles are affected by *different laws of decrement*.

And the symbols, to be perfect, ought to represent each of these divisions clearly and perspicuously.

In the table subjoined to this section, which will point out the relation of the theory of decrements to the different classes of modifications, the various modes of adopting the symbols to particular cases will be fully explained. Whence it will not be necessary here to give more than an outline of the general principle which will regulate their application.

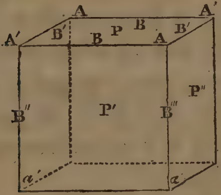
To represent the secondary forms belonging to the first of these divisions, it may not appear strictly necessary to do more than indicate the character of a single plane belonging to any set of similar planes occurring upon the same crystal; but it may tend to prevent ambiguity if we construct our symbol so as to indicate that the secondary planes occur symmetrically on certain edges or angles of the crystal.

We may here remark, that the sets of planes which, in the tables of modifications, replace the solid angles of the cube, tetrahedron, and rhomboid, and *rest,* as it is said, upon the *planes* of those primary forms, are distinguished from those which are said to *rest upon their edges.* But in reference to the theory of decrements, *both these sets of planes are similar in character, and result from simple or mixed decrements on an angle of the primary form.*

The planes which are said to *rest upon the primary planes,* are produced by decrements in which the *number of molecules abstracted in breadth, is greater than the number in height,* while those which are said to rest upon the edges, result from decrements wherein *the number in height exceeds the number in breadth.* The numbers or fractions expressing the *first* of these sets will be always *greater than unity,* as 2, 3, 4, $\frac{3}{2}$, $\frac{4}{3}$, &c.; those expressive of the latter set, will be always *less than unity,* as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c.; and the planes in this latter case are conceived to be carried, as it were, over the solid angle, and made to replace a portion of the adjacent edge.

The Cube.

Fig. 303.



Let us now suppose a cubic crystal, modified on the angles by three planes belonging to class *b* of the modifications of that form; and let us suppose that the modifying planes result from a decrement by two rows in breadth on the angles of the cube. The symbol denoting these planes would be ${}^{2}\overset{2}{A}{}^{2}$, and if this be unaccompanied by any other symbol, it would be implied that all the solid angles were similarly modified. The symbol representing class C might be

$${}^{\frac{1}{2}}\overset{\frac{1}{2}}{A}{}^{\frac{1}{2}}.$$

The planes of modification *a* of the cube might be denoted thus, $\overset{1}{A}$, but for the sake of uniformity with the preceding symbol, they will be represented by the symbol ${}^{1}\overset{1}{A}{}^{1}$.

The planes belonging to class *i* of the cube, do not differ from those belonging to class *d*, except in being three *single* similar planes, instead of three *pairs* of similar planes, as there are in class *d*. To distinguish class *d* therefore, by its symbol, it will be requisite that the symbol should represent one of the *pairs* of planes, and not merely a *single* plane, as might have been sufficient if class *i* had not existed.

Suppose an individual modification belonging to class *d* is to be denoted, and if the decrement producing it be by three molecules on the edge B, one on the edge B′,* and two in height on the angle A, the symbol would be

$$(B\,3\ B'''\,2\ B'\,1 : B\,1\ B'''\,2\ B'\,3),$$

which would imply a pair of planes resting on the plane P. *And the two symbols being both included within a parenthesis, and separated from each other by two dots, implies that both the planes represented result from the same law of decrement, but acting in two different directions.*

If two similar planes belonging to class *f* of the cube, resulting from a decrement by three rows in breadth, occur on all the edges of a cubic crystal, the symbol $\overset{3}{B}$ will be used to denote their existence on one of the edges; and their existence on the other edges is implied, unless their absence be denoted by the characters which will be presently given and explained. This symbol implies that the edge B is replaced by two planes, one of which results from a decrement by three rows in breadth proceeding along the terminal plane, and the other by three rows in breadth proceeding along the lateral plane. The symbol $\overset{1}{B}$ might be sufficient to denote the planes of modification *e*, but for the sake of conformity with the general system of notation, it should be written

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* B′, B″, &c. is read B *dash*, B *two dash*, &c.

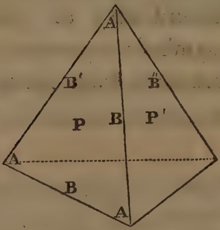
$$\overset{1}{\underset{1}{B}}.$$

When the lateral edge of a prism is modified by two similar planes, the symbol representing them will be ${}^{p}G^{p}$. The G standing *single*, implies that the symbol refers to a *single edge*.

The planes belonging to class *d* of the modifications of the right rectangular prism, may be readily conceived to result from decrements proceeding along either of the planes M or T. If along the plane M, the symbol would be $G'^{p}\ {}^{p}G$; but if the decrement be supposed to have proceeded along the plane T, its symbol would be ${}^{p}G'\ G^{p}$.

The Tetrahedron.

Fig. 304.



Simple and mixed decrements on the angles of the tetrahedron producing planes belonging to class *b*, are supposed to proceed along the plane P; and the symbol by which they are to be represented is

$$\underset{p}{{}^{p}A^{p}}$$

The symbol representing a pair of planes of any particular modification belonging to class *d* would be

$$(B_{p}\ B'_{q}\ B''_{r} : B_{p}\ B'_{r}\ B''_{q})$$

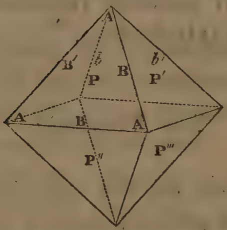
The edges of this primary form are neither perpendicular nor horizontal, and the decrements by which they become modified might therefore be expressed by the symbols which represent the modifications upon either the terminal or lateral edges of prisms. But as the edges of the tetrahedron are more analagous to the lateral, than to the terminal edges of prisms, the symbol ${}^{p}B^{p}$ will be used to denote the modifying planes belonging to class *f*.

The Octahedrons.

The laws of decrement which produce the modifying planes of the octahedrons, are, according to the Abbé Haüy's theory, supposed to take place on parallelopipeds, which would be formed by adding two tetrahedrons to two opposite planes of the octahedron. In the appendix to this treatise, rules will be given for determining directly the laws of decrement on the octahedrons, independently of these added tetrahedrons. And the symbols representing the secondary planes will therefore vary from those adopted by the Abbé Haüy.

The regular Octahedron.

Fig. 305.



The simple and mixed decrements on the angles, which would produce the planes belonging to class *b*, may be represented by the symbol ${}^{p}\overset{p}{A}{}^{p}$; which implies that similar planes occur on the three adjacent angles, and consequently on the fourth.

The intermediary decrements are of two kinds,

1. Those which produce the planes comprehended under class *c*.

   The general symbol to represent these would be $(B_p\ B'_q\ b'_q\ b_r)$.

2. Those which produce the planes comprehended under class *d*.

   The general symbol to represent these would be $(B_p\ B'_q\ b'_r\ b_s : B_p\ B'_r\ b'_q\ b_s)$.

These symbols denote the abstraction of *p*, *q*, *r*, or *s*, molecules from some of the edges B, B′, b′, and b, in the production of the planes belonging to classes *c*, or *d*.

The Rhombic Dodecahedron.

The modifications on the edges of this primary form may be denoted by the general symbol ${}^{p}B^{p}$.

Those on the *acute* solid angles may be represented by symbols analogous in character to those used for the octahedron.

The modifications on the *obtuse* solid angles may be represented by symbols resembling in character those used for the tetrahedron.

The Rhomboid.

The modification on the superior edges may be represented by the general symbol $^{p}B^{p}$, and those on the inferior edges by the general symbol $^{p}D^{p}$.

The modifications *a*, *b*, *c*, *d*, may be represented by symbols of the same character as those adopted for the tetrahedron.

The remaining modifications, on the lateral solid angles, may very obviously be conceived to result from decrements upon either the angles at E, or the angle at O. For if we refer to any of the modications from *e* to *k*, we may perceive that the planes which replace the angles at E, are similar to those which replace the angle at O, but are in an inverted position. It is therefore indifferent, as far as the representative character of the symbol is concerned, whether we refer the planes belonging to any of those modifications, to the angle at E or at O. In either case, the symbols will be similar in character to those which relate to the solid angles of some other parallelopipeds.

In the following tables, both these classes of symbols will be again alluded to, and their differences pointed out in reference to several of the classes of modifications.

In the examples which we have given of the application of symbols to represent the secondary forms of crystals, we have supposed those forms to have been strictly symmetrical, or to have resulted from similar modifications on all the similar edges or angles of the primary form.

It remains now to point out the methods of distinguishing by appropriate symbols, those secondary forms in which similar planes do not occur on all the similar primary edges and angles.

1st. Let us consider the case where an angle or edge is only partially modified. See *cube*, fig. 303, p. 243.

In class *i* of the modifications of the cube, the solid angles are replaced by only three planes, which are found to correspond with the *alternate* planes of class *d*.

But the symbol representing class *d*, is such as to imply that there are *three pairs* of planes on each solid angle. We should therefore construct the symbol which is to represent class *i*, so as to indicate the existence of only *three single* planes on each solid angle; and it should denote the relative positions of the analagous planes on the solid angles at A′, and at A.

The three planes at A′ may be represented by the following symbol, in which q is supposed to be greater than r;

$$(B'_q\ B_p\ B''_r)(B''_q\ B'_p\ B_r)(B'_r\ B''_p\ B_q).$$

And the three planes at A by the following;

$$(B_q\ B'''_p\ B'_r)(B'''_q\ B'_p\ B_r)(B'_q\ B_p\ B'''_r).$$

If these symbols be attentively regarded, they will be observed to express the relative positions of the corresponding planes on each of the solid angles. And by substituting in them b and b′ for B and B′, the planes on the lower solid angles might also be represented, and thus the entire figure would be implied.

The planes belonging to class *k*, which may be said to modify the edges partially, may be thus denoted.

$$\overset{p}{\mathrm{B}}\ \overset{\frac{1}{p}}{\mathrm{B}}{}'\ \mathrm{B}''^{p}\ {}^{p}\mathrm{B}'''.$$

The index not being repeated below, and on both sides of the letters B, *&c. affords an indication that the planes are single upon each edge*; in which respect only, does class *k* differ from class *f*.

2. Let us suppose some angles or edges of a crystal to be modified, while others, which are similar, remain entire.

From what has preceded, it will be apparent, that the character representing these differences may be generally conferred on the symbol, by introducing into it the letters which denote the unmodified edges or angles, and by substituting cyphers in appropriate positions near those letters, for the indices of the symmetrical modifying planes.

An example derived from the defective modifications of the cube, will sufficiently illustrate the characters of these particular forms of symbols; and in the following tables they will be further explained in reference to the different classes of primary forms in which irregular secondary forms occur.

The angles of the lower plane of the cube, corresponding with those marked A and A′ of the upper plane, are, as it has been already stated, supposed to be denoted by a and a′. The planes belonging to class *h* may be represented by the following symbol.

$${}^{p}\overset{p}{\mathrm{A}}{}^{p}\ \ {}^{o}\overset{o}{\mathrm{A}}{}'^{o}\ \ {}^{p}\overset{p}{\mathrm{a}}{}'^{p}\ \ {}^{o}\overset{o}{\mathrm{a}}{}^{o}.$$

This implies the occurrence of the modifying planes on the alternate solid angles only.

If, as it sometimes happens, one of the terminal solid angles of a rhomboid is replaced by a tangent plane, while the other remains entire, the symbol representing the change of figure would be

$${}^{1}\underset{1}{A}{}^{1}\ \ {}^{0}\underset{0}{a}{}^{0}.$$

3. When different decrements take place on two similar angles or edges, the number expressing the law of one of them may be represented by *p*, and the number representing the law of the other, may be substituted for the *o*, by which we have proposed to denote the unmodified angles or edges. Thus, if one terminal solid angle of a rhomboid were replaced by a tangent plane, and the other by three planes belonging to class *b*, resulting from a decrement by two rows in breadth, the symbol would become

$${}^{1}\underset{1}{A}{}^{1}\ \ {}^{2}\underset{2}{a}{}^{2}.$$

It will be convenient when we describe the secondary forms of crystals by means of these symbols, to observe some certain order in their arrangement into what may be termed the theoretical image of the crystal. The Abbé Haüy places the symbols representing the lateral edges of prisms, first; then those which represent the terminal edges; and lastly, those which represent the solid angles. As it is evidently indifferent whether they be taken in this order or in any other, as far as their descriptive character is concerned, I shall observe the same order of arrangement that he has given; although if that had not been established, I should have reversed it, for the sake of conformity with the order in which the modifications are placed in the tables.

The Abbé Haüy has also proposed to designate the secondary planes by small letters, and to place these under the respective symbols of the planes they refer to. And in order to render the character of the symbol more complete, he repeats the letters which designate the primary planes, among those which denote the secondary ones, as in the following example.

Thus, if a *right rhombic prism* should be found containing the modifying planes belonging to the classes *a*, *c*, *e*, *h*, and *g*, and also containing the primary planes, the representative symbol might be this.

$$\begin{array}{ccccccc} {}^{1}\mathrm{G}^{1} & \mathrm{M} & {}^{2}\mathrm{H}^{2} & \overset{1}{\mathrm{B}} & \overset{2}{\mathrm{E}} & \overset{\frac{3}{2}}{\mathrm{A}} & \mathrm{P} \\ \mathrm{h} & \mathrm{M} & \mathrm{g} & \mathrm{e} & \mathrm{c} & \mathrm{a} & \mathrm{P} \end{array}$$

Here the laws of decrement producing the secondary planes are represented by the upper series of characters; and the lower series consists of the letters which are placed on the figure of the crystal, to distinguish the secondary planes.

ON THE RELATION OF THE LAWS OF DECREMENT TO THE DIFFERENT CLASSES OF MODIFICATIONS.

The following tables, which exhibit these relations, will illustrate fully the uses of the symbols we have just described.

It may be remarked, that the letters placed on the figures of the primary forms contained in these tables, differ from those which stand on the corresponding primary forms placed at the heads of the several tables of modifications; and that some new letters have also been added to them.

These changes have been introduced for the purpose of indicating more explicitly, by means of symbols, the changes of figure which any modified primary form may have undergone.

The letters p, q, r, s, are used as the *general indices* of the different classes of modifying planes, and thus represent any numbers whatever; but they will represent different numbers, in relation to the different *individual planes* belonging to each of the classes.

When these letters are used as the indices of the modifying planes produced by intermediary decrements, they represent whole numbers only. But when p denotes the law of a simple or mixed decrement, it may represent any whole number or fraction.

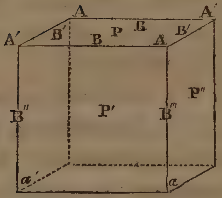
This whole number or fraction may be either greater than 1, which may be thus denoted $p > 1$;
equal to 1, ... $p = 1$;
or less than 1, ... $p < 1$.

The mark $>$ signifying *greater than*,
$=$ . . . . *equal to*,
$<$ . . . . *less than*.

The letter p will generally be used to denote the greater edge of the defect, the letter q the next, and r the least, when there are only three edges to be denoted. But when the symbol represents a modification on the solid angle of an octahedron, s is introduced to denote the fourth and least edge of the defect. Hence the relative values of p, q, r, and s, may be thus expressed, $p > q > r > s$.

The Cube.

Fig. 306.



1. *Symmetrical modifications.*

Simple and mixed decrements on the angles, produce the planes belonging to modifications a, b, and c.

The general symbol to represent these is

$${}^{p}\overset{p}{A}{}^{p}.$$

If $p > 1$, *class b* is represented, and as the value of p increases, the planes b incline more and more on the primary planes.

$p = 1$, *mod. a* is represented.

$p < 1$, *class c* is represented, and as the value of p diminishes, the planes c incline more and more on the primary edges.

Intermediary decrements produce the planes belonging to *class d.*

The general symbol representing these, is

$$(B_p \; B'_q \; B'''_r : B_q \; B'_p \; B'''_r).$$

Decrements on the edges, produce the planes belonging to the *classes e* and *f.*

The general symbol representing these is

$$\overset{p}{\underset{p}{B}}$$

If $p = 1$, *class e* is represented.

$p > 1$, *class f* is represented.

2. *Modifications not strictly conformable with the law of symmetry*; *or, such as have been termed defective modifications.*

The following are the symbols representing these classes.

${}^{0}\overset{0}{A}'^{0} \; {}^{1}\overset{1}{A}^{1} \; {}^{1}\overset{1}{a}'^{1} \; {}^{0}\overset{0}{a}^{0}$ represents *class g.*

${}^{0}\overset{0}{A}'^{0} \; {}^{p}\overset{p}{A}^{p} \; {}^{p}\overset{p}{a}'^{p} \; {}^{0}\overset{0}{a}^{0}$ . . . . *class h.*

$(B'_q \; B_p \; B''_r)(B''_q \; B'_p \; B_r)(B'_r \; B''_p \; B_q)$ represents the planes at A' belonging to *class i.*

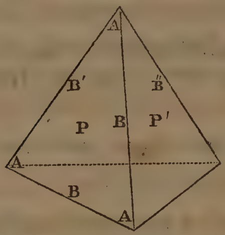
$(B_q \; B'''_p \; B'_r)(B'''_q \; B'_p \; B_r)(B'_q \; B_p \; B'''_r)$ represents the planes at A belonging to the same class.

When the symbolic character is not accompanied by a figure of the crystal, both the preceding symbols should be given; but when there is a figure, it will be sufficient to use the second only.

$\overset{p}{B}\ \overset{\frac{1}{p}}{B'}\ B''^{p}\ {}^{p}B'''$ is the symbol representing *class k*.

The regular Tetrahedron.

Fig. 307.



Simple and mixed decrements on the angles.

General symbol ${}^{p}\underset{p}{A}^{p}$.

If $p > 1$, the symbol represents *class b*.
$p = 1$, ... *mod. a.*
$p < 1$, ... *class c.*

Intermediary decrements.

General symbol, $(B_p\ B'_q\ B''_r)(B_p\ B'_r\ B''_q)$ represents *class d.*

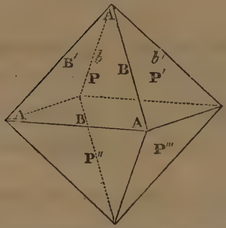
Decrements on the edges.

General symbol, ${}^{p}B^{p}$.

If $p = 1$, the symbol represents *mod. e.*
$p > 1$, ... *class f.*

The regular Octahedron.

Fig. 308.



Simple and mixed decrements on the angles.

General symbol, ${}^{p}\underset{p}{A}{}^{p}$.

If $p = 1$, the symbol represents *mod. a.*
$p < 1$, ... *class b.*

Intermediary decrements

Are of two kinds, and require two general symbols.
1st. $(B_p\ B'_q\ b'_q\ b_r)$ represents *class c*;
2d. $(B_p\, B'_q\, b'_r\, b_s : B_p\, B'_r\, b'_q\, b_s)$ represents *class d.*

Decrements on the edges.

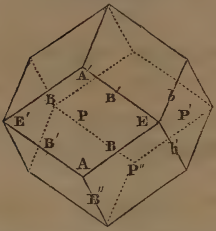
General symbol, ${}^{p}B^{p}$.

If $p = 1$, the symbol represents *mod. e.*
$p > 1$, ... *class f.*

The planes belonging to classes *a* and *k* of the cube, sometimes occur on the same crystal, and when the planes *a* are much enlarged, the secondary form presents the figure of the octahedron modified by two only of the planes *c* of that figure. The secondary crystal may however be referred properly to the cube, so long as it retains any portion of the planes *k*.

The rhombic Dodecahedron.

Fig. 309.



Simple and mixed decrements on the obtuse solid angles.

General symbol $\overset{p}{{}^{p}A^{p}}$, represents the classes *e*, *f*, & *g*.

If $p > 1$, the symbol represents *class f*.

$p = 1$, ... *mod. e.*

$p < 1$, ... *class g.*

Intermediary decrements on the *obtuse* solid angles produce the planes of *class h*, which class may be generally represented thus :

$$(B'_p\ B_q\ B''_r : B'_q\ B_p\ B''_r).$$

Simple and mixed decrements on the *acute* solid angles, produce the planes belonging to classes *a* and *b*.

General symbol $\underset{p}{\overset{p}{{}^{p}E^{p}}}$.

If $p = 1$, the symbol represents *mod. a.*

$p > 1$, ... *class b.*

Intermediary decrements on the *acute* solid angle, consist of two kinds, producing the planes of classes *c* and *d*.

The general symbol representing *class c*, is

$$(B_p\ B'_q\ b'_q\ b_r).$$

The general symbol representing *class d*, is

$$(B_p\ B'_q\ b'_r\ b_s : B_p\ B'_r\ b'_q\ b_s).$$

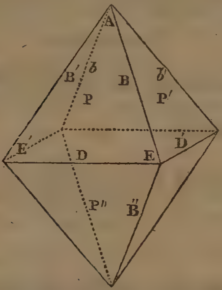
Decrements on the edges, may be represented by the general symbol ${}^{p}B^{p}$,

If $p = 1$, the symbol represents *mod. i.*
$p > 1$. . . . . *class k.*

Some of the secondary crystals of Blende are produced by *defective* modifications of this primary form, and are such as might result from regular modifications of the tetrahedron.

The Octahedron with a square base.

Fig. 310.



Simple and mixed decrements on the terminal edges.

General symbol, ${}^{p}\underset{p}{A}{}^{p}$.

If $p = 1$, the symbol represents *mod. a.*
$p > 1$, ... *class b.*

Intermediary decrements on the terminal solid angles.

These are of two kinds, and require two general symbols.

1st. $(B_p\ B'_q\ b'_q\ b_r)$ represents *class c*;
2d. $(B_p\ B'_q\ b'_r\ b_s : B_p\ B'_r\ b'_q\ b_s)$ represents *class d.*

Simple or mixed decrements on the lateral angles.

General symbol ${}^{p}E^{p}$.

If $p = 1$, the symbol represents *class e.*
$p > 1$, ... *class h.*

Intermediary decrements on the lateral solid angles.

These are of three kinds, and require three general symbols.

1st. $(B_p\ D_q\ D'_q\ B''_r)$ represents *class f.*
2d. $(D_p\ B_q\ B''_q\ D'_r)$ . . . . . . *class g.*
3d. $(B_p\ D_q\ D'_r\ B''_s : B_p\ D_r\ D'_q\ B''_s)$ represents *class i* when $p > q$;
The same symbol represents *class k* when $p < q$.

Decrements on the edges of the pyramids.

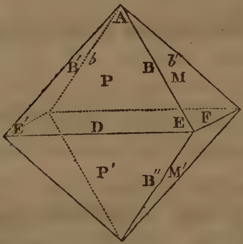
General symbol ${}^pB^p$.
If $p = 1$, the symbol represents *mod. l.*
$p > 1$, ... *class m.*

Decrements on the edges of the base.

General symbol $\overset{p}{\underset{p}{D}}$
If $p = 1$, the symbol represents *mod. n.*
$p > 1$, ... *class o.*

The Octahedron with a rectangular base.

Fig. 311.



Simple and mixed decrements on the terminal angles of the planes P.

General symbol $\overset{p}{\underset{p}{P}}$.
If $p = 1$, the symbol represents *mod. a.*
$p > 1$, ... *class b.*

Simple and mixed decrements on the terminal angles of planes M.

General symbol $^{p}A^{p}$.

If $p = 1$, *mod. a* is again represented, because plane *a* results from a decrement by one row on all the terminal angles.

$p > 1$, *class c* is represented.

Intermediary decrements on the terminal solid angles.

General symbol ($B_p\ B'_q\ b'_r\ b_s$) represents *class d.*

Simple and mixed decrements on the lateral angles of the plane P.

General symbol $E'^{p}\ {}^{p}E$.

If $p = 1$, the symbol represents *mod. e.*

$p > 1$, ... *class h.*

Simple and mixed decrements on the lateral angles of the plane M.

General symbol $^{p}E'\ E^{p}$.

If $p = 1$, *mod. e* is again represented.

$p > 1$, *class i* is represented.

Intermediary decrements on the lateral solid angles.

These are of two kinds.

1. Such as produce the single planes on each angle, which are comprehended under *class g*, or *class f.*

   The general symbol is ($D_p\ B_q\ B''_q\ F_r$).

   If $p > r$, the symbol represents *class f.*

   $p < r$, ... *class g.*

2. Such as produce two planes on each angle belonging to *class k.*

   General symbol ($D_p\ B_q\ B''_r\ F_s$.)

In this symbol, the particular values of either of the indices may be greater or less than either of the others, in reference to particular modifying planes.

Decrements on the terminal edges.

The planes produced by these decrements are all comprehended under *class l*, although they may be said to consist of three varieties.

1st. When the decrements proceed along the plane P.

2d. When the edge at which the planes *l* intersect each other at the base, is parallel to a diagonal of that base.

3d. When the decrements proceed along the plane M.

The general symbol of the 1st, is $B^{\prime p}\ {}^{p}B$.

2d, — ${}^{1}B^{1}$.

3d, — ${}^{p}B^{\prime}\ B^{p}$.

Decrements upon the edge D *of the base.*

General symbol $\overset{p}{\underset{p}{D}}$.

If $p = 1$, the symbol represents *mod. m.*

$p > 1$, ... *class n.*

Decrements upon the edge F *of the base.*

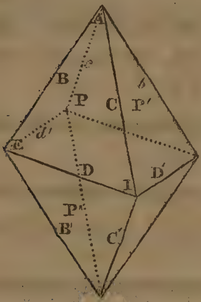
General symbol $\overset{p}{\underset{p}{F}}$.

If $p = 1$, the symbol represents *mod. o.*

$p > 1$, ... *class p.*

The Octahedron with a rhombic base.

Fig. 312.



Simple and mixed decrements on the terminal angles.

General symbol ${}^{p}\underset{p}{A}{}^{p}$.

If $p = 1$, the symbol represents *mod. a.*
$p > 1$, ... *class d.*

Intermediary decrements on the terminal solid angles.

These are of four kinds, and require four general symbols.

We suppose the edges of the upper pyramid, which are opposite to those marked with B and C, to be denoted by b and c.

Class b is represented by $(C_p\ B_q\ b_p\ c_r)$.
Class c . . . . . . $(B_p\ C_q\ c_q\ b_r)$.
Class e . . . $(C_p\ B_q\ b_r\ c_s : C_p\ B_r\ b_q\ c_s)$.
Class f . . . $(B_p\ C_q\ c_r\ b_s : B_p\ C_r\ c_q\ b_s)$.

Simple and mixed decrements on the angle E *at the base.*

General symbol ${}^{p}E^{p}$.

If $p = 1$, the symbol represents *mod. n.*
$p > 1$, ... *class q.*

Intermediary decrements on the acute solid angle at E.

These are of four kinds, and require four general symbols.

Class o is represented by $(B_p\ d'_q\ D_q\ B'_r)$.
Class p . . . . $(D_p\ B'_q\ B_q\ d'_r)$.
Class r . . $(B_p\ D_q\ d'_r\ B'_s : B_p\ d'_q\ D_r\ B'_s)$.
Class s . . $(D_p\ B'_q\ B_r\ d'_s : D_p\ B'_r\ B_q\ d'_s)$.

Simple and mixed decrements on the angle I *at the base.*

General symbol ${}^pI^p$.
If $p = 1$, the symbol represents *mod. g*.
$p > 1$, ... *class k*.

Intermediary decrements on the obtuse solid angle at I.

These are of four kinds, and require four general symbols.

Class h is represented by $(C_p\ D_q\ D'_q\ C'_r)$.
Class i . . . . $(D_p\ C_q\ C'_q\ D'_r)$.
Class l . . $(C_p\ D_q\ D'_r\ C'_s : C_p\ D'_q\ D_r\ C'_s)$.
Class m . . $(D_p\ C_q\ C'_r\ D'_s : D_p\ C'_q\ C_r\ D'_s)$.

Decrements on the acute terminal edges.

General symbol ${}^pB^p$.
If $p = 1$, the symbol represents *mod. v*.
$p > 1$, ... *class x*.

Decrements on the obtuse terminal edges.

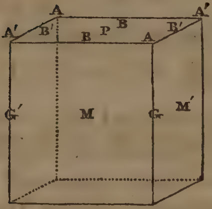
General symbol ${}^pC^p$.
If $p = 1$, the symbol represents *mod. t*.
$p > 1$, ... *class u*.

Decrements on the edges of the base.

General symbol $\overset{p}{\underset{p}{D}}$.
If $p = 1$, the symbol represents *mod. y*.
$p > 1$, ... *class z*.

The right Square Prism.

Fig. 313.



Simple and mixed decrements on the terminal angles.

General symbol $\overset{p}{A}$, represents *class a* generally.
In this symbol p may be > 1.
or $= 1$.
or < 1.

If $p = 1$, an individual plane belonging to the class is represented, whose three edges would be respectively parallel to the diagonals of the adjacent primary planes. This plane may, from its station in the series, be denominated the *middle plane*.

$p > 1$, the planes represented would incline more on P than the *middle plane* does.

$p < 1$, the planes represented would incline more on the edge G.

Simple and mixed decrements on the lateral angles.

General symbol $^{p}A^{p}$, p being > 1.

This symbol represents a series of planes belonging to class b, whose intersections with the planes M and M', are parallel to the diagonals of those planes.

Intermediary decrements.

General symbol $(B_p\ G_q\ B'_r : B_r\ G_q\ B'_p)$, represents the remainder of the series of planes belonging to *class b*.

Decrements on the terminal edges.

General symbol $\overset{p}{B}$.

If $p = 1$, an individual plane belonging to *class c* is represented, which may be termed the *middle plane* of the series comprehended under that class; and it would intersect the lateral planes in lines parallel to one of their *diagonals*.

$p > 1$, the symbol would represent that part of the series of *class c*, which inclines more on the terminal plane than the *middle plane* does.

$p < 1$, the same symbol would represent that part of the series which inclines more on the lateral plane than the *middle plane* does.

Decrements on the lateral edges.

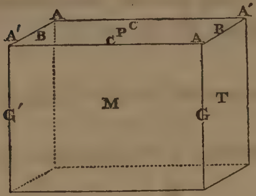
General symbol ${}^{p}G^{p}$.

If $p = 1$, *mod. d* is represented.

$p > 1$, *class e* is represented.

The right Rectangular Prism.

Fig. 314.



Decrements on the angles.

The planes belonging to *class a*, comprehend the following varieties.

1st. Those which result from *simple and mixed decrements* on the

terminal angles, of which the general symbols is $\overset{p}{A}$.
angles of plane M, ... ^{p}A.
angles of plane T, ... A^{p}.

If $p = 1$, in *either* of these symbols, the *same individual plane* belonging to the class is represented by each; the three edges of which are, respectively, parallel to the diagonals of the planes P M and T. This may be termed the *middle* plane of the series.

$p > 1$, the planes represented will incline on the plane P, or M, or T, more than the *middle* plane does.

$p < 1$, the plane represented will incline less on the respective primary planes than the *middle* plane does.

2d. Those which result from *intermediary decrements*, which may be represented by this general symbol,

$$(C_p\ G_q\ B_r).$$

in which p, q, and r, will vary relatively to each other as the decrements proceed along the plane P, M, or T.

Decrements on the terminal edges.

General symbols, $\overset{p}{C}$ represents *class b.*

$\overset{p}{B}$ . . . *class c.*

If $p = 1$, in either of these symbols, a *middle* plane will be represented belonging to each class respectively. And the planes of each class would respectively incline more on the terminal, or on the lateral plane, than its corresponding *middle* plane does, as p is > 1, or < 1.

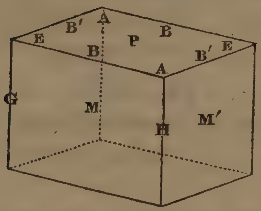
Decrements on the lateral edges, producing the planes of class d.

These are of three kinds, and require three general symbols.

1. $G^{\prime p}\ {}^{p}G$, when the decrement proceeds along the plane M.
2. ${}^{p}G'\ G^{p}$, when the decrement proceeds along the plane T.
3. When $p = 1$, in either of the two preceding symbols, the *middle* plane of the series will be represented.

The right Rhombic Prism.

Fig. 315.



Simple and mixed decrements on the acute terminal angles.

General symbol $\overset{p}{E}$, represents *class c*, generally.

If $p = 1$, the symbol represents the *middle* plane of the series.

$p > 1$. or < 1, the planes represented incline more on plane P, or on the edge G, than the *middle* plane does.

Simple and mixed decrements on the obtuse terminal angles.

General symbol $\overset{p}{A}$ represents *class a* generally.

$\overset{1}{A}$, represents the middle plane.

Simple and mixed decrements on the lateral angles.

1. On those adjacent to E.
   General symbol ${}^{p}E^{p}$, represents one series of planes belonging to *class d*, which intersect the lateral planes parallel to one of their diagonals.
2. On those adjacent to A,
   General symbol ${}^{p}A^{p}$, represents a similar series of planes belonging to *class b*.

Intermediary decrements on the acute and obtuse solid angles.

General symbol $(B'_p\ G_q\ B_r : B'_r\ G_p\ B_p)$ represents a further series of planes belonging to *class d.*

. . . . $(B_p\ H_q\ B'_r : B_r\ B_q\ B'_p)$ represents a further series of planes belonging to *class b.*

Decrements on the terminal edges.

General symbol $\overset{p}{B}$, represents *class e*, generally.

$\overset{1}{B}$, represents its *middle* plane.

Decrements on the lateral edges.

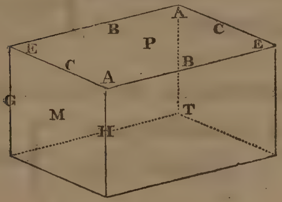
1. On the acute edges.
   ${}^1G^1$ represents *mod. h.*
   ${}^pG^p$ . . . *class i.*

2. On the obtuse edges.
   ${}^1H^1$ represents *mod. f.*
   ${}^pH^p$ . . . *class g.*

The exposition which has been given, in reference to the preceding classes of primary forms, of the relations of the laws of decrement to the several classes of modifications, will, it is presumed, have been sufficiently full, to render more than an outline of those relations unnecessary, in reference to the classes of primary forms which are to follow.

The right Oblique-angled Prism.

Fig. 316.



Decrements on the acute solid angles, are all comprised within class b.

1. *Simple and mixed.*

$\overset{1}{E}$ represents the *middle* plane.
$\overset{p}{E}$ . . . the planes which intersect the plane P parallel to a diagonal.
${}^{p}E$ . . . the planes which in the same manner intersect the plane T.
E^{p} . . . . M.

2. *Intermediary.* General symbol $(B_{p}\ G_{r}\ C_{q})$.

Decrements on the obtuse solid angles are all comprised within class a.

The symbols representing the planes corresponding in character with those above described, are,

$\overset{1}{A}$.
$\overset{p}{A}$.
${}^{p}A$.
A^{p}.
$(C_{p}\ H_{r}\ B_{q})$.

Decrements on the terminal edges.

$$\left.\begin{array}{l} \overset{1}{C},\ \text{symbol of middle plane} \\ \overset{p}{C},\ \text{general symbol}\ \ .\ \ . \end{array}\right\}\text{of } \textit{class d.}$$

$$\left.\begin{array}{l} \overset{1}{B},\ \text{symbol of middle plane} \\ \overset{p}{B},\ \text{general symbol}\ \ .\ \ . \end{array}\right\}\text{of } \textit{class c.}$$

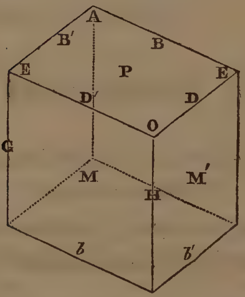
Decrements on the lateral edges.

$$\left.\begin{array}{l} {}^{1}G^{1}\ \text{symbol of middle plane} \\ \left.\begin{array}{l} {}^{p}G\ \ .\ \ .\ \ . \\ G^{p}\ \ .\ \ .\ \ . \end{array}\right\}\text{other planes} \end{array}\right\}\text{of } \textit{class f.}$$

$$\left.\begin{array}{l} {}^{1}H^{1}\ \ .\ \ .\ \ .\ \ \text{middle plane} \\ \left.\begin{array}{l} {}^{p}H\ \ .\ \ .\ \ . \\ H^{p}\ \ .\ \ .\ \ . \end{array}\right\}\text{other planes} \end{array}\right\}\text{of } \textit{class e.}$$

The Oblique Rhombic Prism.

Fig. 317.



Decrements on the acute solid angles.

1. *Simple and mixed* decrements on the angle *A*.

The general symbol $\overset{p}{A}$, represents *class c*.

$\overset{1}{A}$ represents the *middle* plane of that class.

${}^{p}A^{p}$ represents part of the series of *class d.*

2. *Intermediary.*

$(B'_p\ h_q\ B_r : B_p\ h_q\ B'_r)$ represents another part of the series of *class d*.

The corresponding decrements on the *obtuse* solid angles are,

$\overset{p}{O}$.

$\overset{1}{O}$.

$^pO^p$.

$(D'_p\ H_q\ D_r : D'_r\ H_q\ D_p)$.

Decrements on the lateral solid angles, are all comprised within *class e*.

1. *Simple and mixed.*

$\overset{1}{E}$ represents the *middle* plane of the class.

$\overset{p}{E}$ is the symbol, when the intersection of the planess *e* and P, is parallel to the oblique diagonal of P.

When the lateral planes are intersected by the planes *e*, parallel to a diagonal, the symbol will be either ^pE or E^p.

2. *Intermediary.* General symbol $(B_p\ G_q\ D_r)$.

In this symbol the comparative values of *p*, *q*, and *r*, will vary according to the positions of the planes represented.

Decrements on the acute terminal edges.

$\overset{1}{B}$ represents the middle plane of *class g*.

$\overset{p}{B}$ is the general symbol of that class.

Decrements on the obtuse terminal edges.

$\overset{1}{D}$ represents the middle plane of *class f.*

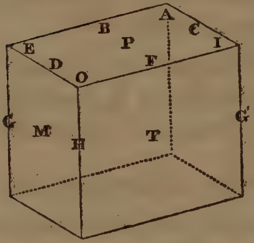
$\overset{p}{D}$ is the general symbol of that class.

Decrements on the edges of the prism.

1. On the lateral edges G.
   - ${}^{1}G^{1}$ represents *mod. k.*
   - ${}^{p}G^{p}$ . . . . . *class l.*
2. On the oblique edges H.
   - ${}^{1}H^{1}$ represents *mod. h.*
   - ${}^{p}H^{p}$ . . . . . *class i.*

The doubly Oblique Prism.

Fig. 318.



Decrements on the solid angles.

The planes comprehended under *class a*, may be represented by the following symbols.

$\overset{p}{O}$, when the decrement proceeds along the plane P.

${}^{p}O$ . . . . M.

O^{p} . . . . T.

$(D_{p}\ H_{q}\ F_{r})$ is intermediary.

The corresponding planes belonging to the other classes, may be represented as follows.

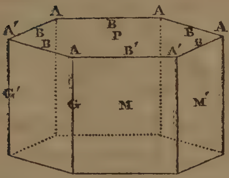
Class b, $\overset{p}{A}$.	*Class c*, $\overset{p}{E}$.	*Class d*, $\overset{p}{I}$.
${}^{p}A$.	${}^{p}E$.	${}^{p}I$.
A^{p}.	E^{p},	I^{p}.
$(B_p\ h_q\ C_r)$.	$(B_p\ G_q\ D_r)$.	$(F_p\ G'_q\ C_r)$.

Decrements on the edges may be expressed as follows.

Class e, $\overset{p}{B}$.	*Class g*, $\overset{p}{D}$.	*Class i*, ${}^{p}H$. H^{p}.
Class f, $\overset{p}{C}$.	*Class h*, $\overset{p}{F}$.	*Class k*, ${}^{p}G$. G^{p}.

Hexagonal Prism.

Fig. 319.



Decrements on the angles.

$\overset{q}{A}$ represents the *middle* plane of the series belonging to *class a*.

$\overset{p}{A}$ . . . the other planes belonging to that class.

${}^{p}A^{p}$ . . . those planes belonging to *class b*, whose edges intersect the planes M parallel to a diagonal.

$(B_p\, G_q\, B'_r : B_r\, G_q\, B'_p)$ represents those planes of *class b* which are produced by *intermediary decrements.*

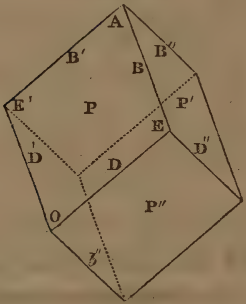
$\overset{p}{B}$ represents *class c.*
${}^{1}G^{1}$ . . . *mod. d.*
${}^{p}G^{p}$ . . . *class e.*

In some crystals of phosphate of lime, the planes belonging to *class b* occur singly. If they result from simple or mixed decrements, their symbol would be ${}^{o}A^{p}$ or ${}^{p}A^{o}$, according as they lie on the left or right of the modified angle. And if they are produced by intermediary decrements, their symbol might be

$$(B_o\, G_o\, B'_o : B_q\, G_p\, B'_r.)$$

The Rhomboid.

Fig. 320.



Simple and mixed decrements on the superior angles.

General symbol ${}^{p}\underset{p}{A}{}^{p}$.

If $p = 1$, the symbol represents *mod. a.*
$p > 1$, ... *class b.*
$p < 1$, ... *class c.*

Intermediary decrements on the superior angles.

$(B'_p\ B_q\ B''_r : B_p\ B'_p\ B''_r)$ represents *class d.*

The inferior *plane angle* at O, and the lateral *plane angle* at E, both belong to the *lateral solid angles*, all of which are *similar*, according to the definitions already given.

The planes modifying the angles at E are therefore similar to those modifying the angle at O, but are reversed in their position on the crystal. The laws of decrement producing both are consequently similar. But if we refer the decrements producing the planes belonging to any of the classes *e*, *f*, *g*, *h*, *i*, *k*, *l*, to *the solid angle at* E, the symbols representing them will differ from those which would represent the same planes, if we refer the decrement *to the solid angle* at O.

A single example will sufficiently illustrate this observation. Let us imagine the lateral solid angles of a rhomboid to be modified by two planes, which intersect the primary planes *parallel to their oblique diagonals*. If the decrement producing these planes were referred to the angle at E, it would appear as a simple or mixed decrement, and its symbol would be $E'^{p}\ {}^{p}E$. But if it be referred to the angle at O, it might be regarded either as a simple or mixed, or as an intermediary decrement, of which latter the symbol would be $(D'_p\ D_q\ b''_p : D_p\ D'_q\ b''_p)$. If we regard these symbols with a little attention, we shall perceive that the variation in their form, does not alter the identity of their character, which is derived from the parallelism of one edge of each of the secondary planes to an oblique diagonal of the primary. But this character is implied in the supposition of a simple or mixed decrement, which the first symbol represents; and it is directly indicated in each branch of the second symbol, by those indices which denote the abstraction of equal numbers of molecules in the direction of the edges $D' b''$, and $D b''$.

The Abbé Haüy has referred some of the planes which modify the lateral solid angles, to the angle at E, and others to the angle at O. It may therefore be convenient to possess the symbols representing those planes, in reference to both angles, and they will accordingly be given below. The symbol on the left represents the modification when the decrement is referred to the angle at O, and that on the right represents the same modification when the decrement is referred to the angle at E.

1. *Simple and mixed decrements on the lateral solid angles.*

1. Producing one plane on each solid angle.

General symbols.

In reference to angle C.	In reference to angle E.
$\overset{\frac{p}{q}}{O}$.	$(D_p\ B_q\ D''_p)$.

If $p = 2q$, in either of these symbols, *mod. e* is represented.

$p < 2q$, ... *class g.*
$p > 2q$, ... *class k.*

2. Producing two planes on each solid angle.

General symbols.

${}^{p}O^{p}$. or, $(D'_p\ b''_p\ D_q : D_p\ b''_p\ D'_q)$.	$E^p\ {}^pE$.

These symbols represent the series of planes belonging to *class h.*

2. *Intermediary decrements on the lateral solid angles.*

The general symbols to represent the modifications produced by these, are,

$(D'_p\ b''_r\ D_q\ :\ D'_q\ b''_r\ D_p)$. | $(D_p\ D''_q\ B_r\ :\ D_q\ D''_p\ B_r)$.

If $r = p$, the symbol represents the planes belonging to *class h*, as described above.

$r > p$, the planes of *class i* are represented.

$r < p$, the planes of *class l*, and *class f*, are represented.

Although these two classes are represented by a common symbol, there is this distinction between them; that the edge produced by the intersection of the planes belonging to class *f*, is always parallel to the vertical axis of the rhomboid; and that their indices *p*, *q*, and *r*, are in a constant ratio to each other, as will be shewn in the appendix; while the planes belonging to class *k* do not intersect each other parallel to the axis of the rhomboid, nor is there any constant ratio between their indices.

Decrements on the superior edges.

General symbol ${}^pB^p$.

If $p = 1$, the symbol represents *mod. m*.

$p > 1$, ... *class n*.

Decrements on the inferior edges.

General symbol ${}^pD^p$.

If $p = 1$, the symbol represents *mod. o*.

$p > 1$, ... *class p*.

It may be remarked, that several of the classes of modifications on the angles of some of the primary forms, comprise planes which are produced by very different laws of decrement. And it may possibly appear to some of my readers, that different classes ought to have been established for the planes produced by the several varieties of laws. But this would have rendered the tables of modifications less generally applicable to the description of secondary forms, independently of the theory of decrements, than they are at present. This will become very obvious if we refer to the classes *a*, *b*, *c*, or *d*, of the modifications of the doubly oblique prism. All that can be known of any individual plane belonging to either of these classes, independently of calculation, is that it belongs to such a class, and inclines on two of the adjacent primary planes at particular angles; and this enables us to record the particular plane.

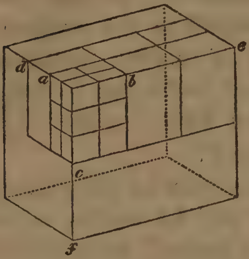
If we refer to p. 274, we may perceive that the planes belonging to either of those classes might be produced by four different kinds of decrement.

Let us suppose that we have observed a plane upon a doubly oblique prism produced by one of those decrements. As it replaces the solid angle O, we refer it without hesitation to our present class, *a*.

But if class *a* had been divided into four classes, we could not, without previous calculation, know to which of those the observed plane ought to be referred; and the measurement of the crystal would not in such case, enable us to describe the secondary form, by the assistance of the tables only.

The symbols used in this volume to represent planes produced by intermediary decrements, contain indices which are always whole numbers; whereas the symbols used by the Abbe Haüy to represent similar planes, frequently contain fractional indices.

Fig. 321.



There is, however, no real difference in the character conferred on the plane by the two methods of representing it.

The only difference between them consists in this; the indices used in this volume simply give the character of the compound molecule by whose continual abstraction the new plane is produced, while the Abbé Haüy's symbol supposes this molecule compounded of several other compound molecules.

This will be readily understood by a reference to the above figure, which we shall suppose a doubly oblique prism, with an intermediary decrement on the solid angle at O.

Let *d e c* represent a compound molecule consisting of three molecules in height, four in the direction *a d*, and six in the direction *b e*, and let us suppose this the molecule abstracted from the first plate superimposed on the terminal plane, and let us also suppose that two of these would be abstracted from the second plate, and so on, as explained in p. 22 and 23.

The edge *d a*, in the above figure, corresponds with the edge D of the primary form, *b e* with F, and *c f* with H.

The symbol we should use to represent the plane produced by this decrement, would be (D4 H3 F6), but the Abbé Haüy's symbol would be $(D2\ \overset{\frac{2}{3}}{O}\ F3)$, and would be understood to imply that the compound molecules abstracted in the production of the new plane, consisted of smaller compound ones, each of these being three molecules in height, and two in breadth, repeated twice on the edge D, and three times on the edge F.

From this exposition of the difference between the two symbols, it will be readily perceived that if in the Abbé Haüy's symbol, we substitute for the letter denoting the angle on which the decrement is conceived to take place, that which denotes the edge upon which the angle of the new plane may be said to rest, and place the number used by him to express the decrement in height, which is in this case the *denominator* of his fraction, after it as its proper index; and if we multiply at the same time his other indices by the number he uses to express the decrement in breadth, which is in this case the *numerator* of his fraction, the new symbol will be similar in character to those which are contained in this volume.

This method of converting the form of the one symbol into that of the other, may be considered general, and by reversing the process, the symbols given in the preceding pages, may be converted into the form of those which he has used.

APPENDIX.

APPENDIX.

CALCULATION OF THE LAWS OF DECREMENT.

In the preceding sections, some general rules have been given for determining the class of primary forms to which any given secondary crystal belongs, and for describing the secondary crystal by means of the position of its secondary planes, and of the angles at which those planes respectively incline on the primary.

The following is an outline of the method of applying the theory of decrements to determine the relations between the secondary and primary forms of crystals.*

The application of this theory will embrace the following problems.

First,—*To determine the law of decrement by which any secondary plane is produced, the elements of the primary form being known, and the angles at which the secondary plane inclines on the adjacent primary planes, being also known.*

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* The reader of this appendix is supposed to be acquainted with the elements of plane and spherical trigonometry, and with the use of the tables of logarithms.

Second,—*To determine the angles at which the secondary plane inclines upon the adjacent primary planes, the elements of the primary form, and the law of decrement by which the secondary plane is produced, being known.*

Or, to determine the particular values of the general indices given in the tables at p. 254, the inclination of the secondary planes to the primary being known; and to determine those inclinations when the indices are known.

The *elements* of the several classes of primary forms consist of

1st. The angles at which the primary planes incline to each other. These may be ascertained by means of the goniometer, if not already known.

2d. The plane angles of the primary planes. When these angles cannot be ascertained by other means, they may be deduced by spherical trigonometry, from the known inclination of the primary planes to each other.

3d. The comparative lengths of the primary edges, and of such other lines upon or within any crystal as may be required for facilitating our calculations of the laws of decrement, or for delineating its primary or any of its secondary forms. The methods of deducing such of these elements as cannot be ascertained by measurement of the crystal, will be described where the elements of the several classes of primary forms are described.

In the tables at p. 254, &c. the letter p is used to represent *any whole number, or fraction.* But it will be more convenient for our present purpose to represent *simple and mixed decrements by the general fractional index* $\frac{p}{q}$; *p expressing the decrements in breadth, and q the decrements in height.*

It has been already remarked, that one half the number of planes by which any crystal is bounded, are generally shewn in front of the engraved figure of that crystal. And as we know that the opposite angles, edges, and planes, which are supposed to form the back of the engraved figure, are respectively similar to those which appear on its front, if the decrements on these be described, the decrements on the hidden or back planes may be conceived to be described also. And again, as the law of symmetry requires that *all similar angles and edges* shall be *similarly* modified, if among the modified angles and edges, which are supposed to be in *front* of the figure, there be *two* or *more*, *similar* to each other, it is obviously sufficient to investigate the decrement upon *one* of these, in order to determine the character of the modifying planes upon the others.

Decrements, as we have already seen, take place on the edges or angles of crystals, and are of two principal kinds; one of which produces planes intersecting the primary planes, in lines, of which one at least, is parallel to an edge or diagonal of one of those planes; and the planes produced by the other intersecting the primary planes in lines, not any of which are parallel to an edge or diagonal of any of those planes.

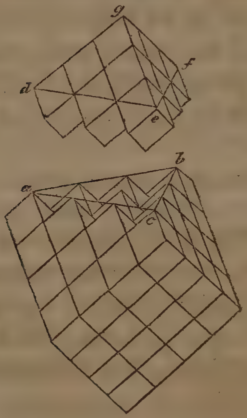
The effect of both these classes of decrements upon the primary form, is similar to that which would take place, if we conceive the enlarged crystal to have been completed, and the whole of the omitted molecules to have been then removed from it in one mass.

This will be readily perceived, if we refer for an example to modification *a* of the rhomboid. Let us conceive a rhomboid of a given dimension to have been formed; and during its further increase in bulk, a row of molecules to have been abstracted at the angle A of the primary form, from the first plate of molecules added to the plane P, and an additional row to have been abstracted from each succeeding plate.

As the three plane angles which concur to produce the solid angle at A, are *similar*, a *similar abstraction* of molecules would take place simultaneously from the plates superimposed on *each* of the three adjacent planes, and the result would be the production of a tangent plane, presenting at its surface the terminal solid angles of the molecules belonging to those plates which had been added to the smaller rhomboid.*

Let us next suppose that instead of any abstraction of molecules from the superimposed plates, those plates had been added entire, and a perfect enlarged rhomboid had been produced.

Fig. 322.



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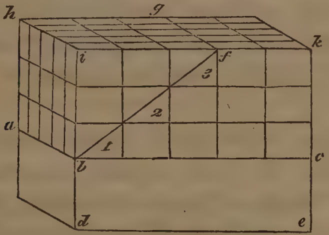
* It will be recollected that the molecules are so small, as to occasion no perceptible difference in the character of the secondary plane,

We may conceive it possible to reduce this entire rhomboid to the state of the modified one, by removing, in one mass, the triangular pyramid of molecules *d e f g*, fig. 322, in which the supposed modified crystal is deficient.

The mass of molecules, therefore, in which any secondary form is deficient, when compared with its primary form, is equal to the number of molecules abstracted in the production of that secondary form, arranged in the same order as they would have been, if they had completed the enlarged primary form.

This mass, so arranged, being all the addition to the secondary form which would be required to complete the primary, will be called the *defect of the primary form*, and it will be shewn presently *that the edges of this defect may be used in every instance, to determine the decrement by which the secondary plane is produced.*

Fig. 323.



For the purpose of illustrating this proposition further, let us observe the change which would have taken place, if a parallelopiped of any kind, either right or oblique, as *a b c d e*, fig. 323, had been modified on one of its edges, by a decrement consisting of a single row of molecules.

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whether that plane exposes the edges, solid angles, or planes of the molecules.

Let us suppose the edge *i k*, of the primary form, to be to the edge *i d*, in the ratio of *m* to *n*, *m* and *n* being any numbers whatsoever.

It follows from what has been before stated, that the ratios of the corresponding edges of the molecules which compose this form, will also be as *m* to *n*; and consequently that the primary edges are composed of equal numbers of edges of molecules, and may therefore be regarded as multiples of *m* and *n*, by some indefinite whole number.

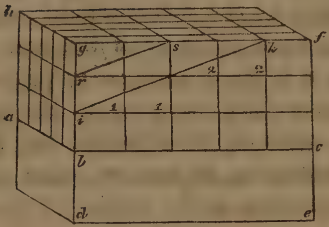
Let us further suppose that the decrement had begun to act at the edge *a b*, and had proceeded along the plane *a b c*.

In the first plate of molecules superimposed on that plane, the row 1 would have been omitted. In the second plate, the *additional* row 2. In the third plate, the additional row 3, and so on.

Now the evident result of these abstractions from the several superimposed plates, would have been the production of a new plane, *a b g f*, replacing the edge *h i*, of the enlarged crystal; and the triangular prism, whose base is the triangle *b i f*, represents the *defect of the primary form* occasioned by this decrement.

But it is obvious from the figure, that the ratio of the lines *i f* to *i b* of the *defect*, is as 3 *m* to 3 *n*, or as *m* to *n*. Hence when a decrement by 1 row of molecules takes place on the edge of any parallelopiped, the ratio of the edges of the *defect*, corresponding to *i f*, *i b*, is similar to the ratio of those edges of the primary form, of which these are respectively supposed to be portions. And as the edge *b f*, of the new plane, coincides with a diagonal of the molecules, it is evidently parallel to a diagonal of the plane *d i k e*.

Fig. 324.



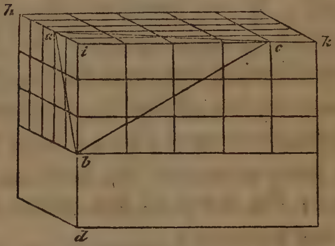
Let us now suppose a decrement by 2 rows in breadth to have taken place on the edge of a similar parallelopiped. If we imagine the first plate of molecules which is superimposed on the primary plane to be deficient in two rows of single molecules; and if we imagine two additional rows of molecules abstracted from the second plate, and so on, the plane *i k* would be produced, and the lines *i g*, *g k*, would be the edges of the *defect* of the primary form occasioned by this decrement.

But it is evident that the line *g k* is to the line *g i*, as 4 *m* is to 2 *n*, or as 2 *m* to *n*, this being the ratio which the number of molecules abstracted in the direction *g f*, bears to the number deficient in the direction of *g d*.

From these examples we find that whenever a decrement takes place on the edges of any parallelopiped, replacing that edge by a plane, the edges of the *defect* will be to those edges of the primary form, of which they are respectively parts, in the ratio of the numbers of molecules abstracted from each superimposed plate in the direction of the same edges respectively; and that such ratio will express the law of decrement by which the new plane has been produced.

We may perceive from the figures 323 and 324, that if we had conceived the new plane to be produced by the superposition of *a single plate* of molecules, the edges of the *defect* would still be in the same ratio to each other as if the new plane were produced by a *series of decreasing plates*. We may therefore express the character of this plane by the ratio of the edges of the *defect* of the first *plate* of molecules, consisting of one or more molecules in thickness, according to the nature of the decrement.

Fig. 325.



Let us derive another illustration of this proposition from a decrement on the angle of a parallelopiped; and, to render the example more general, let us suppose an intermediary decrement acting on that angle to have produced a plane *a b c*, fig. 325, by the abstraction of a *compound molecule*, consisting of three molecules in height, two in the direction of *i h*, and four in the direction of *i k*. If we suppose the lines *i k*, and *i d*, to be to each other in the ratio of *m* to *n*, and the line *i h*, to be as *o*, the corresponding edges of the compound molecules would consequently be in the same ratio, and the edges, *i c*, *i b*, *i a*, of the *defect*, would be as 4 *m*, 3 *n*, and 2 *o*, and would, when divided by *m*, *n*, and *o*, express the law of decrement by which the new plane is produced.

From these examples it appears that the *edges of the defect of the primary form are multiples of the corresponding edges of the molecules;* and the *ratios of the edges of the defect are consequently multiples of the ratios of the corresponding edges of the primary form.*

For let the ratio of *i k* : *id*, which is that of $m : n$, be represented by the fraction $\frac{m}{n}$ and the ratio of *i b* : *i c* being that of $4\,m : 3\,n$, be represented by the fraction $\frac{4\,m}{3\,n}$.

It is evident that the ratio $\frac{4\,m}{3\,n}$ is a multiple of $\frac{m}{n}$ by $\frac{4}{3}$.

Hence *the problem of ascertaining the law of decrement producing any secondary plane, is reduced to that of ascertaining the ratios of the edges of the defect of the primary form occasioned by such decrement,* and dividing these ratios by the ratios of the corresponding edges of the primary form.

We may also discover the law of decrement in some particular cases, by dividing the ratios of the edges of the defect, by the ratio of an edge to some other line upon the crystal.

Whatever ratio we may use for this purpose, will be termed *the unit of comparison.*

This *unit of comparison* is, generally, the *ratio of certain edges or other lines, either on the surface, or passing through the interior of crystals, of which, proportional parts would be intercepted by any new plane, resulting from a decrement by one row of molecules.*

According to the theory already explained, the molecules of all parallelopipeds are similar parallelopipeds, and their edges are consequently proportional to the corresponding edges of the primary form.

Hence, when through the operation of a decrement on an edge of any parallelopiped, a single row of molecules is abstracted, the parts which are removed from the two edges adjacent to that on which the decrement has taken place, will be proportional to those edges respectively; and the ratio of those edges may therefore constitute the *unit of comparison* for decrements on the edges of parallelopipeds.

If a decrement take place by one row on an angle of a parallelopiped, a single molecule is first abstracted from its solid angle; and the parts thus abstracted from the three edges which meet at the solid angle, are respectively proportional to those edges.

The ratios of the three adjacent edges of any parallelopiped, therefore, may be taken as the units of comparison for determining the various laws of decrement on the angles of that class of primary forms.

And, by analogy, we may take the ratios of the *three or four edges adjacent to the solid angles of any class of primary forms*, to express the ratios of the edges of the *defect* occasioned by a decrement by one row of molecules.

But other lines may be traced on some of the classes of primary forms, proportional parts of which will also be intercepted by decrements by one row of molecules.

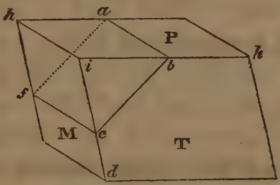
The ratios of these may therefore be taken *as the units of comparison*, if we find them more convenient for our calculations than those of the primary edges.

When the edges, or other lines, from which the unit of comparison is to be derived, are equal, their ratio will be = I, *and in this case the lowest whole numbers which will express the ratios of the edges of the defect of the primary form, will also express the law of decrement.*

And whenever an edge of any primary form is replaced by two similar secondary planes, as in mod. *f* of the cube or regular octahedron, or *g*, or *i*, of the right rhombic prism, &c. the lines whose ratio constitutes the unit of comparison in such cases, will always be equal.

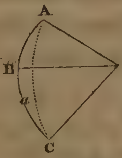
And the units of comparison for determining any law of intermediary decrement, will always be the ratios of the edges which meet at the solid angle on which the decrement has taken place.

Fig. 326.



To ascertain the ratio of *the edges of the defect of the primary form*, when a decrement takes place on an *edge* of any parallelopiped, fig. 326, we must suppose the inclination of the primary planes to each other to be known, and the inclination of the modifying plane *a b c f*, to the primary planes P and T.

Fig. 327.

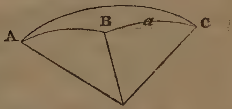


Now to determine the ratio which the line *i b*, bears to *i c*, we require the angles of the plane triangle *i b c*. These may be obtained by means of a spherical triangle, fig. 327, whose angle A is the supplement of the inclination of P on the plane *a b c f*, the angle B the inclination of P on T, and the angle C the supplement of the inclination of T on the plane *a b c f*.

From this spherical triangle we deduce the side *a*, containing the required angle *i b c*, by the known formula

$$\sin.\tfrac{1}{2}\,a = R\sqrt{\frac{-\cos.\tfrac{1}{2}(A+B+C).\cos.\tfrac{1}{2}(B+C-A)}{\sin.B.\ \sin.C}}$$

Fig. 328.



and by applying the same formula to a second spherical triangle, fig. 328, whose angle
C is similar to that of the preceding,
B is the inclination of M on T,
A the supplement of M on the plane *a b c f*,
derived from actual measurement, or deduced from the known inclination of P on the plane *a b c f*, and of P on M, we may again obtain the side *a*, which contains the other required angle, *i c b*.

Having thus determined the two plane angles, *i b c*, *i c b*, the ratio of *i b* to *i c* is known from the analogy between the sines of the angles of triangles, and the sides subtending those angles, thus,

$$i\,b : i\,c :: \sin.\vee\,i\,c\,b : \sin.\vee\,i\,b\,c.^{*}$$

Let us suppose *i k* : *i d* :: *m* : *n*; *m* and *n* being any whole numbers whatever, and being already known by means which will be pointed out in a later part of this appendix.

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* This mark $\vee$ is used to denote the word angle.

If the new plane has resulted from a decrement on the edge $h\,i$, by one row of molecules, the lines $i\,b$, $i\,c$, must also be to each other as m to n, and we should then have

$$\sin.\ \vee\, i\,c\,b : \sin.\ \vee\, i\,b\,c :: m : n.$$

But if the new plane has resulted from a decrement by unequal numbers of molecules in height and breadth, the ratio of $i\,b$ to $i\,c$, should be as $p\,m$ to $q\,n$; the letter p representing the number of molecules abstracted in the direction of the edge $i\,k$, and q representing the number abstracted in the direction of the edge $i\,d$.

The ratio of $p\,m : q\,n$, may be expressed by the fraction of $\frac{p\,m}{q\,n}$, which is evidently the product of $\frac{p}{q} \times \frac{m}{n}$. We may therefore obtain the values of p and q, whatever may be the particular values of $\frac{p\,m}{q\,n}$ and $\frac{m}{n}$, if we divide $\frac{p\,m}{q\,n}$, which expresses the ratio of the edges of the defect by $\frac{m}{n}$, which expresses the ratio of the corresponding edges of the molecules, or, which is the same thing, of the corresponding edges of the primary form.

There are two methods by which this division may be effected.

The *first* is by finding the absolute values of m and n, and of $p\,m$ and $q\,n$, by means of the tables of natural sines, &c. and then reducing those ratios to their lowest denominations in whole numbers; and after dividing the one fraction by the other, reducing the quotient to its lowest denomination in whole numbers. The quotient so reduced would express the law of decrement.

As an example of this method, let us suppose we have found

$$i\,k : i\,d :: 11 : 8$$

therefore $$\frac{m}{n} = \frac{11}{8}.$$

And let us suppose the ratio of

$$i\,b : i\,c :: \text{sin.} \vee i\,c\,b : \text{sin.} \vee i\,b\,c :: 33 : 16;$$

then we should have $\frac{p\,m}{q\,n} = \frac{33}{16}$.

If we divide the second fraction by the first, the quotient will be $\frac{33}{16} \times \frac{8}{11} = \frac{264}{176} = \frac{3}{2}$, which would give a law of decrement by three rows in breadth, or in the direction of $i\,k$, and two in height, or in the direction of the edge $i\,d$.

If we now suppose the edges $i\,k$, and $i\,f$, to be equal, it is evident that $\frac{m}{n}$ becomes equal to 1.

Under this supposition the ratio of $i\,b$ to $i\,c$ might be expressed by a fraction of the form $\frac{p}{q}$.

Let us now imagine the ratio of $i\,b : i\,c$ to have been found as $1 : 3$.

This would indicate a decrement proceeding along the terminal plane by 3 rows of molecules in height.

If we find $i\,b : i\,c :: 4 : 3$, the law of decrement producing the plane from which that ratio is deduced, is by 4 rows in breadth, and 3 in height, on the terminal plane.

The *second* method of dividing the proposed fraction $\frac{p\,m}{q\,n}$ by $\frac{m}{n}$ is by means of the logarithms of the quantities from whence those ratios are deduced.

Let us suppose we have found $i\,k : i\,d :: \mathrm{R} : \sin.\, a$, we should then have $\frac{m}{n} = \frac{\mathrm{R}}{\sin.\, a.}$

and . . . $\mathrm{Log.}\,\frac{m}{n} = \mathrm{Log.\,R} - \mathrm{Log.}\,\sin.\, a.$

Let us also suppose $i\,b : i\,c :: \sin. \surd i\,c\,b : \surd \sin.\, i\,b\,c$,

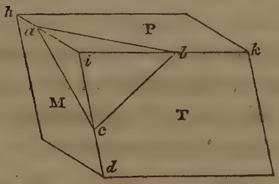
then . . $\frac{p\,m}{q\,n} = \frac{\sin. \surd\, i\,c\,b}{\sin. \surd\, i\,b\,c};$

and $\mathrm{Log.}\,\frac{p\,m}{q\,n} = \mathrm{Log.}\,\sin. \surd i\,c\,b - \mathrm{Log.}\,\sin. \surd i\,b\,c.$

The division of $\frac{p\,m}{q\,n}$ by $\frac{m}{n}$ is effected by subtracting the logarithm of the latter fraction from that of the former. And the natural decimal number corresponding to the resulting logarithm, will bear the same ratio to 1·0, 1·00, 1·000, &c. according to the number of decimal planes in the number found, as the decrement in breadth bears to that in height.

Examples of the application of this method of deducing the values of p and q, will occur in the course of this appendix.

Fig. 329.



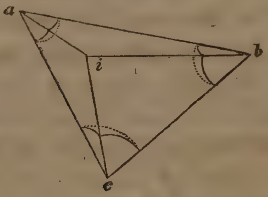
Let us now enquire how we may determine the ratios of the three edges, $i\,b$, $i\,c$, $i\,a$, fig. 329, of the *defect*, occasioned by a decrement on one of the angles of a parallelopiped.

We are supposed to know the inclination of the primary planes to each other, and let

P on M be called I_1
P — T . . . . I_2
M — T . . . . I_3

We may from these readily deduce the plane angles at i by means of a spherical triangle; and having measured the inclination of the plane $a\,b\,c$ on P, M, and T, we may discover the plane angles at a, b, and c, by means of the three spherical triangles marked on fig. 330.

Fig. 330.



In these triangles we known only the angles, which are those at which the primary planes incline to each other, and the supplements of those at which the secondary plane inclines on the adjacent primary planes.

The plane angles at a, b, and c, being found, we may readily discover the ratios of $i\,b : i\,c$, and $i\,b : i\,a$, which will give the law of decrement by which this modifying plane has been produced.

Let us still suppose, $i\,k : i\,d :: m : n$,
and $i\,k : i\,h :: m : o$,

our units of comparison here would be $\frac{m}{n}$ and $\frac{m}{o}$

and let $i\,b : i\,c :: p\,m : q\,n$
$i\,b : i\,a :: p\,m : r\,o$.

After effecting our division of $\frac{p\,m}{q\,n}$ by $\frac{m}{n}$, and of $\frac{p\,m}{r\,o}$ by $\frac{m}{o}$, we should find

$$i\,b = p,$$
$$i\,c = q,$$
$$i\,a = r,$$

which would imply a decrement by p molecules in the direction of $i\,k$, q molecules in that of $i\,d$, and r molecules in the direction of $i\,h$.

If we suppose fig. 329 to be a doubly oblique prism, the letter to denote the edge

$i\,h$ would be D,
$i\,k$ . . . F,
$i\,d$ . . . H,

and the symbol of the plane $a\,b\,c$, would then be

$$(D_r\ E_p\ H_q).$$

It may be remarked here, that tangent planes are generally the result of a decrement by 1 row of molecules, whether they replace the angles or edges of those classes of the primary forms in which they occur.

By this general method of proceeding we may, when we know the inclination of the primary planes to each other, and of the secondary plane on one or more of the primary, discover the law of decrement by which any secondary plane has been produced on any of the classes of parallelopipeds; and it may be adapted also to all the other classes of primary forms.

We shall now apply it to the several classes of those forms in succession, and the calculation will be found to become much more simple in its application to many of those classes.

As it will not be necessary to repeat even the formulæ in all the cases which are to follow, it may not be useless again to observe that when a law of decrement producing any plane is to be determined, the general symbol of that plane is to be first discovered, and then the particular values of its indices to be found.

In *simple or mixed decrements*, these values are deduced from the ratio of radius to tangent a, or of sin. a to sin. b, as we have already seen; a and b representing the particular angles in each particular case.

The following may be regarded as the general process for determining the law of an *intermediary decrement.*

1st. To measure the inclination of one of the secondary planes on two of the adjacent primary planes.

2d. To determine the two plane angles at the termination of the greater edge of the *defect* of the primary form occasioned by the plane we have measured.

3d. From a knowledge of these plane angles, and of the plane angles of the primary planes, to deduce the ratios of the edges of the *defect.*

4th. When the primary edges are unequal, to divide these ratios by the ratios of the corresponding edges of the primary form, and thus to deduce the law of decrement.

5th. If the intermediary decrement has taken place on an octahedron, to determine the *fourth* edge of the defect by a method which will be described when we apply our calculations to the regular octahedron.

It will be recollected that in *framing* the general symbol of any secondary plane, we are generally to consider $p > q > r > s$.

By carefully observing the position of the plane we have measured, and whose law of decrement we require to know, we shall feel no difficulty in adapting an appropriate symbol to it. And having found our general symbol, we may readily find the *particular* values of these indices by the methods already described, or by such as will be detailed in the succeeding part of this appendix.

From what has preceded, the method will be readily perceived by which we may determine the ratios of the primary edges of crystals, if we assume some observed secondary plane replacing an edge of those forms whose terminal edges are equal, or replacing an angle of those whose terminal edges are unequal, to have been produced by some given law of decrement.

If we assume that a plane replacing an angle or edge of any primary form, has resulted from a decrement by one row of molecules, we determine the ratio of the primary edges by discovering the ratio of the edges of the defect occasioned by that plane. And if we assume any other law of decrement to have produced the given plane, the ratios of the primary edges may evidently be determined, by dividing the ratios of the edges of the defect by the assumed law of decrement.

THE CUBE.

Its elements.

The inclination of any two adjacent planes at their common edge = 90°.
Plane angles = 90°.
Edges all equal.
Inclination of an edge to an axis = 54° 44′ 8″.
Ratio of an edge : ½ a diagonal :: 2 : $\sqrt{2}$.
. . . . . an edge : an axis :: 1 : $\sqrt{3}$.

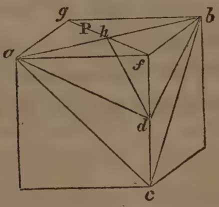
Its units of comparison.

In reference to decrements on the edges, the unit is = 1.
. . . . . simple and mixed decrements on the angles, it is $= \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$.

Simple and mixed decrements on the angles.

The law of a *simple or mixed decrement* on any *angle* of a *cube*, may be computed by means of an edge of the primary form, and half a diagonal of one of its planes. The ½ *diagonal* being used to measure the decrements *in breadth*, and the *edge* those *in height*.

Fig. 331.



Let us suppose a cube represented by figure 331, and let us imagine a simple decrement to have taken place by 1 *row* of molecules on the angle *a f b*. The edges of the *defect* of the primary form, would, in this case, be, as we have already seen, proportional to the *corresponding edges* of the primary form, and might consequently, if the secondary plane were sufficiently enlarged, be *equal* to those edges. The edges of the new plane might therefore coincide with the lines *a b*, *a c*, *b c*.

If we now draw the diagonal *g f*, on the terminal plane, we shall observe that one half of it is intercepted at the point *h*, by the edge *a b* of the secondary plane.

As a decrement by 1 *row* therefore intercepts the *half diagonal f h*, at the same time that it intercepts the *whole of the edges f a*, *f b*, *f c*, the ratio of $f h : f c$ may be assumed as the unit of comparison for determining the law of a simple or mixed decrement on the angle *a f b*.

For let us suppose a decrement to have taken place on that angle, by 2 rows in breadth; if this decrement be conceived to be continued until the edges *a f*, and *b f*, are again intercepted by the new plane, it is obvious from what has been already stated, that only one half of the edge *f c* would be intercepted by the same plane.

Here then the law of decrement would be expressed by the ratio of $f h : \frac{1}{2} f c$, or $2 f h : f c$; and if $f h : f c$ be represented by $m : n$, the ratio of $f h : f d$, should be as $2 m : n$, and would thus give the required law of decrement by 2 rows in breadth on the angle *a f b*.

But the ratio of $f h : f d$, is that of radius : tang. of the angle *f h d*; and *f h d* is the supplement of the

CUBE.

angle $g\ h\ d$, the inclination of the primary to the secondary plane.

The planes belonging to classes b and c of the modifications of the cube, result from simple and mixed decrements.

Let the inclination of P on the plane b adjacent to it, or on the plane c which rests on the edge between P′ and P″,* be measured and called I_1.

If p, as before, be used to represent the decrement in breadth, and q the decrement in height, then

$$\frac{p}{q} = \frac{R}{\text{tang.}\,(180^\circ - I_1)}$$

Intermediary decrements.

The general symbol representing a single plane belonging to class d, would be ($B_p\ B'''_q\ B'_r$). And the law of decrement producing a particular plane of that class, would be discovered by finding the values of p, q, and r, in relation to that particular plane.

Let us suppose $q > r$.

If we refer to the tables of modifications, we shall observe that two of the planes which have the d placed upon them, rest on the edge between P and P′.

Let that which inclines most on P′, be measured on P and P′.

Let P on $d = I_2$.

P′ . . $d = I_3$.

Let the plane angle of the defect corresponding to $i\ a\ c$, fig. 329, be called A_1, and that corresponding to $i\ a\ b$ of the same fig. be called A_2.

---

* See the tables of modifications whenever the classes are referred to.

CUBE.

We shall have $\cos. A_1 = \frac{\text{R}. \cos. (180° - I_2)}{\sin. (180° - I_3)}$

and $\cos. A_2 = \frac{\text{R}. \cos. (180° - I_3)}{\sin. (180° - I_2)}$

The plane angles being thus found, we have

$ai : ic :: p : q :: \text{R} : \text{tang}. A_1$

$ai : ib :: p : r :: \text{R} : \text{tang}. A_2$.

We may determine these particular values of the ratios of $p : q$, and $p : r$, by means of the tables of natural tangents, or by logarithms.

Decrements on the edges of the cube.

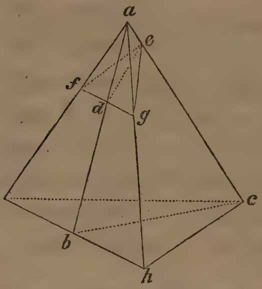
Let the inclination of the plane P on the plane f, or k, adjacent to it, be called I_4.

and $\frac{p}{q} = \frac{\text{R}}{\text{tang}. (180° - I_4)}$.

Throughout the remainder of this appendix, when the law of a simple or mixed decrement is expressed by $\frac{p}{q}$, *the letter p will always be understood to denote the decrement in breadth, and q the decrement in height.*

THE REGULAR TETRAHEDRON.

Fig. 332.



Its elements.

The mutual inclination of any two adjacent planes at their common edge = 70° 31′ 44″, and may be called I_1.

Plane angles = 60°, and may be denoted by A_1.

Edges all equal.

Inclination of an edge *a c* to *a* perpendicular, *a b* = 54° 44′ 8″, and may be called A_2.

Inclination of an edge to an axis = 35° 15′ 52″, and may be called A_3.

Inclination of a perpendicular *a b* to an axis = 19° 28′ 16″, and may be called A_4.

Ratio of a perpendicular
a b : an edge *a c* :: $\sqrt{3} : 2$.

Its units of comparison.

The unit is = $\frac{\sqrt{3}}{2}$, in relation to simple or mixed decrements on the angles.

. . . . = 1, in relation to decrements on the edges.

REGULAR TETRAHEDRON.

Simple and mixed decrements on the angles.

To determine the law of a simple or mixed decrement on an angle of the regular tetrahedron, we may assume as the unit of comparison, the ratio to an edge *a c*, of a perpendicular *a b* upon the base, fig. 332, drawn from the angle *a*. The line *a b* measuring the decrement in breadth, and the edge *a c* measuring the decrement in height.

The ratio of $ab : ac$ is known, from the relation of the tatrahedron to the cube, to be as $\sqrt{3} : 2$.

Let *f g e*, fig. 332, represent a secondary plane belonging to class *b*, whose inclination to the primary plane, which is obviously equal to the angle *b d e*, has been determined by measurement; we may call this angle I_2.

In the triangle *a d e*, we have the following angles,

$\vee\, ade = (180^\circ - I_2)$,

$\vee\, dae = (90^\circ - \frac{1}{2} I_1) = 54^\circ\, 44'\, 8''$, which we have called A_2.

$\vee\, aed = (I_2 - A_2)$.

Whence

$$ad : ae :: \sin.(I_2 - A_2) : \sin.(180^\circ - I_2) :: pm : qn.$$

In the fraction $\frac{pm}{qn}$, $\frac{m}{n}$ represents $\frac{\sqrt{3}}{2}$.

Dividing therefore $\frac{\sin.(I_2 - A_2)}{\sin.\, 180^\circ - I_2)}$ by $\frac{\sqrt{3}}{2}$, we shall find the values of *p* and *q*, *p*, representing the decrement in breadth on the angle *a*, proceeding along the plane *a b*, and *q* the number of molecules in height corresponding to the line *a e*.

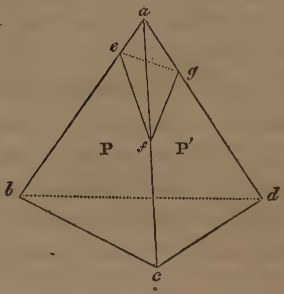
If the inclination on P, of any plane belonging to class *c*, which rests on the edge between P′ and the

REGULAR TETRAHEDRON.

back plane of the figure, be known and called I_3, the preceding formula will give the law of decrement producing that particular modification.

Intermediary decrements.

Fig. 333.



The law of an *intermediary decrement* on an *angle* of the *tetrahedron*, is determined by the ratios of the *defect* or intercepted portions $a e$, $a f$, $a g$, of the three primary edges $a b$, $a c$, $a d$.

Let $e f g$, fig. 333, be one of the six planes produced by a modification of the tetrahedron belonging to class *d*.

The general symbol representing a single plane belonging to this class is (B_p B'_q B''_r), p representing the number of molecules contained in $a f$, q the number contained in $a g$, and r the number contained in $a e$.

To determine the ratios of $a e : d f : a g$, we require the plane angles $a f e$, $a f g$; from which, as we know the angles $e a f$, $f a g$, we may deduce the angles $a e f$, $a g f$.

To obtain the plane angles $a f e$, $a f g$, we may have recourse to a spherical triangle.

REGULAR TETRAHEDRON.

The plane representing class *d*, on the figure in the tables of modifications, which corresponds to the plane *e f g* of fig. 333, is that which rests on the edge between P and P′ and inclines on P′.

Let the inclination of this plane on P be called I_4.
. . . . P′ . . . I_5.
and let the plane angle *a f e* be called A_5.
. . . . *a f g* . . . A_6.

we shall have sin. ½ A_5 =

$$\sqrt{\frac{-\cos.\frac{1}{2}[I_1+(180°-I_4)+(180°-I_5)]\cos.\frac{1}{2}[I_1+(180°-I_4)-(180°-I_5)]}{\sin.\ I_1\ \sin.\ I_4}}$$

and sin. ½ A_6 =

$$\sqrt{\frac{-\cos.\frac{1}{2}[I_1+(180°-I_5)+(180°-I_4)]\cos.\frac{1}{2}[I_1+(180°-I_5)-(180°-I_4)]}{\sin.\ I_1\ \sin.\ I_5}}$$

Having from these formula deduced the angles A_5 and A_6, we have

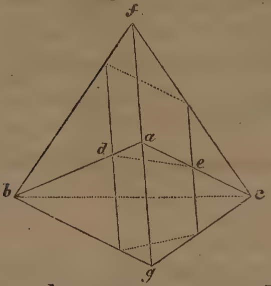
$$a f : a g :: p : q :: \sin.\ (120°-A_6) : \sin.\ A_6$$
$$:: 1 : \frac{\sin.\ A_6}{\sin.\ (120°-A_6)}$$
$$a f : a e :: p : r :: \sin.\ (120°-A_5) : \sin.\ A_5$$
$$:: 1 : \frac{\sin.\ A_5}{\sin.\ (120°-A_5)}$$

Hence the particular values of *p*, *q*, and *r*, being found, and substituted for those letters in the general symbol $(B_p\ B'_q\ B''_r)$,
we shall obtain a symbol representing the particular plane we have observed.

REGULAR TETRAHEDRON.

Decrements on the edges.

Fig. 334.



The law of any decrement on an *edge* of the *regular tetrahedron*, may be ascertained by means of the two lines *a b*, *a c*, drawn from the angles *b* and *c*, perpendicularly on the edge *f g*, on which the decrement is supposed to take place.

The primary planes of this figure being *equilateral* triangles, the lines *a b*, and *a c*, which are *perpendicular* to the edge *f g*, are *equal.*

The angle *b a c*, is that at which the adjacent primary planes incline to each other, and is already known and called I_1.

Let the new plane *d e*, be one of the planes belonging to *mod. f* of the tetrahedron, and inclining on the primary plane P′ at an angle which we shall call I_6. And let *d a*, *a e*, be the portions of the lines *a b*, *a c*, intercepted by the new plane.

In the triangle *d a e*, the angle *a e d*, is $=(180°-I_6)$, and consequently the angle *a d e*, is (I_6-I_1).

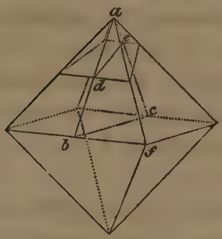
Wherefore,

$a\,e : a\,d :: p : q ::$ sin. (I_6-I_1) : sin. $(180°-I_6)$.

We may determine *p* and *q*, those being the *lowest whole numbers* which will express that ratio, by the means of either the natural sines, or their logarithms; and when determined, they will express the law of decrement by which the new plane has been produced.

THE REGULAR OCTAHEDRON.

Fig. 335.



Its elements.

The inclination of any two adjacent planes at their common edge = 109° 28′ 16″, may be called I_1.

Plane angles = 60°, may be called A_1.

Edges all equal.

Inclination of edge to edge measured over the solid angle 90°.

Inclination of plane to plane measured over the solid angle = 70° 31′ 44″, and may be called I_2.

Ratio of a perpendicular

$a\,b$: an edge $a\,f$ :: $\sqrt{3} : 2$.

. . . an edge : $\frac{1}{2}$ an axis :: $\sqrt{2} : 1$.

Its unit of comparison

Is 1, in reference to simple and mixed decrements on the angles, and also to decrements on the edges.

Simple and mixed decrements on the angles.

The law of a *simple or mixed decrement* on the *angle* of a *regular octahedron* may be determined by means of the perpendiculars $a\,b$, $a\,c$, drawn from the angle a upon the edges of the base; which perpen-

REGULAR OCTAHEDRON.

diculars are, from the nature of the figure, equal. We know the angle *b a c*, which we call I_2, and we are supposed to know the angle *b d e*, which is the inclination of one of the planes of class *b* to the adjacent primary planes. This inclination we shall call I_3, and from these angles we may deduce the angle *d e a*, which we may call I_4.

Hence we have

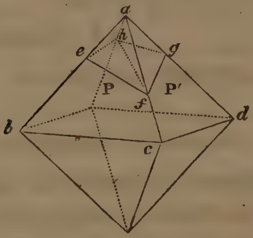
$$a\,d : a\,e :: \sin.\ I_4 : \sin.\ (180^\circ - I_3) :: p : q,$$

which gives the law of decrement by *p* rows in breadth, and *q* rows in height, on the angle *a*, proceeding along the plane *a b*.

Intermediary decrements.

The *intermediary* decrements of the *regular octahedron* are, as we have already seen, of two kinds; the one producing the modifications class *c*, and the other class *d*, of that form.

Fig. 336.



In the modifications class c, the general symbol representing which is $(B_p\ B'_q\ b'_q\ b_r)$, the edges *a e*, *a g*, of the defect of the primary form, are equal, *a h* is less than *a e*, or *a g*, and *a f* greater.

It will be sufficient therefore to determine the ratios of *a f* to either *a e* or *a g*, and to *a h*. For this purpose let us imagine the *defect* of the primary form to be divided by a plane passing through the edges $a\,h$, $a\,f$, and let $a\,h\,f\,g$ represent *one half of this defect.*

The inclination is known of P on P′, and called I_1, and of the new plane on P′, which we shall call I_5. Hence from a spherical triangle whose angles are 90°, $\frac{1}{2}\,I_1$, and $(180°—I_5)$, we may deduce the plane angles $a\,f\,g$, $a\,f\,h$, which we shall call A_2 and A_3.

The formulæ for this purpose are the following:

$$\cos.\ A_2 = \frac{\cot.\ \frac{1}{2}\,I_1 .\ \cot.\ (180°—I_5)}{\mathrm{R}}$$

$$\cos.\ A_3 = \frac{\mathrm{R}.\ \cos.\ (180°—I_5)}{\sin.\ \frac{1}{2}\,I_1}$$

we know the angle $f\,a\,g = 60°$

$f\,a\,h = 90°$

whence we deduce

$$a\,f : a\,g :: \sin.\ (120°—A_2)\ \sin.\ A_2$$

$$:: 1 : \frac{\sin\ A_2}{\sin.\ (120°—A_2)} :: p : q$$

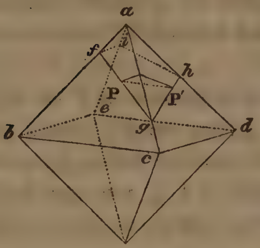
and $a\,f : a\,h :: \mathrm{R} : \mathrm{tang.}\ A_3$

$$:: 1 : \frac{\mathrm{tang.}\ A_3}{\mathrm{R}} :: p : r$$

and by substituting for p, q, and r, in the general symbol of the class of modifications to which this plane belongs, these particular values, we shall obtain the symbol of the particular plane we have observed.

REGULAR OCTAHEDRON.

Fig. 337.



The law of an intermediary decrement producing any modification belonging to *class d*, may be thus discovered.

Let *f g h i*, fig. 337, be one of the planes of that class, and let its inclination on P, and on P′, which is supposed to be known, be called I_6 and I_7.

The symbol to represent this plane would be

$$(\mathrm{B}_p\ \mathrm{b}'_q\ \mathrm{B}'_r\ \mathrm{b}_s).$$

p representing the number of molecules contained in the line *a g* of the above fig.

q . . . . . *a h*,

r . . . . . *a f*,

s . . . . . *a i*.

The spherical triangle marked on the fig. at P P′, will give the plane angles *a g f* and *a g h*, which we may denote by A_4 and A_5, by means of the following formulæ.

$$\sin.\tfrac{1}{2} A_4 =$$

$$\sqrt{\frac{-\cos.\frac{1}{2}[(180^\circ-I_6)+I_1+(180^\circ-I_7)]\cos.\frac{1}{2}[(180^\circ-I_6)+I_1-(180^\circ-I_7)]}{\sin.(180^\circ-I_6)\sin.I_1.}}$$

$$\sin.\tfrac{1}{2} A_5 =$$

$$\sqrt{\frac{-\cos.\frac{1}{2}[(180^\circ-I_6)+(180^\circ-I_7)+I_1)]\cos.\frac{1}{2}[(180^\circ-I_7)+I_1-(180^\circ-I_6)]}{\sin.(180^\circ-I_7)\sin.I_1.}}$$

REGULAR OCTAHEDRON.

Whence we may deduce the particular values of p, q, and r, from the ratios of $a\,g : a\,h$, and $a\,g : a\,f$.

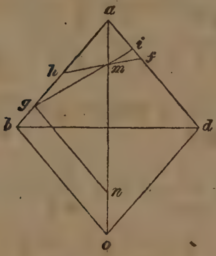
$$a\,g : a\,h :: \sin.\,(120^\circ - A_5) : \sin A_5$$
$$:: 1 : \frac{\sin.\,A_5}{\sin.\,(120^\circ - A_5)} :: p : q.$$
$$a\,g : a\,f :: \sin.\,(120^\circ - A_4) : \sin A_4$$
$$:: 1 : \frac{\sin.\,A_4}{\sin.\,(120^\circ - A_4)} :: p : r.$$

The value of s may be found from the following equation,

$$\frac{1}{s} = \frac{1}{q} + \frac{1}{r} - \frac{1}{p}$$

This equation may be thus derived.

Fig. 338.



Let $a\,i\,e\,c$, fig. 338, represent a section passing through the axis of any octahedron whose terminal edges are equal, and through four of those edges. And let the lines $g\,i$, $h\,f$, be equal to lines which might be drawn on the plane $f\,g\,h\,i$, fig. 337, from g to i, and from h to f. Both these lines will evidently cut the axis at the same point, which we may call m.

If we draw $g\,n$ parallel to $a\,i$, we have

$$g\,n = g\,a : a\,i :: an - a\,m : a\,m.$$

REGULAR OCTAHEDRON.

But from the structure assigned to the octahedron, it is evident that the axis $a\,o$ represents double the number of molecules that are represented by an edge $a\,b$, and we may therefore in relation to the numbers of molecules represented, consider

$$a\,n = 2\,g\,a.$$

Hence $g\,a : a\,i :: 2\,g\,a - a\,m : a\,m.$

and $g\,a + a\,i : a\,i :: 2\,g\,a : a\,m.$

From this ratio we find

$$a\,m = \frac{2\,g\,a\,.\,a\,i}{g\,a + a\,i.}$$

And by a similar proceeding we shall find

$$a\,m = \frac{2\,h\,a\,.\,a\,f}{h\,a + a\,f.}$$

Therefore $\dfrac{2\,g\,a\,.\,a\,i}{g\,a + a\,i} = \dfrac{2\,h\,a\,.\,a\,f}{h\,a + a\,f.}$

And $2\,g\,a\,.\,a\,i : g\,a + a\,i :: 2\,h\,a\,.\,a\,f : h\,a + a\,f$

$$2\,a\,i : 1 + \frac{a\,i}{g\,a} :: 2\,a\,f : 1 + \frac{a\,f}{h\,a}$$

$$1 : \frac{1}{a\,i} + \frac{1}{g\,a} :: 1 : \frac{1}{a\,f} + \frac{1}{h\,a}$$

Therefore $\dfrac{1}{a\,i} + \dfrac{1}{g\,a} = \dfrac{1}{a\,f} + \dfrac{1}{h\,a}$

and, $\dfrac{1}{a\,i} = \dfrac{1}{a\,f} + \dfrac{1}{h\,a} - \dfrac{1}{g\,a}$

whence $\dfrac{1}{s} = \dfrac{1}{r} + \dfrac{1}{q} - \dfrac{1}{p}.$*

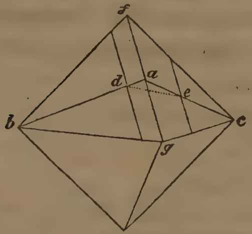
---

* This formula was mentioned to me in conversation by Mr. Levy, some months before it occurred to me as the result of the investigation given in the text. But it was mentioned without any allusion to the means by which it had been obtained; and these, I since learn from Mr. L., were different from those employed above.

REGULAR OCTAHEDRON.

Decrements on the edges of the regular octahedron.

Fig. 339.



The inclination of one of the planes of class *f*, on the primary plane adjacent to it, being known and called I_8, the angle *a e d*, fig. 339, becomes

$$(180^\circ - I_8).$$

and $a\,e : a\,d :: \sin.\,(I_8 - I_1)\ \sin.\,(180^\circ - I_8) :: p : q$, which will give the law of decrement producing the particular plane we have measured.

THE RHOMBIC DODECAHEDRON.

Its elements.

The inclination of any two adjacent planes at their common edge = 120°, to be called I_1.

Obtuse plane angles = 109° 28′ 16″, may be called A_1.

Acute plane angles = 70° 31′ 44″, may be called A_2.

Edges all equal.

Inclination of edge to edge measured over the summit = 109° 28′ 16″, may be called A_3.

Inclination of plane on plane measured over the summit = 90°.

Inclination of an edge to the adjacent lesser diagonal = 125° 15′ 52″, may be called A_4.

Ratio of an edge : $\frac{1}{2}$ a greater diagonal :: $\sqrt{3} : \sqrt{2}$.
an edge : $\frac{1}{2}$ a lesser diagonal :: $\sqrt{3} : 1$.
an edge : greater axis :: $\sqrt{3} : 4$.
an edge : lesser axis :: $1 : 2$.
greater diagonal : greater axis :: $\sqrt{2} : 2$.
lesser diagonal : lesser axis :: $1 : \sqrt{3}$.

Its units of comparison.

For simple and mixed decrements on the acute angles, or on the edges, the unit is = 1.

For simple or mixed decrements on the obtuse angles, it is $\frac{1}{\sqrt{3}}$, being the ratio of $\frac{1}{2}$ a lesser diagonal to an edge.

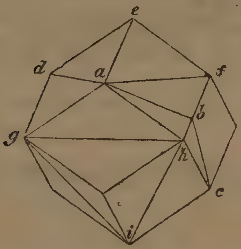
After the ample illustration which has been given of the methods proposed for determining the laws of decrement, it will not be necessary in future to do

RHOMBIC DODECAHEDRON.

much more than to indicate the results of their application to new cases.

This may generally be effected, by giving the general symbol of each particular class of planes, and the formulæ which are required for determining the particular values of the general indices of each class respectively.

Fig. 340.



Simple and mixed decrements on the obtuse angles.

General symbol $\overset{\frac{p}{q}}{\frac{p}{q}A\frac{p}{q}}$

Let the inclination of P on a plane of class f adjacent to it, or on the upper of the three planes on which g is placed, be known and called I_2, and

$$\frac{p\,m}{q\,n} = \frac{\sin.\,(I_2 - A_4)}{\sin.\,I_2}$$

If we divide $\frac{p\,m}{q\,n}$ by $\frac{1}{\sqrt{3}}$, being the particular value of $\frac{m}{n}$ in this case, we shall obtain the particular values of p and q, and hence the law of decrement producing the plane we have measured.

RHOMBIC DODECAHEDRON.

Intermediary decrements on the obtuse solid angles.

On referring to class *h* of the modifications of this form, we shall perceive that two planes rest on the edge between P and P″.

Let the plane which inclines most on P be measured on P and on P″. Call its inclination on P, I_3,
on P″, I_4.

The symbol representing this plane would be

$$(B_p\ B'_q\ B''_r).$$

To determine *p*, *q*, and *r*, we must, as we have done for the tetrahedron, find the two plane angles of the defect, which we shall call A_5 and A_6, by the formulæ

$$\sin.\tfrac{1}{2}A_5 = \sqrt{\frac{-\cos.\frac{1}{2}[(180°-I_3)+I_1+(180°-I_4)]\cos.\frac{1}{2}[(180°-I_3)+I_1-(180°-I_4)]}{\sin.(180°-I_3)\sin.I_1}}$$

$$\sin.\tfrac{1}{2}A_6 = \sqrt{\frac{-\cos.\frac{1}{2}[(180°-I_4)+I_1+(180°-I_3)]\cos.\frac{1}{2}[(180°-I_4)+I_1-(180°-I_3)]}{\sin.(180°-I_4)\sin.I_1}}$$

Whence $p : q :: \sin.[180°-(A_1+A_5)] : \sin.A_5$
$p : r :: \sin.[180°-(A_1+A_6)] : \sin.A_6$

And the particular values of *p*, *q*, and *r* thus found, being substituted for those letters in the above general symbol, we shall obtain the particular symbol of the observed plane.

It thus appears that the formulæ used for determining the laws of decrement on the obtuse solid angles of this figure, are similar in character to those applied to the determination of decrements on the solid angles of the tetrahedron.

This results from an analogy which subsists between these solid angles in the two forms. For the lines

RHOMBIC DODECAHEDRON.

g h. g i, h i, in fig. 340, may be regarded as edges of the base of an irregular tetrahedron, of which the obtuse solid angle would be the summit.

Simple and mixed decrements on the acute angles.

An analogy similar to that which subsists between the obtuse solid angle of this form and the solid angle of the regular tetrahedron, will be found to subsist between its acute solid angle and the terminal solid angle of an octahedron. For the lines *d a*, *a f*, may be regarded as edges of the base of an irregular octahedron, and obviously of one with a square base.

Let the inclination of P on an adjacent plane of class *b*, be known and called I_5

The inclination of P on P′ is 90°.

whence $\frac{p}{q} = \frac{R}{\text{tang. } (180° - I_5)}$

Intermediary decrements on the acute solid angles.

These, like those on the octahedron, are of two kinds, and produce the planes of mod. *c* and *d*.

Let P on one of the planes belonging to mod. *a* be measured, and its angle called I_6.

Its symbol might be $(B_p\ B'_q\ b'_q\ b_r)$, and those indices may be determined by the method adapted to the corresponding plane on the regular octahedron; observing, however, that as the plane angles, and the mutual inclination of the primary planes, vary from those of the regular octahedron, there will be a corresponding variation in the terms of the ratios which give the values of *p*, *q*, and *r*.

RHOMBIC DODECAHEDRON.

The symbol representing the upper plane belonging to class *b*, of the two on which *b* is placed, and which rest on the edge between P and P'', is

$$(\mathrm{B}_p\ \mathrm{B}'_q\ \mathrm{b}_r\ \mathrm{b}'_s).$$

Let the inclination of this plane on P be called I_7
P'' . . . I_8

A spherical triangle will give the values of angles corresponding to A_4 and A_5 of the regular octahedron, and from these, the particular values of *p*, *q*, *r*, and *s*, may be deduced by the methods pointed out in reference to the analagous decrements on that form.

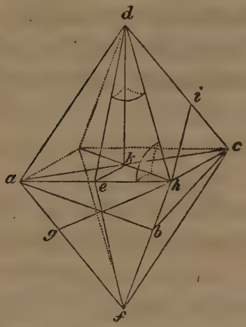
Decrements on the edges.

Let the inclination of plane P on one of the planes *k*, adjacent to it, be called I_9.

and $\frac{p}{q} = \frac{\sin.\ (I_9 - 120^\circ)}{\sin.\ (180^\circ - I_9)}$

THE OCTAHEDRON WITH A SQUARE BASE.

Fig. 341.



Its elements.

The inclination of the planes at the terminal edges will differ in different minerals, and may be represented generally by I_1.

Inclination of the planes at the edges of the base will also vary in different minerals, and may be called I_2.

Plane angles at the summit will also vary, but they may be denoted generally by A_1, and may be thus deduced,

$$\cos. \tfrac{1}{2} A_1 = \frac{\text{R. cos. } 45^\circ}{\sin. \tfrac{1}{2} I_1}$$

It is by means of the *upper* spherical triangle marked on the fig. that we are enabled to determine the angle A_1.

For if *d k* represent $\frac{1}{2}$ the axis, *d e* a perpendicular on the base, and *h k* $\frac{1}{2}$ a diagonal of the base, the angles of the spherical triangle will be 90°, 45°, and $\frac{1}{2} I_1$. And thence the angle *e d h*, which we have called $\frac{1}{2} A_1$ is deduced.

OCTAHEDRON WITH A SQUARE BASE.

The plane angles at the base are consequently $= 90^\circ - \frac{1}{2} A_1$, and may be called A_2.

Terminal edges are all equal.

Edges of the base are all equal.

Ratio of a terminal edge : an edge of the base :: sin. A_2 : sin. A_1.

Ratio of an edge *d h* : a perpendicular on the base, *d e*, :: R : cos. $\frac{1}{2} A_1$.

Inclination of an edge on the axis may be called A_3, and may be thus found.

$$\cos. A_3 = \cos. \tfrac{1}{2} I_1$$

The same spherical triangle from which we have determined the angle A_1, may be used to determine A_3, which in the above fig. is the angle *k d h*, and is the hypothenuse of that triangle.

The known formula to determine A_3 is

$$\cos. A_3 = \frac{\cot. \frac{1}{2} I_1 \cot. 45^\circ}{R}$$

But as cot. $45^\circ = R$, this evidently becomes

$$\cos. A_3 = \cot. \tfrac{1}{2} I_1.$$

Inclination of edge on edge over the summit will consequently be $= 2 A_3$.

Ratio of $\frac{1}{2}$ a diagonal of the base : $\frac{1}{2}$ the axis :: tang. A_3 : R.

The relation of I_1, I_2, and A_2, may be thus expressed,

$$\cos. \tfrac{1}{2} I_2 = \frac{\cos. A_2 \text{ tang. } \frac{1}{2} I_1}{R}$$

We may observe that the angles of the spherical triangle marked at the base of the fig. are $\frac{1}{2} I_1$, $\frac{1}{2} I_2$, and 90°, and that the side *d h e*, which we have called A_2, is the hypothenuse of the triangle; and hence results the relation we have given.

From which, if any two of these angles be known, the third may be immediately found.

Hence it is apparent that the angle I_1 being known, all the other elements of the form may be known also.

Its units of comparison.

The unit of comparison is = 1 in reference to the following decrements.

1st. Those which affect the terminal angles, the four edges of the pyramid being equal.

2d. Simple and mixed decrements on the lateral angles; these being measured by the proportions intercepted of the equal lines *h g*, *h i*, drawn from the angle *h* perpendicularly on the edges *a f* and *d c*.

3d. All decrements on the terminal edges, which are measured by the proportions intercepted of the equal lines *b a*, *b e*.

4th. Those on the edges of the base, which are measured on the lines *e d*, *e f*.

But in relation to intermediary decrements on the lateral solid angles, the unit is constituted of the ratio of the terminal and lateral edges; which is evidently that of sin. A_1 : sin. A_2.

Simple and mixed decrements on the terminal angles.

If we measure the inclination of P to the adjacent plane belonging to class *b*, and call the angle I_3, we have

$$\frac{p}{q} = \frac{\sin.\,(I_3 + I_2 - 180^\circ)}{\sin.\,(180^\circ - I_3)}$$

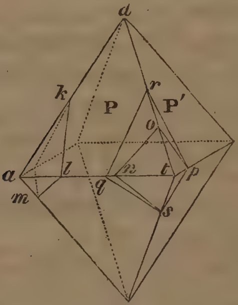
OCTAHEDRON WITH A SQUARE BASE.

Intermediary decrements on the terminal solid angles.

If we measure the inclination of one of the planes of class *c*, on P, or of class *d* on P and P′, we may determine its law of decrement by means of formulæ similar to those which have been used for the analogous modifications of the regular octahedron.

The calculations relative to decrements on the lateral angles of this form, require a little additional illustration.

Fig. 342.



Let the inclination on P of the plane *k l m*, fig. 342, which represents one of the planes of class *f*, be denoted by I_4.

A spherical triangle will give the plane angle of the *defect* $a\ k\ l$, which we may call A_5, by means of the formula

$$\cos. A_5 = \frac{\cot. \frac{1}{2} I_1 \cot. (180^\circ - I_4)}{R}$$

whence $a\ k : a\ l :: p : q$ may be readily found, and $a\ m = r$ may be thus determined.

$$\frac{1}{r} = \frac{2}{q} - \frac{1}{p}$$

Let $n\ o\ p\ s$ represent a plane belonging to class *g*, whose inclination on P is known and called I_5.

OCTAHEDRON, SQUARE BASE.

We now require the plane angle *o n t* of the *defect*, which being denoted by A_6, may be thus found,

$$\cos. A_6 = \frac{\cot. \frac{1}{2} I_2 . \cot. (180 - I_5)}{R}$$

From this angle and *o t n*, which we have called A_2, the ratio may be found of $t\,n : t\,o :: p : q$, and *r* may be determined as before.

The planes of class *h* result from simple and mixed decrements on the lateral angles. Let I_6 represent the inclination on P of one of the planes *h* adjacent to it, and the law of decrement may be thus determined,

$$\frac{p}{q} = \frac{\sin. (I_1 + I_6 - 180^\circ)}{\sin. (180^\circ - I_6)}$$

Let *q r p s*, fig. 342, represent a plane belonging to class *i* or *k*, whose general symbol would be

$$(B_p\ D_q\ D'_r\ B''_s : B_p\ D_r\ D'_q\ B''_s)$$

and its inclination on P be called I_7, and on P′, I_8. A spherical triangle will give the plane angles *r t q*, *r t p*, and hence the following ratios are known,

$$t\,r : t\,q :: p\,m : q\,n$$
$$t\,r : t\,p :: p\,m : r\,n$$

And if we divide $\frac{p\,m}{q\,n}$ and $\frac{p\,m}{r\,n}$ by $\frac{\sin. A_2}{\sin. A_1}$, the particular values of *p*, *q*, and *r* will be found.

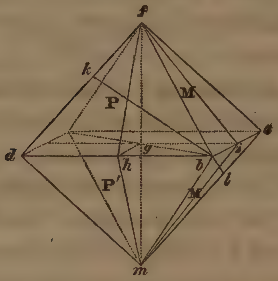
The fourth index *s*, may also be known from the equation,

$$\frac{1}{s} = \frac{1}{q} + \frac{1}{r} - \frac{1}{p}$$

The decrements on the edges may be determined by the general methods applied to analagous decrements on the regular octahedron.

THE OCTAHEDRON WITH A RECTANGULAR BASE.

Fig. 343.



Its elements.

The inclination of the planes at the terminal edges may be denoted by I_1.
inclination of the plane P on P′, may be called I_2.
. . . . M on M′, . . . I_3.
Plane angle $d f b$, fig. 343, may be called A_1, and may be thus known,

$$\text{tang.}\ \tfrac{1}{2} A_1 = \frac{\sin.\ \frac{1}{2} I_2 .\ \cot.\ \frac{1}{2} I_3}{\text{R}}$$

Plane angle $b\ f\ e$, being called A_2, may be thus determined,

$$\text{tang.}\ \tfrac{1}{2} A_2 = \frac{\sin.\ \frac{1}{2} I_3\ \cot.\ \frac{1}{2} I_2}{\text{R}}$$

The first of these equations is thus derived.

The angle $h f b$ in the above fig. represents $\tfrac{1}{2} A_1$.
$h f g$ . . . . $90° - \tfrac{1}{2} I_2$.
$b f i$ . . . . $\tfrac{1}{2} A_2$.
$g f i$ . . . . $90° - \tfrac{1}{2} I_3$.

But $fg : fh :: \text{R} : \sec.\ (90° - \tfrac{1}{2} I_2)$
$fg : hb = gi :: \text{R} : \text{tang.}\ (90° - \tfrac{1}{2} I_3)$

OCTAHEDRON, RECTANGULAR BASE.

Whence

$fh : hb :: \sec.(90^\circ - \frac{1}{2}I_2) : \text{tang}.(90^\circ - \frac{1}{2}I_3)$

and $fh : hb :: R : \text{tang}. \frac{1}{2}A_1$

Therefore

$$\text{tang}. \tfrac{1}{2}A_1 : R :: \text{tang}. (90^\circ - \tfrac{1}{2}I_3) : \sec. (90^\circ - \tfrac{1}{2}I_2)$$

$$\text{tang}. \tfrac{1}{2}A_1 = \frac{R. \text{tang}. (90^\circ - \frac{1}{2}I_3)}{\sec. (90^\circ - \frac{1}{2}I_2)}$$

But as $\dfrac{R}{\sec.} = \dfrac{\cos}{R}$,

and $\text{tang}. 90^\circ - a = \cot. a$,

the equation becomes

$$\text{tang}. \tfrac{1}{2}A_1 = \frac{\cos. (90^\circ - \frac{1}{2}I_2) \cot. \frac{1}{2}I_3}{R}$$

$$= \frac{\sin. \frac{1}{2}I_2 . \cot. \frac{1}{2}I_3}{R}$$

The second equation must evidently be similar in its character to the first, but substituting I_3 and I_2 for I_2 and I_3.

The terminal edges equal.

Ratio of a terminal edge : a perpendicular on the base of plane P :: R : cos. $\frac{1}{2}A_1$.

Ratio of a terminal edge : a perpendicular on the base of plane M :: R : cos. $\frac{1}{2}A_2$.

Ratio of a terminal edge : a greater edge of the base :: R : 2 sin. $\frac{1}{2}A_1$.

Ratio of a terminal edge : a lesser edge of the base :: R : 2 sin. $\frac{1}{2}A_2$.

Edge $d\,b$ of the base : edge $b\,e$ :: cot. $\frac{1}{2}I_3$: cot. $\frac{1}{2}I_2$.

For $g\,i = \frac{1}{2}d\,b :: g\,h = \frac{1}{2}b\,e$

$:: \text{tang}. (90 - \frac{1}{2}I_3) : \text{tang}. (90^\circ - \frac{1}{2}I_2)$

$:: \cot. \frac{1}{2}I_3 : \cot. \frac{1}{2}I_2$

OCTAHEDRON, RECTANGULAR BASE.

The inclination of a terminal edge to the axis may be called A_3, and may be thus found,

$$\cos. A_3 = \frac{\sin. \tfrac{1}{2} I_2 \ \cos. \tfrac{1}{2} A_1}{R}$$

If we suppose a spherical triangle to be represented by a segment $h g b f$ of the octahedron, it would obviously be right angled at h; and we know the side

$$h f b = \tfrac{1}{2} A_1$$
$$h f g = 90^\circ - \tfrac{1}{2} I_2$$

Whence we find cos. $\vee\, g f b$, which we call A_3, by the known formula

$$\cos. A_3 = \frac{\cos. (90^\circ - \tfrac{1}{2} I_2) \cos. \tfrac{1}{2} A_1}{R}$$

$$= \frac{\sin. \tfrac{1}{2} I_2 \ \cos. \tfrac{1}{2} A_1}{R}$$

Ratio of ½ a diagonal of the base : ½ the axis :: sin. A_3 : cos. A_3.

The relation between I_1, I_2, and I_3 may be thus discovered,

$$\cos. I_1 = \frac{-\cos. \tfrac{1}{2} I_2 \ \cos. \tfrac{1}{2} I_3}{R}$$

If we suppose the angle $h b i = 90$, to be the side of a spherical triangle, the angles of the same triangle would be I_1, $\tfrac{1}{2} I_2$, and $\tfrac{1}{2} I_3$.

A general equation to discover I_1 would be

$$\cos. I_1 = \frac{\cos. \vee h b i . \sin. \tfrac{1}{2} I_2 . \sin. \tfrac{1}{2} I_3}{R^2} - \frac{\cos. \tfrac{1}{2} I_2 . \cos. \tfrac{1}{2} I_3}{R}$$

But as cos. $\vee\, h b i = o$, the equation becomes that which has been given.

From which formula, any two of the angles being known, the third angle may be found.

OCTAHEDRON, RECTANGULAR BASE.

Its units of comparison.

The unit is = 1 in reference to all decrements on the terminal angles, and to decrements on the edges of the base.

For simple or mixed decrements on the lateral angles, the unit will be the ratio of the perpendiculars $b\,k$, $b\,l$, drawn from the angle at b perpendicularly on the edges $f\,d$ and $e\,m$. The ratio of those lines may be thus determined.

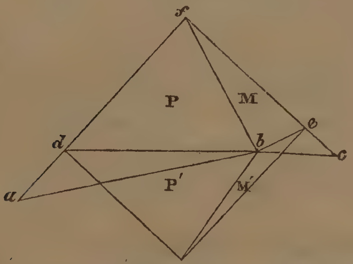
$$f\,b : k\,b :: \mathrm{R} : \text{sin.} \vee k\,f\,b = A_1$$

$$b\,m = f\,b : b\,l :: \mathrm{R} : \text{sin.} \vee b\,m\,l = A_2$$

whence . . . . $k\,b : b\,l :: \text{sin.}\,A_1 : \text{sin.}\,A_2$

When the decrement is conceived to proceed along the plane P, the unit of comparison will be $\frac{\text{sin.}\,A_1}{\text{sin.}\,A_2}$ but when the decrement proceeds along the plane M, the decrement in breadth will evidently be measured by the line $b\,l$, and the unit will then become $\frac{\text{sin.}\,A_2}{\text{sin.}\,A_1}$.

Fig. 344.



The laws of decrement on the terminal edges may be determined from the lines $b\,a$, $b\,c$, drawn perpendicular to the edge $f\,b$, on which a decrement is supposed to have taken place, and meeting the edges

OCTAHEDRON, RECTANGULAR BASE.

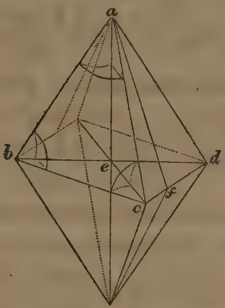
f d, *f e*, produced to *a* and *c*. These lines are to each other as tang. A_1 : tang. A_2.

When the decrement proceeds along the plane P, the unit of comparison is $\frac{\text{tang. } A_1}{\text{tang. } A_2}$, but when it is conceived to proceed along the plane M, the unit becomes $\frac{\text{tang. } A_2}{\text{tang. } A_1}$.

Knowing the elements of the primary form, the unit of comparison, and the symbol of any plane whose law of decrement we require, we are to measure the inclination of that plane on one or more of the primary planes, and then to adapt such formulæ to the particular case, as will give the particular values of the general indices of the plane in question.

THE OCTAHEDRON WITH A RHOMBIC BASE.

Fig. 345.



Its elements.

The inclination of the planes at the obtuse terminal edges may be called I_1.

Inclination of the planes at the acute terminal edges may be called I_2.

Inclination of the planes at the edges of the base may be called I_3.

Plane angles at the summit, being called A_1, may be thus found,

$$\cos. A_1 = \frac{\cot. \frac{1}{2} I_1 \ \cot. \frac{1}{2} I_2}{R}$$

A spherical triangle is marked on the upper part of the above figure, the angles of which are 90°, $\frac{1}{2} I_1$, and $\frac{1}{2} I_2$, and of which the required side *b a c* is the hypothenuse, which is found by the preceding formula.

The two spherical triangles marked at the base of the figure, will give the following formulæ; the required side being in each instance the hypothenuse of the assumed triangle.

OCTAHEDRON, RHOMBIC BASE.

The greater lateral plane angle *a c b*, may be called A_2, and may be thus known,

$$\cos. A_2 = \frac{\cot. \tfrac{1}{2} I_1 \ \cot. \tfrac{1}{2} I_3}{R}$$

Most acute of the lateral plane angles, *a b c*, may be called A_3, and may be thus found,

$$\cos. A_3 = \frac{\cot. \tfrac{1}{2} I_2 \ \cot. \tfrac{1}{2} I_1}{R}$$

Angles of the rhombic base may be thus determined. Let the lesser angle be called A_4, and

$$\cos. \tfrac{1}{2} A_4 = R \ \frac{\cos. \tfrac{1}{2} I_2}{\sin. \tfrac{1}{2} I_3}$$

The required angle $\tfrac{1}{2} A_4$, is the angle *e b c* of the above figure, and is that side of the spherical triangle nearest to *b*, which is opposite the angle $\tfrac{1}{2} I_2$; and hence the formula which is given.

Ratio of $\tfrac{1}{2}$ the greater diagonal of the base : $\tfrac{1}{2}$ the lesser diagonal :: R : tang. $\tfrac{1}{2} A_4$.

Obtuse terminal edges are equal.

Acute terminal edges are equal.

Inclination of the obtuse terminal edge to the axis being called A_5, may be thus found,

$$\cos. A_5 = R \ \frac{\cos. \tfrac{1}{2} I_2}{\sin. \tfrac{1}{2} I_1}$$

Inclination of the acute terminal edge to the axis may be called A_6, and may be thus found,

$$\cos. A_6 = R \ \frac{\cos. \tfrac{1}{2} I_1}{\sin. \tfrac{1}{2} I_2}$$

The angles A_5 and A_6, are those sides of the upper triangle marked on the figure, which subtend the angles $\tfrac{1}{2} I_1$ and $\tfrac{1}{2} I_2$. And hence the formulæ for their determination.

OCTAHEDRON, RHOMBIC BASE.

The ratio of an obtuse terminal edge
: an acute terminal edge :: sin. A_3 : sin. A_2.
Ratio of $\frac{1}{2}$ the axis : $\frac{1}{2}$ the greater diagonal of base :: R : tang. A_6
$\frac{1}{2}$ the axis : $\frac{1}{2}$ the lesser diagonal of base :: R : tang. A_5
$\frac{1}{2}$ the axis : the obtuse terminal edge :: cos. A_5 : R.
$\frac{1}{2}$ the axis : the acute terminal edge :: cos. A_6 : R.
obtuse terminal edge : perpendicular af :: R : sin. A_2.
acute terminal edge : perpendicular af :: R : sin. A_3.

Its units of comparison.

The unit is = 1, in reference to the following decrements.

1. Simple and mixed on the terminal angles, producing mod. *b*.
2. . . . . obtuse lateral angles, producing mod. *k*.
3. . . . . acute lateral angles, producing mod. *q*.
4. . . . . obtuse terminal edges, producing mod. *u*.
5. . . . . acute terminal edges, producing mod. *x*.
6. . . . . edges of the base, producing mod. *z*.

OCTAHEDRON, RHOMBIC BASE.

The units of comparison for determining the laws of intermediary decrements will be the ratios of those edges, of which portions are intercepted by the particular plane we are examining.

Having determined the unit, and the symbol, and measured the inclination of the modifying plane on one or more of the primary planes, we may proceed to discover the law of decrement by some of the methods already described.

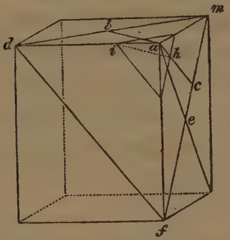
THE RIGHT SQUARE PRISM.

Its elements.

The inclination of any two adjacent planes at their common edge = 90°.
Plane angles = 90".
Terminal edges equal.
Lateral edges equal.

Its units of comparison.

Fig. 346.



Let the terminal edge be to the lateral edge as $m : n$; this will be the unit of comparison for decrements on the terminal edges. For simple and mixed decrements on the angles of the terminal planes, the ratio of $a\,b : a\,f$ becomes the unit, which is $\frac{1}{2}\,\frac{\sqrt{2\,m^2}}{n} = \frac{m}{n\sqrt{2}}$; and for similar decrements on the lateral angles the ratio of $a\,e$, being ½ the diagonal of the lateral plane, to the terminal edge $a\,d$, may be regarded as the unit. But we cannot immediately determine the ratio of the intercepted portions of the lines $a\,e$, $a\,d$, from the inclination of the secondary plane on the lateral primary plane. It must be deduced from the ratio to the edge $a\,d$, of a perpendicular $a\,c$ upon the diagonal $m\,f$.

It is very obvious that this perpendicular possesses the character assigned to those lines from which the unit of comparison may be derived. For it would be wholly intercepted by a decrement by 1 row, when the edge $a\,d$ is so intercepted. And the propriety of adopting this unit will be apparent, if we recollect that the inclination of two planes to each other is measured upon lines perpendicular to their common edge.

Let the lines $i\,h$, $h\,c$, be perpendicular to the edge of the secondary plane at the point h. The angle $i\,h\,c$ is all that can be known from actual measurement of the crystal; but from this we know the angle $i\,h\,a$, and hence the ratio of $a\,h$ to $a\,i$. The constant relation of $a\,c$ to $a\,d$, may be readily found.

We are supposed to know the ratio of $a\,m$ to $a\,f$; and $a\,m : a\,f :: \mathrm{R} : \text{tang.} \vee a\,m\,f$
call $\vee a\,m\,f$, A_1;
And because $a\,c$ is perpendicular upon $f\,m$, we have

$$a\,c : a\,m \text{ or } a\,d :: \sin A_1 : \mathrm{R}.$$

Hence $\frac{\sin. A_1}{\mathrm{R}}$ becomes the unit of comparison in reference to simple and mixed decrements on the lateral angles of the prism, the symbol representing which would be $^{p}A^{p}$.

For decrements on the lateral edges the unit is $= 1$, and for intermediary decrements, the unit will be the ratios of the particular edges affected by each law of decrement.

The general methods adopted for determining the laws of decrement on the cube, may be applied to this prism; observing however the differences in the several units of comparison afforded by the two forms.

THE RIGHT RECTANGULAR PRISM.

Its elements.

The inclination of any two adjacent planes at their common edge = 90°.

Plane angles = 90°.

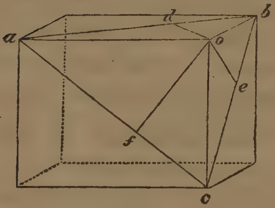
Parallel edges are always equal.

Three adjacent edges always unequal.

This inequality cannot be determined but by the means of secondary planes.

Its units of comparison.

Fig. 347.



Let us suppose the edge $o\,a : o\,b :: m : n$

$o\,a : o\,c :: m : o$

$\frac{n}{o}$ will evidently be the unit of comparison for determining the laws of decrement producing the planes of class *b*.

$\frac{m}{o}$, for those of class *c*.

$\frac{m}{n}$, for those of class *d*, whose symbol is $G^p\,{}^pG$.

$\frac{n}{m}$, ... , . ${}^pG\,G^p$

It has been shewn in p. 267, that the planes comprehended under class *a*, might be produced by four

RIGHT RECTANGULAR PRISM.

different kinds of decrement, of which the general symbols would be

$\overset{p}{A}$, ${}^{p}A$, A^{p}, and $(C_{p}\ G_{q}\ B_{r})$.

The unit of comparison will be different in each of these cases.

Let *a b*, *a c*, *b c*, fig. 347, be three diagonals, and *o d*, *o f*, *o e*, three perpendiculars drawn from those diagonals to the angle *o*.

The edges *a b*, *a c*, *b c*, might be the edges of a plane produced by a decrement by 1 row of molecules, which plane would intercept the three edges *o a*, *o b*, *o c*, together with the three perpendiculars drawn on the diagonals. The ratios of those perpendiculars to the edges may therefore be assumed as the units of comparison for simple and mixed decrements on the angles. Thus for decrements on the angle *a o b*, whose symbol is $\overset{p}{A}$, our unit will be $o\,d : o\,c$. For those on the angle *a o c*, whose symbol is ${}^{p}A$, the unit is $o\,f : o\,b$. And for those on the angle *b o c*, whose symbol is A^{p}, the unit is $o\,e : o\,a$.

To find a constant ratio between these perpendiculars and the primary edges, let us suppose *m*, *n*, and *o* known, and from these quantities, the plane angles *o a b* being also known and called A_1

o a c . . . . A_2

o b c . . . . A_3

we have $o\,a : o\,d :: \mathrm{R} : \sin. A_1$

$o\,a : o\,c :: \mathrm{R} : \mathrm{tang.}\ A_2$

$\therefore o\,d : o\,c :: \sin. A_1 : \mathrm{tang.}\ A_2$

We also find $o\,f : o\,b :: \sin. A_2 : \mathrm{tang.}\ A_1$

$o\,e : o\,a :: \sin. A_3 : \mathrm{tang.}\ (90^\circ - A_1)$

The units for intermediary decrements are the ratios of *m*, *n*, and *o*, and the methods for determining the several laws of decrement will be similar to some of those already employed.

THE RIGHT RHOMBIC PRISM.

Its elements.

The inclination of terminal to lateral planes = 90°.

Inclination of lateral planes to each other varies in different minerals; call the greater angle at which they meet I_1.

Terminal plane angles being equal to the angles of the prism, are consequently $= I_1$, and $(180° - I_1)$. But being plane angles, we shall designate the greater by A_1, and the lesser by A_2.

Half the greater diagonal of the terminal plane : half the lesser diagonal :: tang. $\frac{1}{2} A_1$: R.

Lateral plane angles = 90°.

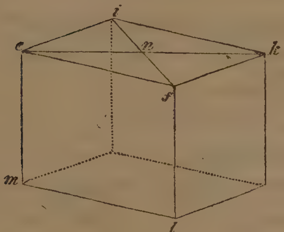
Terminal edges equal.

Lateral edges equal.

Ratio of a lateral to a terminal edge is determinable from secondary planes only.

Its units of comparison.

Fig. 348.



The unit of comparison for determining the different laws of decrement by which this class of primary forms may be affected, will be different for nearly all the different classes of modifications.

RIGHT RHOMBIC PRISM.

Let the ratio of the terminal to the lateral edge be known and expressed by the ratio of $m : n$ or of $1 : \frac{n}{m}$.

The general expression of the unit of comparison will become

$\frac{m \cos. \frac{1}{2} A_1}{n \mathrm{R}}$ in reference to simple or mixed decrements on the terminal angle A of the primary form. This is evidently derived from the ratio

$$ef : fn :: \mathrm{R} : \cos. \tfrac{1}{2} A_1 :: 1 : \frac{\cos. \frac{1}{2} A_1}{\mathrm{R}}$$

$$ef : fl :: 1 : \frac{n}{m}$$

hence

$$fn : fl :: \frac{\cos. \frac{1}{2} A_1}{\mathrm{R}} : \frac{n}{m}$$

$$:: m \cos. \tfrac{1}{2} A_1 : n \mathrm{R}.$$

$\frac{m \sin. \frac{1}{2} A_1}{n \mathrm{R}}$, in reference to similar decrements on the angle E of the primary form.

For let

$$ef : en :: \mathrm{R} : \sin. \tfrac{1}{2} A_1 :: 1 : \frac{\sin. \frac{1}{2} A_1}{\mathrm{R}}$$

and

$$ef : em :: 1 : \frac{n}{m}$$

therefore

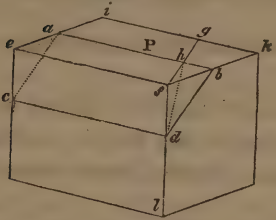
$$en : em :: \frac{\sin. \frac{1}{2} A_1}{\mathrm{R}} : \frac{n}{m}$$

$$:: m \sin. \tfrac{1}{2} A_1 : n \mathrm{R}.$$

The ratios of the edges may be used as the units for determining the laws of decrement producing the planes belonging to the classes *b* and *d* of the modifications of this form.

RIGHT RHOMBIC PRISM.

Fig. 349.



The unit of comparison for decrements on the terminal edges, is the ratio of the line fg, fig. 349, to fl; fg being perpendicular to ik, and consequently to ab. For the law of decrement is to be determined from the inclination of the plane $abcd$, to one of the primary planes, let us suppose to P, which inclination would be measured on the lines gh, hd.

It is evident that the line fg falls within the description already given of the lines from which the unit of comparison may be derived; for, whatever the law of decrement may be which produces the new plane $abed$, we must have

$$fh : fb :: fg : fk.$$

The ratio of fg to fl may be thus discovered. We are supposed to have found

$$fk : fl :: m : n :: 1 : \frac{n}{m}$$

and knowing the angle efk, which we have called A_1, we known the angle $gfk = A_1 - 90°$.

hence $$fk : fg :: \text{R} : \cos.\ (A_1 - 90°)$$

$$:: 1 : \frac{\cos.\ (A_1 - 90°).}{\text{R}}$$

wherefore $$fg : fl :: \frac{\cos.\ (A_1 - 90°)}{\text{R}} : \frac{n}{m}$$

$$:: m \cos.\ (A_1 - 90°) : n\text{R}.$$

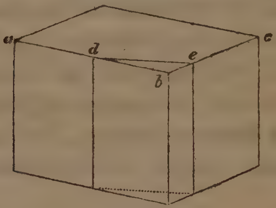
RIGHT RHOMBIC PRISM.

The required unit is therefore $\frac{m \cos.(A_1-90^\circ)}{n \mathrm{R}}$

For decrements on the lateral edges of the prism, the unit is $= 1$.

The methods of applying these several units to the determination of the laws of decrement, will be similar to some of those already described.

Fig. 350.



If we require the law of decrement producing the plane *d e*, fig. 350, and we know the angle *a d e*, which is the inclination of a primary plane on the plane *d e*, and may be called I_3, we shall find the ratio of the edges of the defect,

$$b\,d : b\,e :: \sin.(I_3-A_1) : \sin.(180^\circ-I_3) :: p : q,$$

which gives the required law of decrement.

THE RIGHT OBLIQUE-ANGLED PRISM.

Its elements.

The inclination of terminal on lateral planes = 90°.

Inclination of lateral planes to each other varies in the different individuals belonging to the class. Call the greater angle at which they incline to each other, I_1.

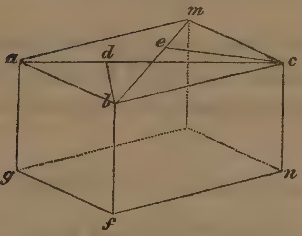
Terminal plane angles are consequently I_1, and (180°—I_1), but they will be called A_1 and A_2.

Lateral plane angles = 90°.

The adjacent edges unequal. Their ratio, and that of the diagonals of the terminal planes, can be known from secondary planes only.

Its units of comparison.

Fig. 351.



When the edges of any plane of class *a* or *b*, of the modifications of this form, which intersect the plane P, are parallel to a diagonal of that plane, the modifying plane results from a simple or mixed decrement.

The unit of comparison for determining the laws of simple or mixed decrements belonging to class *a*,

RIGHT OBLIQUE-ANGLED PRISM.

is the ratio of $b\,d$, a perpendicular on the diagonal, to the edge $b\,f$, which may be thus found.

Let the ratios of the edges be as follows,

$$b\,c : b\,a :: m : n :: 1 : \frac{n}{m}$$

$$b\,c : b\,f :: m ; o :: 1 : \frac{o}{m}$$

$$b\,a : b\,f :: n : o :: 1 : \frac{o}{n}$$

The angle $a\,b\,c$ has been called A_1

$b\,c\,m$ . . . . . . A_2

From the ratios of the edges and the angles A_1 and A_2, we may find the angles

$b\,a\,c = a\,c\,m$, call this A_3

$a\,b\,m = b\,m\,c$, . . . A_4

thus . . $b\,a : b\,d :: \mathrm{R} : \sin A_3 :: 1 : \frac{\sin. A_3}{\mathrm{R}}$

therefore $b\,d : b\,f :: \frac{\sin. A_3}{\mathrm{R}} : \frac{o}{n} :: n \sin. A_3 : o\,\mathrm{R}$.

The unit for determinining the laws of simple and mixed decrements belonging to class b is the ratio of $e\,c$, a perpendicular on the diagonal, to an edge $c\,n$, or $b\,f$.

We have $m\,c = a\,b : e\,c :: \mathrm{R} : \sin. A_4 :: 1 : \frac{\sin. A_4}{\mathrm{R}}$

and because $c\,n = b\,f$, we must have

$$e\,c : b\,f :: \frac{\sin. A_4}{\mathrm{R}} : \frac{o}{n} :: n \sin. A_4 : o\,\mathrm{R}.$$

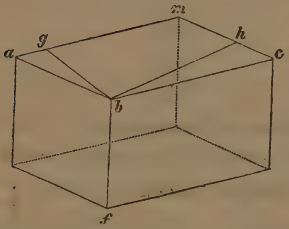
The ratios of the primary edges may be assumed as the units for determining the decrements producing the remainder of the planes belonging to class a and b; and they should be so adapted to each particular

RIGHT OBLIQUE-ANGLED PRISM.

case, as to give the particular values of the indices of each individual plane.

The following are the units of comparison for determining the laws of decrement on the edges.

Fig. 352.



For those which produce the planes *a*, the unit is the ratio of the perpendicular *g b*, fig. 352, to an edge *b f*; and for those which produce the planes *d*, the unit is the ratio of a perpendicular *b h*, to an edge *b f*.

To find $b\,g : b\,f$, we have

$$b\,a : b\,g :: \mathrm{R} : \sin. A_2 :: 1 : \frac{\sin. A_2}{\mathrm{R}}$$

and as . . . . $b\,a : b\,f :: 1 : \frac{o}{n}$

we must have $b\,g : b\,f :: n \sin. A_2 : o\,\mathrm{R}$.

To find $b\,h : b\,f$, we have

$$b\,c : b\,h :: 1 : \frac{\sin. A_2}{\mathrm{R}}$$

$$b\,c : b\,f :: 1 : \frac{o}{m}$$

$$b\,h : b\,f :: m \sin. A_2 : o\,\mathrm{R}.$$

RIGHT OBLIQUE-ANGLED PRISM.

For decrements on the lateral edges, where the symbol is

^{p}G the unit is $\frac{m}{n}$

G^{p} . . . . $\frac{n}{m}$

^{p}H . . . . $\frac{n}{m}$

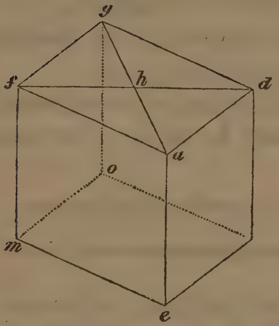
H^{p} . . . . $\frac{m}{n}$.

The application of these units to the determination of the several laws of decrement, will be similar to many of the examples already given.

THE OBLIQUE RHOMBIC PRISM.

Its elements.

Fig. 353.



The inclination of P on M, or M′, fig. 317, varies in different minerals; call it I_1.

Inclination of M on M′ also varies in different minerals, and may be called I_2.

Plane angle *f a d*, fig. 351, may be called A_1.

Let *g a e*, *g a d*, *d a e*, be the three sides of a spherical triangle, whose angles would be 90°, I_1, and $\frac{1}{2} I_2$. The side *g a d*, which is $\frac{1}{2} A_1$, may be thus found,

$$\cos. \tfrac{1}{2} A_1 = \frac{\text{R.} \cos. \frac{1}{2} I_2}{\sin. \frac{1}{2} I_1}$$

Plane angle *f a e*, or *d a e*, may be called A_2.

This angle is the hypothenuse of the triangle from which we have derived the preceding formula, and may therefore be thus known,

$$\cos A_2 = \frac{\cot. I_1 \cot. \frac{1}{2} I_2}{\text{R}}$$

Terminal edges equal.

Lateral edges equal.

OBLIQUE RHOMBIC PRISM.

The inclination of the oblique diagonal $a\,g$, to an edge $a\,e$, being called I_3, may be thus found,

$$\cos. I_3 = \frac{\text{R}.\cos. I_1}{\sin. \frac{1}{2} I_2}$$

The angle $g\,a\,c$ is the third side of the triangle already used.

Ratio of a terminal to a lateral edge can be known only by means of secondary planes.

Its units of comparison.

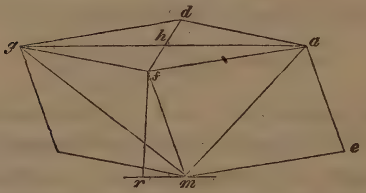
Let us suppose the ratio of $a\,f : a\,e :: m : n :: 1 : \frac{n}{m}$

The unit of comparison in reference to the decrements producing the planes of classes a and c, is the ratio of half an oblique diagonal $a\,h$ to a lateral edge $a\,e$, which may be thus found.

$$a\,f : a\,h :: \text{R} : \cos. \tfrac{1}{2} A_1 :: 1 : \frac{\cos. \frac{1}{2} A_1}{\text{R}}$$

Therefore $a\,h : a\,e :: m \cos. \frac{1}{2} A_1 : n\,\text{R}$.

Fig. 354.



The unit for determining the decrements producing those planes belonging to class e, of which the representative symbol is $\overset{\text{P}}{\text{E}}$, is the ratio of $\frac{1}{2}$ a horizontal diagonal $f\,h$ to the line $f\,r$ drawn from the solid angle at f perpendicularly on $r\,m$; the line $r\,m$ being parallel to the diagonal $g\,a$, and touching the solid angle at m. For the line $f\,r$ is evidently in a plane perpendicular to $f\,d$, and if the modifying planes we are considering were to be produced, they would cut $f\,m$ and $f\,r$ proportionally. The ratio of $f\,h$ to $f\,r$, is thus found.

$$f\,g : f\,h :: \mathrm{R} : \sin.\tfrac{1}{2} A_1 :: 1 : \frac{\sin.\tfrac{1}{2} A_1}{\mathrm{R}}$$

$$f\,g : f\,m :: 1 : \frac{n}{m}$$

Therefore $f\,h : f\,m :: m \sin.\tfrac{1}{2} A_1 : n\,\mathrm{R}.$

$$:: \frac{m \sin.\tfrac{1}{2} A_1}{n\,\mathrm{R}} : 1$$

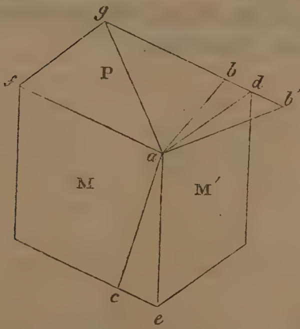
But $f\,r : f\,m :: \cos.\,(I_3 - 90^\circ) : \mathrm{R}$

$$:: \frac{\cos.\,(I_3 - 90^\circ)}{\mathrm{R}} : 1$$

Therefore $f\,h : f\,r :: \dfrac{m \sin.\tfrac{1}{2} A_1}{n\,\mathrm{R}} : \dfrac{\cos\,(I_3 - 90^\circ)}{\mathrm{R}}$

$$:: m \sin.\tfrac{1}{2} A_1 : n \cos.\,(I_3 - 90^\circ).$$

Fig. 355.



The unit for determining the laws of decrement producing the planes of classes f and g is the ratio of $a\,b$ to $a\,e$, fig. 355, when the angle $f\,a\,d$ is obtuse, or

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OBLIQUE RHOMBIC PRISM.

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of $a\,b'$ to $a\,e$, when that angle is acute; $a\,b$, or $a\,b'$ being perpendicular on the edge $g\,d$, and $a\,e$ perpendicular on the edge $e\,c$.

This ratio may be thus deduced.

$$a\,d : a\,b :: \text{R. cos. } (A_1-90^\circ) :: 1 : \text{cos. } \frac{(A_1-90^\circ)}{\text{R}}$$

$$a\,d : a\,e :: 1\ \frac{n}{m}$$

$$a\,b : a\,e :: m \text{ cos. } (A_1-90^\circ) : n\,\text{R}$$

$$a\,c : a\,e :: \text{cos. } (A_2-90^\circ) : \text{R}$$

$$a\,b : a\,c :: \frac{m \text{ cos. } (A_1-90^\circ)}{n\,\text{R}} : \frac{\text{cos. } (A_2-90^\circ)}{\text{R}}$$

$$:: m \text{ cos. } (A_1-90^\circ) : n \text{ cos. } (A_2-90^\circ).$$

The unit is equal to 1, in relation to decrements producing the planes i and l.

The units for determining the several laws of decrement producing the planes of classes b and d, and such of class e as are not represented by the symbol $\overset{\text{P}}{\text{E}}$, are the ratios of the primary edges; and the methods of determining the laws of decrement producing any modification of this form, will be analogous to some of those already described.

THE DOUBLY OBLIQUE PRISM.

Its elements.

The inclination of the primary planes is unequal at *any three adjacent edges*, and is different in different minerals.

Three adjacent plane angles unequal.

Three adjacent edges unequal, and the ratios of these inequalities are to be deduced only from some secondary planes.

The laws of decrement which produce the modifying planes of this class of primary forms, may be determined by the general methods already described at p. 295, and the units of comparison will then be the ratios of the primary edges.

The Hexagonal Prism.

Its elements.

The inclination of the adjacent lateral planes to each other = 120°.

Inclination of the terminal on the lateral planes = 90°.

Plane angles of the summit = 120°.

Lateral plane angles = 90°.

Ratio of the terminal to the lateral edge of each particular prism can be deduced only by means of some secondary plane.

Let us suppose it known, and expressed by

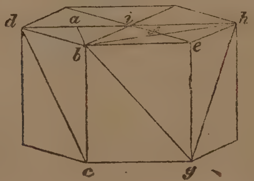
$$m : n, \text{ or by } 1 : \frac{n}{m}$$

Its units of comparison.

For decrements on the angles of the terminal plane, the unit is $\frac{m}{2n}$; and for decrements on the terminal edges it is $\frac{m \sin. 60^\circ}{n \text{R}}$, as will be shewn below.

For decrements on the lateral edges it is = 1.

Fig. 356.



The diagonals drawn on the terminal plane divide that plane into six equilateral triangles.

The law of any simple or mixed decrement on an angle of the prism is deduced from the ratio of

HEXAGONAL PRISM.

$e f : e g$, but $e f$ is half an edge of either of the equilateral triangles $b\ i\ c$ or $h\ i\ c$, whence

$$e f : e g :: \tfrac{1}{2} m : n :: m : 2n,$$

which thus becomes the unit of comparison for simple or mixed decrements on the angles of this prism.

The laws of *intermediary decrements* may be determined by means of spherical triangles adapted in the manner already described.

Decrements on the terminal edges.

A decrement by 1 row on the edge $b\ e$, fig. 356, would intercept proportional parts of the edges $b\ d$, $b\ c$, and consequently if the whole of $b\ d$ were intercepted by the new plane, the whole of $b\ c$, $e\ g$, and $e\ h$, would be intercepted also, and $d\ h$ would be the edge of the new plane $d\ h\ c\ g$. And we observe that the entire of the line $b\ a$, which is perpendicular to $d\ h$, would also be intercepted by the same plane. The ratio of $b\ a : b\ c$ may therefore be taken as the unit of comparison for determining the laws of decrement on the terminal edges of the hexagonal prism.

But $b\ a$ is perpendicular to $d\ i$, the base of the equilateral triangle $d\ i\ b$;

whence $d\ b : b\ a :: \mathrm{R} : \sin. 60^\circ :: 1 : \dfrac{\sin. 60^\circ}{\mathrm{R}}$

But . . . $d\ b : b\ c :: 1 : \dfrac{n}{m}$

Therefore $b\ a : b\ c :: m \sin. 60^\circ : n\,\mathrm{R}$.

The law of decrement on the *lateral edges* of the prism, will be represented by the units contained in the ratio of the edges of the *defect* occasioned by such decrement.

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The individual or particular prisms belonging to the seven preceding classes are, as we have seen, distinguishable from each other by the comparative lengths of two or three of their adjacent edges, or by the particular values of some of their plane angles.

These plane angles may be determined by means of spherical trigonometry from the inclination of the primary planes to each other; and this inclination may be ascertained by measurement with the goniometer.

But the comparative lengths of the edges can be deduced in no other manner than from some secondary plane, which, for that purpose, must be supposed to have been produced by a given law of decrement.

If for example we assume that any known secondary plane has been produced by a decrement by 1 row of molecules, the ratio of the edges of the *defect* of the primary form would, as we have already seen, be equal to the ratio of those edges of the primary form of which they are respectively portions.

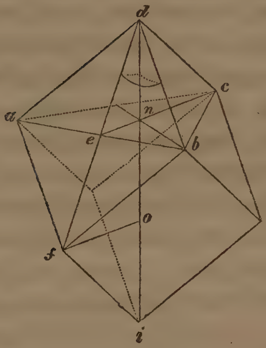
If therefore we determine the ratio of the edges of the *defect* occasioned by the interference of the secondary plane, which we suppose to have been produced by a decrement by 1 row of molecules, we shall, if our supposition be correct, evidently obtain the ratio of the corresponding primary edges.

But it may happen that the plane which we have supposed to result from a decrement by 1 row of molecules, is really produced by some other law of decrement.

The only method we possess of discovering whether we have determined the true dimensions of the prism, is to use those dimensions for ascertaining the laws of decrement producing *other secondary planes*; and if, when so applied, we find that the laws of decrement producing those planes are simple, we may conclude that we have determined the dimensions of our prism rightly; but if the ratios we have assigned to the primary edges, suppose those planes produced by irregular and extraordinary laws of decrement, we may conclude that the plane which we have first observed, has resulted from some other law than by 1 row of molecules, and we must proceed to assign new dimensions to the prism, in conformity with the new law, by which we now suppose the plane first observed to have been produced.

The Rhomboid.

Fig. 357.



In this fig. *d f* is the oblique diagonal,
a b . . . horizontal diagonal,
d i . . . axis.

If we imagine *a b*, *a c*, *b c*, to be edges of a plane passing through the solid, that plane would be perpendicular to the axis, and the line *c n e* drawn upon it, would consequently be perpendicular to the axis.

But the point *e* at which this perpendicular touches the plane *d a f b*, is the middle of the diagonal *d f*. If therefore we draw *f o* parallel to *e n*, we shall have $o\,n = n\,d$. But $f\,o = c\,n$, and $f\,i = d\,c$; therefore $i\,o = n\,d = o\,n$. Hence perpendiculars upon the axis of a rhomboid, drawn from two adjacent lateral solid angles, divide the axis into three equal parts.

Its elements.

The inclination of the adjacent planes at the superior edges will be different in different minerals, but may be designated generally by I_1.

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This angle is supposed generally to be measurable by the goniometer; but it may sometimes require to be deduced from the inclination of secondary planes to each other or to the primary. If two planes modifying the edge itself be used to determine the angle I_1, we must know the inclination of those planes to each other, and that of one of them on the adjacent primary plane; and hence the inclination of the primary planes is known.

If we know the inclination of the plane *a* of the modification of the rhomboid, to the plane P, and call it I, we may determine I_1 from the following equation,

$$\cos.\ \tfrac{1}{2}\, I_1 = \frac{\cos.\ 30°.\ \sin.\ (180°-I)}{R}$$

It is apparent from the above fig. that if $(180°-I)$ and $\frac{1}{2}\, I_1$ be taken as two angles of a spherical triangle, the third angle must be 90°, and the side subtending the angle $\frac{1}{2}\, I_1$ must be 30°, and hence the given equation is derived.

If we know the inclination to the plane P, of that plane of mod. *e* which replaces the inferior angle of the plane P, we may still deduce I_1 from that inclination, by the preceding formula. For plane *e* on P is obviously $270°-I$, and consequently $I = 270°-(e \text{ on } P)$.

The inclination of the adjacent planes at the inferior edges will consequently be $(180°-I_1)$ and may be called I_2.

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The superior plane angles may be called A_1, and may be thus found,

The angle $\frac{1}{2} A_1$ is one of the sides of the spherical triangle marked on fig. 357, whose angles are evidently 90°, 60°, and $\frac{1}{2} I_1$. The general formula to determine $\frac{1}{2} A_1$ is

$$\cos. \tfrac{1}{2} A_1 = \frac{\text{R}. \cos. 60^\circ}{\sin. \frac{1}{2} I_1}$$

but as $\cos. 60^\circ = \frac{\text{R}}{2}$, the formula becomes

$$\cos. \tfrac{1}{2} A_1 = \frac{\text{R}^2}{2 \sin. \frac{1}{2} I_1}$$

Lateral plane angles will be $(180^\circ - A_1)$ and may be called A_2.

Inclination of the superior edge to the axis may be called A_3, and may be thus found,

The angle *b d n*, fig. 357, is the inclination required, and is the hypothenuse of the triangle already used, whence

$$\cos. A_3 = \frac{\cot. 60^\circ \ \cot. \frac{1}{2} I_1}{\text{R}}$$

Inclination of the oblique diagonal to the axis may be denoted by A_4, and may be thus found,

The angle *e d n* which we call A_4, is the third side of the triangle marked on the figure, and may consequently be determined from the formula,

$$\cos. A_4 = \frac{\text{R}. \cos. \frac{1}{2} I_1}{\sin. 60^\circ}$$

Sum of the two preceding angles, being the inclination of an oblique diagonal to a superior edge, when measured over the summit, may be denoted by A_5.

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The inclination of an oblique diagonal to an adjacent edge, measured over the inferior angle, would be 180°—A_5, and may be denoted by A_6.

Ratio of a perpendicular upon the axis drawn from a lateral solid angle, to the axis itself, is as $\frac{1}{3}$ tang. A_3 : R.

We have already seen that $d\,n$ is $\frac{1}{3}$ of the axis $d\,i$,

and $d\,n : n\,b :: \text{R} : \text{tang. } A_3$

Therefore $d\,i = 3\,d\,n : a\,b :: 3\,\text{R} : \text{tang. } A_5$

$:: \text{R} : \frac{1}{3}\,\text{tang.} A_3$.

Ratio of $\frac{1}{2}$ the oblique diagonal, to $\frac{1}{2}$ the horizontal diagonal, as R : tang. $\frac{1}{2}\,A_1$.

Ratio of $\frac{1}{3}$ of the axis : $\frac{1}{2}$ half the oblique diagonal :: cos. A_4 : R.

Ratio of $\frac{1}{3}$ of the axis : $\frac{1}{2}$ the horizontal diagonal :: cot. A_4 : tang. 60°.

For $d\,n : n\,e :: \text{tang. } (90° - A_4) : \text{R}$

$:: \text{cot. } A_4 : \text{R}$

and $e\,b : n\,e :: \text{tang. } 60° : \text{R}$

Therefore $d\,n : e\,b :: \text{cot. } A_4 : \text{tang. } 60°$.

Ratio of $\frac{1}{2}$ an oblique diagonal : an edge :: cos. $\frac{1}{2}\,A_1$: R.

Ratio of $\frac{1}{2}$ a horizontal diagonal : an edge :: sin. $\frac{1}{2}\,A_1$: R.

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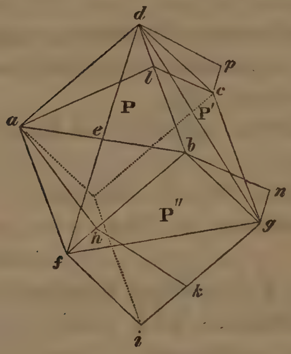
Its units of comparison.

Fig. 358.



The following are the units of comparison in relation to the decrements producing the several classes of modifications contained in the tables.

Class *b*, *c*, *e*, *g*, *k*, unit $\frac{\cos.\ \frac{1}{2}A_1}{R}$, being the ratio of ½ an oblique diagonal : an edge.

Class *h*, the unit is $\frac{\sin.\ \frac{1}{2}A_1}{\sin.\ A_5}$, when the rhomboid is acute.

and . . . $\frac{\sin.\ \frac{1}{2}A'_1}{\cos.\ (A_5-90^\circ)}$, when it is obtuse.

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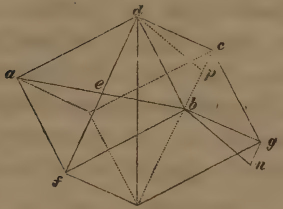
Fig. 359.



This unit is the ratio of $e\,b : b\,n$, fig. 398 and 399, $b\,n$ being perpendicular on $g\,n$, which is parallel to $d\,f$. Fig. 398 represents an acute rhomboid,

in which $e\,b : b\,f$, or $b\,g :: \text{sin. } \frac{1}{2} A_1 : \text{R}$

$b\,n : b\,g :: \text{cos. } (90^\circ - A_5) : \text{R} :: \text{sin. } A_5 : \text{R}$

therefore $e\,b : b\,n :: \text{sin } \frac{1}{2} A_1 : \text{sin. } A_5$.

Fig. 399 represents an obtuse rhomboid,

in which $e\,b : b\,g :: \text{sin. } \frac{1}{2} A_1 : \text{R}$

$b\,n : b\,g :: \text{cos. } (A_5 - 90^\circ) : \text{R}$

and . . . $e\,b : b\,n :: \text{sin. } \frac{1}{2} A_1 : \text{cos. } (A_5 - 90^\circ)$.

If in fig. 358 we suppose $c\,p$ a portion of the oblique diagonal produced, and in both 398 and 399, $d\,p$ parallel to $b\,n$, the assumed value of $b\,n$ will be readily perceived.

The unit of comparison, in relation to decrements producing the classes *d*, *f*, *i*, and *l*, is the ratio of the edges and is consequently $= 1$.

In relation to class *n*, it is that of two equal perpendiculars $l\,a$, $l\,c$, on a superior edge, drawn from two lateral angles; and in relation to class *p*, it is that of two equal perpendiculars $h\,a$, $h\,k$, on an inferior edge, drawn from the parallel superior edges, and it is consequently in both cases $= 1$.

The determination of the laws of decrement affecting the rhomboid, and the developement of the various relations between that primary form and its numerous secondary forms, occupy a considerable portion of the Abbé Haüy's crystallographical researches. Some, however, of the relations he has demonstrated, though curious in themselves, are not immediately useful to the mineralogist for determining the mineral species to which a given crystal belongs.

Little more will be attempted here than to give an outline of a method of determining the laws of decrement, similar to that which has been applied to the other classes of primary forms; and it is hoped that this will supply the mineralogist with as much assistance as his purposes will generally require.

Simple and mixed decrements on the superior and inferior angles.

The planes produced by these are contained in the *classes b, c, e g*, and *k*.

Let us be supposed to have measured the inclination of one of the planes *b* on an adjacent primary plane, or of one of the planes *c*, over the summit on a primary plane, and if we call the measured angle I_3, the ratio of the portions of the oblique diagonal and edge contained in the *defect*, would be as

$$\text{sin.}\ (I_3 - A_5) : \text{sin.}\ (180^\circ - I_3),$$

and if we divide this ratio by $\frac{\text{cos.}\ \frac{1}{2} A_1}{\text{R}}$, we shall obtain the law of decrement producing the plane we have measured.

Let us suppose the inclination of one of the planes belonging to the *classes e, g, or k*, on that primary plane which is intersected by the modifying plane parallel to its diagonal, to be called I_4, the ratio of the edges of the defect will be as

$$\sin. (I_4 - A_6) : \sin. (180° - I_4).$$

This being divided as before by cos. $\frac{1}{2} A_1 : R$, will give the law of decrement producing the plane we have measured.

Simple and mixed decrements on the lateral angles.

Let us suppose the inclination known of the primary plane P, to one of the adjacent planes of mod. *h*, and let this be called I_5. The angle measured, would be in the plane *e b n*, fig. 358. And as *e b* is perpendicular to *b n*, the ratio of the edges of the defect would be as radius to tang. of the supplement of the measured angle; and this being divided by cos. $\frac{1}{2} A_1 : R$, will give the required law of decrement.

Intermediary decrements on the terminal solid angles.

The general symbol representing a single plane of mod. *d* is $(B_p \; B'_q \; B''_r)$.

The values of the indices *p*, *q*, and *r* may be discovered from the inclination of the particular plane represented by that symbol, on the two adjacent primary planes, by means of a spherical triangle, and the plane triangles, adapted in the manner already described.

Intermediary decrements on the lateral solid angles.

These, as we have already seen in our account of the symbols representing the planes produced by them, may be referred to the angle at O, or to that at E. Let the plane from which we are about to

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deduce our required law of decrement be one of those which appears on the angle at O.

There are, as may be seen in the tables, three classes of modifications produced by these decrements, being *classes f, i, and l.*

The distinction between these three classes has been already pointed out in the table of modifications of the rhomboid, and in p. 279, where their several relations to the theory of decrements are given.

The general symbol representing one of the planes at the angle O, belonging to either of these modifications, is $(D'_q\ b''_r\ D_p)$, and the particular value of the indices may be discovered, in relation to any particular modification belonging to either class, by measuring the inclination to the two primary planes adjacent to the edge b″, of the plane represented by the above symbol; and finding the plane angles of the *defect* adjacent to the edge b″, by means of a spherical triangle, and thence deducing the ratio of the edges of the defect in the manner already described.

The indices of the individual modifications belonging to class *f* will be found in a constant ratio to each other. This results from the condition that the edge at which the modifying planes intersect each other shall be parallel to the axis of the rhomboid.

Let the index p be $> q$, and $q > r$.

The relations between p, q, and r, may be thus stated,

$$p = q\ (q-1) = q\ r.$$

$$q = \frac{1 + \sqrt{4\,p+1}}{2} = \frac{p}{r}.$$

$$r = \frac{\sqrt{(4\,p+1)} - 1}{2} = q - 1 = \frac{p}{q}.$$

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And the manner of deducing these may be given as an example of one of the methods* of analysis applicable to these investigations.

The general equation of a plane in relation to three co-ordinate axes, is known to be

$$A\,x + B\,y + C\,z + D = o.$$

Let the distances from the origin at which the plane cuts the three axes, be represented by p, q, and r.

We shall then have the following equations of the points where the axes are cut by the plane.

For the point on the axis x we have $x = p$,

y . . . . $y = q$,

z . . . . $z = r$.

To find the values of the co-efficients A B C D, in function of the quantities p, q, and r, with a view to substitute those quantities in the general equation for the co-efficients A, B, C, and D,

Let $y = o$, $z = o$, and $x = \dfrac{-D}{A}$

$$x = o,\ z = o,\ \therefore y = \frac{-D}{B}$$

$$x = o,\ y = o,\ \therefore z = \frac{-D}{C}$$

Therefore $\dfrac{-D}{A} = p$, whence $A = \dfrac{-D}{p}$

$$\frac{-D}{B} = q,\ \therefore\ B = \frac{-D}{q}$$

$$\frac{-D}{C} = r,\ \therefore\ C = \frac{-D}{r}$$

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* This *method* of determining the relations that may exist among crystals, has been used in a paper published by Mr. Levy in the Edinburgh Philosophical Journal, relative to another object which will be referred to in a later part of this Appendix.

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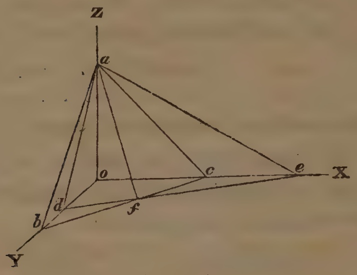
The general equation may therefore be thus expressed,

$$\frac{-\mathrm{D}\,x}{p} + \frac{-\mathrm{D}\,y}{q} + \frac{-\mathrm{D}\,z}{r} = -\mathrm{D}$$

or, dividing all the terms by —D, it may be reduced to this general form,

$$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1.$$

Fig. 360.



Let the plane *a d e*, fig. 360, represent one of the planes belonging to class *f*, whose indices are *p*, *q*, and *r*; and let

$o\,e = p.$
$o\,d = q,$
$o\,a = r;$

the equation of this plane will then be,

$$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1.$$

But as the line *a f* at which the two planes of mod. *f* intersect each other, is parallel to the axis of the rhomboid, and passes through one of its superior edges, it might obviously be on the surface of some plane belonging to mod. e. The equation of this plane may be thus expressed,

$$\frac{x}{p'} + \frac{y}{p'} + \frac{z}{r} = 1.$$

From the character of the planes of mod. e, the index p' is always $= 2\,r$.

Knowing the relation of p' to r, we may discover the relations of p and q to r, by finding their relations to p'; and these relations may be known from the equations of the traces of the two planes on the plane of the $x\,y$, when referred to the point f.

The equations of these traces are obtained by making $z = o$ in the two preceding equations, whence

the equation of the trace $d\,e$, is $\frac{x}{p} + \frac{y}{q} = 1$ (1)

and of $b\,c$ . . . . $\frac{x}{p'} + \frac{y}{p'} = 1$ (2)

But as both traces pass through the point f, the values of x and of y must be equal in both equations.

Hence from equation (1), $y = q - \frac{q\,x}{p}$

(2), $y = p' - x$

Therefore . . . $q - \frac{q\,x}{p} = p' - x$

$$x - x\frac{q}{p} = p' - q$$

$$x\left(1 - \frac{q}{p}\right) = p' - q$$

$$x = \frac{p' - q}{1 - \frac{q}{p}}$$

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In the like manner we may find

$$y = \frac{p' - p}{1 - \frac{p}{q}}$$

But at the point f, $x = y$,

therefore . . . $\frac{p' - q}{1 - \frac{q}{p}} = \frac{p' - p}{1 - \frac{p}{q}}$

whence . . . $p' = \frac{2\,p\,q}{p+q}$

$$p = \frac{p'\,q}{2\,q - p'}$$

$$q = \frac{p\,p'}{2\,p - p'}$$

As we know that $p' = 2\,r$, and that r cannot be less than 1, p' cannot be less than 2; and it must, in relation to any particular plane of mod. e, be either 2, 4, 6, 8, or some greater even number, according to the number of molecules supposed to be contained in the defect occasioned by that plane.

It may be easily seen that when $p' = 2$, we must have $q = \frac{3}{2}$.

But as the indices of planes produced by intermediary decrements must be whole numbers, it follows that the planes $a\,b\,c$, and $a\,d\,c$, cannot both pass through the point f, unless p' be greater than 2.

Let $p' = 4$, and $4 = \frac{2\,p\,q}{p+q}$

whence $p = \frac{2\,q}{q-2}$

If we regard the figure 360, we may perceive that if $o\,b$, which we have called p', be considered equal to 4, the line *o d*, which we have called q, must be greater than 2; for as the line *b c* is equally divided at the point *f*, if $d\,o = d\,b$, the line *d f* would be parallel to *o c*.

Therefore when $p' = 4$, we must have $q = 3$, and consequently $p = 6$.

And as $r = \frac{p'}{2}$, if we suppose the value of q to be successively increased to 4, 5, 6, &c. we shall have the following series of indices to represent the series of planes of class *f*.

$p = 6$	$q = 3$	$r = 2$
. . 12	. . 4	. . 3
. . 20	. . 5	. . 4
. . 30	. . 6	. . 5
. . 42	. . 7	. . 6
&c.		

From what has been stated in the preceding pages, it will be readily perceived, that when, in addition to the inclination of the primary planes to each other, we know the unit of comparison, and the inclination of the secondary plane to the primary plane along which the decrement is conceived to proceed, we may immediately determine the law of decrement. For we can from these data directly deduce the ratio of the lines of the defect corresponding with those from whence we derive our unit; and if we divide this ratio by our assumed unit, we obtain, as we have before observed, the law of decrement producing the plane we have measured.

The preceding sketch of the methods of discovering the laws of decrement, will, it is hoped, be generally found sufficient for that purpose, when the angles at which the secondary planes incline on the primary are known, and where the ratios of the edges or other lines already described are also known.

But it very frequently happens that the whole of the primary planes are obliterated by such an extension of the secondary planes, as produces an entire secondary crystal. In these cases we must recur to cleavage for determining the relative positions of the primary and secondary planes, and for measuring the angle at which they meet. The cleavage planes which we may adopt as the *primary* set, if more than one set be discoverable, should be those which are most compatible with the observed secondary forms.

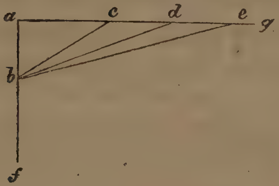
Having thus given an outline of the solution of our first problem, by shewing how the laws of decrement may be determined from certain data, we shall proceed to examine the second, and to ascertain *how the angles may be determined at which the secondary planes incline on the primary, the elements of the primary form, and the law of decrement, being known.*

The methods used for determining these angles, will be nearly similar to those already described for determining the laws of decrement.

The plane triangles which have been used for determining the laws of decrement, have been both *right-angled* and *oblique.*

Where a law of decrement is expressed by means of the ratio of the sides of *a right-angled triangle*, the angles are readily found by *reducing the ratio to that of radius, and tangent of the required angle.*

Fig. 361.



Let fig. 361 represent a section of any crystal whose planes *a f*, *a g*, are perpendicular to each other, and let the lines *b c*, *b d*, *b e*, be sections of planes modifying the edge or angle *g a f*.

Thus let us suppose that we have the law of decrement given by which the plane *b c* has been produced; and let the required angle *g c b* be called *I*.

Let the ascertained ratio of the edges *a g*, *a f*, be as 5 : 4, and the law of decrement producing the plane *b c*, be 1 row of molecules.

It follows that $a\,c : a\,b :: 5 : 4 :: 1 : \frac{4}{5}$.

But we also have $a\,c : a\,b ::$ R : tang. $(180° - I)$

therefore $\frac{4}{5} = \cdot 8 =$ tang. $(180° - I)$

and ·8, in the table of natural tangents, is the tang. of angle 38° 40′ nearly, $= 180° - I$;
therefore $I = 180° - (38° \; 40') = 141° \; 20'$.

If we suppose *b d* the section of a plane resulting from a decrement by 2 rows in breadth, we should obviously have

$$a\,d : a\,b :: 10 : 4 :: 5 : 2 :: 1 : \frac{2}{5}.$$

And if we call the angle *g d b*, *I′*, we must have

$$a\,d : a\,b :: \text{R. tang.} \, (180° - I')$$

whence $\frac{2}{5} = \cdot 4 = \tan\ (180^\circ - I') = 21^\circ\ 48'$ nearly,

hence $I' = 180^\circ - (21^\circ\ 48') = 158^\circ\ 12'$.

By this method we may in all similar cases determine any required angle, whatever may be the ratio of *a g* : *a f*.

Where the planes *a c*, *a b*, are not at right angles to each other, the angle *c a b*, may be either acute or obtuse. In either case knowing the angle *c a b*, and the particular values of *a c* and *a b*, deduced from the known ratio of *m* : *n*, and from the given law of decrement, we may obtain the angle *a b c* from the formula

$$\text{tang. } \tfrac{1}{2}\, d = \frac{d'.\ \text{tang. } \tfrac{1}{2}\, s}{s'}$$

where d = difference of required angles,
d' = difference of given sides,
s = sum of required angles = 180°—given angle.
s' = sum of given sides.

Where spherical triangles have been used for determining the law of decrement, they may also be used for determining the angles of the secondary planes with the primary, the law of decrement being known; with this difference however, that where in the former examples we have sought the sides of those triangles, knowing the angles, we have now to determine the angles from the given sides: and the sides are known from the plane angles of the primary crystal, and from the ratio of the edges of the *defect* of the primary form, as deduced from the ratios of the corresponding primary edges, and the law of decrement.

The preceding parts of this section suppose the angles known at which the secondary plane whose law of decrement is required, inclines on one or more of the primary planes. But it may sometimes occur that the inclination of the secondary on the primary planes cannot be directly obtained. In certain cases, where this happens, the laws of decrement may be deduced from the inclination of the secondary planes to each other.

We shall suppose in the following examples of one or two particular and simple cases of this nature, that the unit of comparison is expressed by $\frac{m}{n}$, and the ratio of the edges or other lines of the defect by $\frac{p\,m}{q\,n}$. Whence $\frac{p}{q}$ will express the law of decrement by p molecules in breadth and q molecules in height.

It has been already stated that where an edge is replaced by two *similar* planes, m will always be found to equal n, and the fraction $\frac{p\,m}{q\,n}$, or its equivalent $\frac{p}{q}$, when reduced to its lowest terms in whole numbers, will express the ratio of the edges of the defect.

1. Let us suppose the edge of a cube replaced by 2 *similar* planes as in mod. *f*, or the lateral edge of a right square prism, as in mod. *e*. And let the inclination of the secondary planes to each other be called I. We shall find

$$\frac{p}{q} = \frac{\text{R}}{\text{tang.}\ (\frac{1}{2}\,I - 45^\circ)}$$

and the inclination of either of the new planes on the adjacent primary plane would be $= 225° - \frac{1}{2} I$.

2. Let the lateral edges of the right or oblique rhombic prism, or any edge of the rhomboid be modified by 2 *similar* planes, and let the inclination of the primary planes to each other $= I$, and that of the secondary planes to each other $= I'$.

Then $$\frac{p}{q} = \frac{\sin.\ [180° - (\frac{3}{2} I' + \frac{1}{2} I)]}{\sin.\ (\frac{1}{2} I' - \frac{1}{2} I)}$$

And the inclination of either of the new planes on the *adjacent* primary planes would be $= 180° + \frac{1}{2} I - \frac{1}{2} I'$.

The following application to a particular case of the proposed methods of calculation, will probably be sufficient to illustrate their general use.

It will be found convenient, if we have to determine the laws of decrement producing secondary planes upon any primary form, to determine in the first place the particular values of such of the elements of that form as we may require for ascertaining those laws of decrement; and the values so determined may be reserved for any future occasion.

Let it be required to determine the elements of the rhomboid of carbonate of lime :

Here the angle
$I_1 = 105^\circ\ 5'$
$I_2 = 74^\circ\ 55'$
$A_1 = 101^\circ\ 55'$.

We have seen that $\cos.\ \tfrac{1}{2} A_1 = \dfrac{R^2}{2 \sin.\ \tfrac{1}{2} I_1}$

By means of the *tables of logarithms* we find the angle $\tfrac{1}{2} A_1$, thus,

$$\begin{array}{lrr}
\log.\ R^2 \ \ldots\ldots\ldots\ldots & & = 20\cdot \text{———} \\
\log.\ 2 \ \ldots\ldots\ldots & = 0\cdot3010300 & \\
\log.\ \sin.\ \tfrac{1}{2} I_1 = & & \\
\quad \log.\ \sin.\ 52^\circ\ 32'\ 30'' & = \underline{9\cdot8997088} & \\
 & & 10\cdot2007388 \\
\text{Therefore} \ \ldots\ \log.\ \cos.\ \tfrac{1}{2} A_1 & = & \underline{\overline{9\cdot7992612}}
\end{array}$$

Therefore $\tfrac{1}{2} A_1 = 50^\circ\ 57'\ 30''$, and consequently $A_1 = 101^\circ\ 55'$.

$A_2 = 78^\circ\ 5'$
$A_3 = 63^\circ\ 44'\ 45''$, which may be found thus from the

$$\text{formula} \ldots \cos.\ A_3 = \frac{\cot 60^\circ\ \cot.\ \tfrac{1}{2} I_1}{R} = \frac{\cot.\ 60^\circ\ \cot.\ 52^\circ\ 32'\ 30''}{R}$$

$$\begin{array}{lr}
\log.\ \cot.\ 60^\circ \ \ldots & = 9.7614394 \\
\log.\ \cot.\ 52^\circ\ 32'\ 30'' & = \underline{9.8843264} \\
 & 19.6457658 \\
\log.\ R \ \ldots\ldots & = \underline{10.\ \text{———}} \\
\log.\ \cos.\ A_3 \ \ldots & = \underline{9.6457658}
\end{array}$$

Therefore angle $A_3 = 63^\circ\ 44'\ 45''$.

$A_4 = 45° \, 23' \, 26''$, which may be thus found.

$$\cos. A_4 = \frac{R. \cos. \frac{1}{2} I_1}{\sin. 60°} = \frac{R. \cos. 52° \, 32' \, 30''}{\sin. 60°}$$

$$\begin{array}{lr}
\log. R \; . \; . \; . \; . \; . = & 10\cdot \text{———} \\
\log. \cos. 52° \, 32' \, 30'' = & 9\cdot7840353 \\
\hline
 & 19\cdot7840353 \\
\log. \sin. 60° \; . \; . \; . = & 9\cdot9375306 \\
\hline
\log. \cos. A_4 \; . \; . = & 9\cdot8465047 \\
\hline
\end{array}$$

Therefore angle $A_4 = 45° \, 23' \, 26''$.
$A_5 = 109° \, 8' \, 11''$.
$A_6 = 70° \, 51' \, 49''$.

The following ratios may be known from the tables of natural sines, &c. when we require the nearest whole numbers by which they may be represented; or their logarithms may be taken from the tables, when we use logarithms only in our calculations.

Perpendicular on axis drawn from lateral solid angle : axis :: $\frac{1}{3}$ tan.A_3 : R :: $\frac{1}{3}$ tang. 63° 44′ 45″ : R :: $\frac{1}{3}$ 2·0274279 : 1 :: ·6758093 : 1.

This may be reduced with sufficient accuracy to its lowest equivalent terms in whole numbers, by means of a common sliding rule, and will be found as 23 : 34 very nearly.

Or if the logarithms be required, we have

$$\begin{array}{lr}
\log. \text{tang. } 63° \, 44' \, 45'' = & 10\cdot3069454 \\
\log. 3 \; . \; . \; . \; . \; . = & 0\cdot4771213 \\
\hline
 & 9\cdot2298241 \\
\log. R \; . \; . \; . \; . \; . = & 10 \text{ ———} \\
\hline
 & -\;0\cdot0701759 \\
\hline
\end{array}$$

$\frac{1}{2}$ oblique diagonal : $\frac{1}{2}$ horizontal diagonal :: R : tang. $\frac{1}{2}A_1$ :: R : tang. 50° 57′ 30″ :: 1 : 1·2330626 :: 17 : 21 very nearly.

or log. R . . . . . = 10· ———
log. tang. 50° 57′ 30″ = 10·0909851
— 0·0909851

$\frac{1}{3}$ axis : $\frac{1}{2}$ oblique diagonal :: cos. A_3 : R :: cos. 63° 44′ 45″ : R :: ·4423539 : 1 :: 31 : 70 very nearly.

or log. cos. 63° 44′ 45″ = 9·6457698
log. R . . . . = 10· ———
— 0·3542302

$\frac{1}{3}$ axis : $\frac{1}{2}$ horizontal diagonal :: cos. $\frac{1}{2}A_3$: tang. $\frac{1}{2}A_1$ :: cos. 31° 52′ 22″ : tang. 50° 57′ 30″ :: ·849227 : 1·2330626 :: 11 : 16 nearly.

or log. cos. 31° 52′ 22″ = 9·9290216
log. tang. 50° 57′ 30″ = 10·0909851
— 0·1619635

$\frac{1}{2}$ oblique diagonal : edge :: cos. $\frac{1}{2}A_1$: R :: cos. 50° 57′ 30″ : R :: ·6298254 : 1 :: 12 : 19 nearly.

or log. cos. 50° 57′ 30″ = 9·7992615
log. R . . . . = 10· ———
— 0 2007385

$\frac{1}{2}$ horizontal diagonal : edge :: sin. $\frac{1}{2}A_1$: R :: sin. 50° 57′ 30″ : R :: ·7766881 : 1 :: 7 : 9 nearly.

or log. sin. 50° 57′ 30″ = 9·8902466
log. R . . . . = 10· ———
— 0·1097534

$\frac{1}{2}$ horizontal diagonal : perpendicular bn (fig. 358 & 9) $:: \sin. \frac{1}{2} A_1 : \cos. (A_5 - 90°) :: \sin. 50° 57' 30'' : \cos. 19° 8' 11'' :: \cdot 7766881 : \cdot 9447409 :: 46 : 56$ nearly.

$$
\begin{array}{lcr}
\text{or log. sin. } 50° 57' 30'' & = & 9 \cdot 8902466 \\
\text{log. cos. } 19° 8' 11'' & = & 9 \cdot 9753128 \\
\hline
 & - & 0 \cdot 0850662 \\
\hline
\end{array}
$$

Having thus determined the elements of the rhomboid of carbonate of lime, which we may remark are all deduced from the single angle I_1, we may proceed to determine the laws of decrement producing any of its observed secondary planes.

Let us now suppose that we have measured the inclination to the plane P, of a plane belonging to class b of the rhomboid, and that we have found it 143° 28'.

We have already seen in p. 366, that the ratio of those lines of the defect occasioned by the planes b, from which the law of decrement is to be deduced, may be expressed by the fraction

$$\frac{\sin. (I_3 - A_5)}{\sin. (180° - I_3)}$$

In relation to the plane we have measured we find

$$
\begin{array}{lcr}
 & I_3 = & 143° 28' \\
\text{and we have found . } & A_5 = & 109° \ 8' 11'' \\
\hline
\text{therefore } I_3 - A_5 \quad . \ . & = & 34° 19' 40'' \\
\hline
\text{and } 180° - I_3 \text{ is evidently} & = & 36° 32'.
\end{array}
$$

The ratio of those lines of the defect from which the law of decrement may be deduced, is therefore in this particular case

$$\frac{\sin. 34° 19' 49''}{\sin. 36° 32'}$$

If we recur to the tables of *natural sines*, we shall find the numbers constituting this ratio to be nearly $\frac{5639}{5952}$, which fraction being reduced to its lowest terms will be $\frac{18}{19}$.

This, as we have already shewn, is to be divided by the unit of comparison; which in this instance is the ratio of $\frac{1}{2}$ the oblique diagonal to an edge, and has been found equal to $\frac{12}{19}$.

But to divide $\frac{18}{19}$ by $\frac{12}{19}$, we must invert the terms of the latter fraction, and then multiply the first by it.

Hence $\frac{18}{19} \times \frac{19}{12} = \frac{18}{12} = \frac{3}{2}$, which gives a law of decrement by 3 rows in breadth and 2 in height proceeding along the plane P.

If however instead of using the natural sines, &c. we use only logarithms, the law of decrement may be thus determined.

$$\begin{array}{lr} \text{log. sin. } 34^\circ\ 19'\ 49'' = & 9{\cdot}7512503 \\ \text{log. sin. } 36^\circ\ 32' = & 9{\cdot}7747288 \\ \hline & -\ 0{\cdot}0234783 \\ \hline \end{array}$$

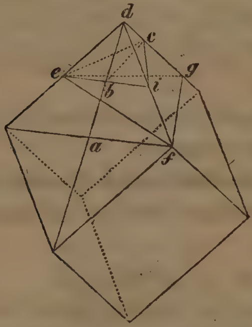
To divide this by the unit of comparison, we must subtract the logarithm of that unit, which is given in p. 381, from the above logarithm of the ratio of the edges of the defect; this may be done by the ordinary method of subtracting algebraic quantities, by changing the sign of the quantity to be subtracted and then adding;

hence if to — 0·0234783
we add . . + 0·2007385

we have + 0·1772602 as the logarithm of the natural number which is to determine the law of decrement.

But the natural number corresponding to 0·1772602 is 1·504, and which, if we disregard the last figure, 4, may obviously be expressed by the fraction $\frac{15}{10} = \frac{3}{2}$, a result similar to that which has been already found by means of the natural sines.

Fig. 362.



Let us next require the law of an intermediary decrement producing a particular modification of the rhomboid belonging to class *d*. And let us suppose that we have measured the inclination to P and P′, of that plane with the letter *d* upon it, which rests on P′, see fig. p. 204.

Let *d* on P be found = 132° 13′
d . . P′ . . . . . = 145° 57′

Hence 180°—132° 13′ = 47° 47′, which we may call I_6
and 180°—145° 57 = 34° 3′ . . . . I_7
I_1 it will be recollected is 105° 5′.

We first require the plane angles *e f d*, *d f g*, fig. 362, which may be thus found.

$\sin. \tfrac{1}{2} \vee e f d =$

$$R \sqrt{\frac{-\cos. \tfrac{1}{2}(I_1+I_6+I_7) \cos. \tfrac{1}{2}(I_1+I_6-I_7)}{\sin. I_1 \sin. I_6}}$$

$\sin. \tfrac{1}{2} \vee d f g =$

$$R \sqrt{\frac{-\cos. \tfrac{1}{2}(I_1+I_7+I_6) \cos. \tfrac{1}{2}(I_1+I_7-I_6)}{\sin. I_1 \sin. I_7}}$$

But $I_1 = 105^\circ\ 5'$
$I_6 = 47^\circ\ 47'$

$152^\circ\ 52' + I_7,\ 34^\circ\ 3' = 186^\circ\ 55'$, $\tfrac{1}{2}$ of which is $93^\circ\ 27'\ 30$

and . . . $152^\circ\ 52' - 34^\circ\ 3' = 118^\circ\ 49'$, $\tfrac{1}{2}$ of which is $59^\circ\ 24'\ 30$

Again $I_1 = 105^\circ\ 5'$
$I_7 = 34^\circ\ 3'$

$139^\circ\ 8' + I_6,\ 47^\circ\ 47' = 186^\circ\ 55'$ $\tfrac{1}{2}$ of which is $93^\circ\ 27'\ 30''$

and . . . . $139^\circ\ 8' - 47^\circ\ 47' = 91^\circ\ 21'$, $\tfrac{1}{2}$ of which is $45^\circ\ 40'\ 30''$.

The preceding general formulæ therefore become,

$\sin. \tfrac{1}{2} \vee e f d =$

$$R \sqrt{\frac{-\cos. 93^\circ\ 27'\ 30'' \ \cos. 59^\circ\ 24'\ 30''}{\sin. 105^\circ\ 5' \ \sin. 47^\circ\ 47'}}$$

$\sin. \tfrac{1}{2} \vee d f g =$

$$R \sqrt{\frac{-\cos. 93^\circ\ 27'\ 30'' \ \cos. 45^\circ\ 40'\ 30''}{\sin 105^\circ\ 5' \ \sin. 34^\circ\ 3'}}$$

These equations may be resolved by the assistance of the table of logarithms in the following manner.

log.—cos. 93° 27′ 30″=l. cos. 86° 32′ 30″ = 8·7804792
log. cos. 59° 24′ 30″ . . . . = 9·7066463

18·4871255

log. sin. 105° 5′=l. sin. 74° 55′=9·9847740
log. sin. 47° 47′ . . . . . =9·8695891
19.8543631

To extract the square root of this quotient,—1.3672376

divide it by 2, and ½ is = — 0·6836188
log. R. . . . . = 10·

log. sin. ½ ∨ *efd* . . . . = 9·3163812

therefore ½ ∨ *efd* = 11° 57′ 28″, and consequently
∨ *efd* = 23° 54′ 56″.

log.—cos. 93° 27′ 30″=l. cos. 86° 32′ 30″ = 8.7804792
log. cos. 45° 40′ 30″ . . . . = 9·8443079

18·6247871

log. sin. 105° 5′=l. sin. 74° 55′=9·9847740
log. sin. 34° 3′ . . . . . =9·7481230
19·7328970

divide by 2 . . — 1·1081099

and ½ is . . . . — 0·5540549
log. R . . . . = 10·

log. sin. ½ ∨ *dfg* . . . . = 9·4459451

Therefore ½ ∨ *dfg* = 16° 12′ 49″, and consequently
∨ *dfg* = 32° 25′ 38″.

Now as we know the angle *edf* = *fdg* = 101° 55′
we therefore know the angle *def* = 45° 10′ 4″
and *dgf* = 45° 39′ 22″

and hence *df* : *de* :: sin. 54° 10′ 4″ : sin. 23° 54′ 56″
:: 8107 : 4054
:: 4 : 2

and . . $df : dg :: \sin. 45° 39' 22'' : \sin. 32° 25' 38''$
$:: 7152 : 5360$
$:: 4 : 3$

If we again dispense with the use of natural sines, we may still derive the same result by means of the logarithms of those sines.

For log. sin. 54° 10′ 4″ = 9·9088794
log. sin. 23° 54′ 56″ = 9·6078695

$$\begin{array}{r} 9{\cdot}9088794 \\ 9{\cdot}6078695 \\ \hline 0{\cdot}3010099 \end{array}$$

The natural number corresponding to this resulting log. is 2 very nearly, which may be represented by the fraction $\frac{2}{1} = \frac{4}{2}$.

And log. sin. 45° 39′ 22″ = 9·8544077
log. sin. 32° 25′ 38″ = 9·7293493

$$\begin{array}{r} 9{\cdot}8544077 \\ 9{\cdot}7293493 \\ \hline 0{\cdot}1250584 \end{array}$$

The natural number corresponding to this resulting log. is $1{\cdot}3337 = \frac{1{\cdot}3337}{1{\cdot}0000} = \frac{4}{3}$, which gives the same law of decrement as that already found.

The *general* symbol to represent the plane *e f g*, would be $(B'_r\ B_p\ B''_q)$, and its particular symbol will be found by substituting in this general symbol, for the letters *p*, *q*, and *r*, the particular values of the indices as we have just found them. We shall then have the symbol (B′2 B4 B″3), which represents this particular plane, and signifies that the compound molecule abstracted in the production of this plane belongs to a treble plate, or is 3 molecules in height, 2 in the direction of the edge B′, and 4 in the direction of the edge B.

Let us now require the inclination to the primary planes, of the planes whose law of decrement we have just determined.

And first of the plane *e i c*, fig. 362, whose symbol is $\underset{\frac{3}{2}}{A}$.

The inclination of this plane to the primary is equal to the angle *a b c*, fig. 362; to obtain which we must first know the angle *d b c*.

The law of decrement being 3 molecules in breadth and 2 in height, and the decrement in breadth being measured by $\frac{1}{2}$ an oblique diagonal and an edge, it follows that the ratio of the lines of the defect may be thus expressed,

$d\,b : d\,c ::$ 3 half oblique diagonals : 2 edges.

But we have before seen that

$\frac{1}{2}$ oblique diagonal : edge :: 12 : 19

we have therefore $d\,b : d\,c :: 3 \times 12 : 2 \times 19 :: 36 : 38$.

The sum of the sides *d b*, *d c*, of the triangle *d b c* is therefore $38 + 36 = 74$; and their difference is $38 - 36 = 2$.

The angle *d b c* which we require, is evidently the greater of the two angles *d b c* and *d c b*.

Now the sum of these two angles is $180° - A_5 = 180° - 109°\,8'\,11'' = 70°\,51'\,49''$ of which $\frac{1}{2} = 35°\,25'\,54''$.

But to find the greater angle, we must also know their difference, which we may discover by means of the general formula given in p. 376.

$$\text{tang. } \tfrac{1}{2}\, d = \frac{d' \text{ tang. } \frac{1}{2}\, s}{s'}$$

which formula in relation to this particular case becomes

$$\text{tang. } \tfrac{1}{2}\, d = \frac{2 \text{ tang. } 35°\,25'\,54''}{74}$$

From the tables of logarithms we find

log. 2 . . . . . . = 0·3010300
log. tang. 35° 25′ 54″ = 9·8521719

10·1532019
log. 74 . . . . = 1·8692317

log. tang. $\frac{1}{2}$ *d* . . . = 8·2839702

Therefore
$\frac{1}{2}$ difference of the angles *d b c* and *d c b* = 1° 6′ 6″
and $\frac{1}{2}$ their sum being . . . . . . = 35° 25′ 54″

the greater angle *d b c* . . . . . . = 36° 32′

and ∨ *c b a* is consequently 180°—36° 32′=143° 28′.

If we turn to p. 382, we may observe that this angle is the same we are supposed to have found by measurement, and from which we have deduced the law of decrement.

We shall now deduce the inclination of the plane *e f g* to each of the adjacent primary planes, from the known law of decrement producing it, and from the known angles I_1 and A_1.

The symbol of this plane being as we have already seen (B′2 B4 B″3), the edges *d e*, *d f*, *d g*, of fig. 362, are as follows,

d e = 2
d f = 4
d g = 3

The angle *e d f*, or *f d g*, corresponds to A_1, which has been found = 101° 55′.

Consequently the sum of the unknown angles *d f e* and *d e f*, or *d f g* and *d g f*, is = 78° 5′.

We require the plane angles $d f e$, and $d f g$, which are evidently less than $d e f$ and $d g f$.

1. To find the angle $d f e$, from the formula

$$\text{tang. } \tfrac{1}{2} d = \frac{d' \text{ tang. } \frac{1}{2} s}{s'}.$$

we have $d f + d e = 4 + 2 = 6$
$d f - d e = 4 - 2 = 2$
and $\frac{1}{2}$ the sum of the unknown angles$=39° 2' 30''$

$$\text{therefore tang. } \tfrac{1}{2} d = \frac{2 \text{ tang. } 39° 2' 30''}{6}$$

log. 2 . . . . . = 0·3010300
log. tang. 39° 2′ 30″ = 9·9090149

10·2100449
log. 6 . . . . = 0·7781513

long. tang. $\frac{1}{2} d$. = 9·4318936

Therefore $\frac{1}{2}$ the difference of the unknown angles $= 15° 7' 38''$
and consequently $\vee d f e = 39° 2' 30'' - 15° 7' 38''$
$= 23° 54' 52''$.

2. To find the angle $d f g$,
we have $d f + f g = 4 + 3 = 7$
$d f - f g = 4 - 3 = 1$
and $\frac{1}{2} s$ as before $= 39° 2' 30''$

$$\text{therefore tang. } \tfrac{1}{2} d = \frac{\text{tang. } 39° 2' 30''}{7}$$

log. tang. 39° 2′ 30″ = 9·9090149
log. 7 . . . . . = 0·8450980

log. tang. $\frac{1}{2} d$ . . . = 9·0639169

therefore $\frac{1}{2} d = 6° 36' 31''$
and $\vee d f g = 39° 2' 30'' = 6° 36' 31'' = 32° 25' 59''$.

Having thus found the plane angles $d\,f\,e$, $d\,f\,g$, which may be regarded as the sides of a spherical triangle, we may from these and the angle I_1, deduce the values of the angles subtended by these sides.

Let us call the angle subtended by the side $d\,f\,g$, I_6, and that subtended by $d\,f\,e$, I_7.

We have $\vee\, d f g + \vee\, d f e = 32^\circ\, 25'\, 59'' + 23^\circ\, 54'\, 52''$
$= 56^\circ\, 20'\, 51''$
$\vee\, d f g - \vee\, d f e = 32^\circ\, 25'\, 59'' - 23^\circ\, 54'\, 52''$
$= 8^\circ\, 31'\, 7''$

and $\frac{1}{2}\, 56^\circ\, 20'\, 51'' = 28^\circ\, 10'\, 25''$
$\frac{1}{2}\ \ 8^\circ\, 31'\ \ 7'' = \ \ 4^\circ\, 15'\, 33''$
$\frac{1}{2}\, I_1 = 52^\circ\, 32'\, 30''$.

Having thus two of the sides and an angle, of a spherical triangle, whose other angles are I_6 and I_7, we may find $\frac{1}{2}$ the sum, and $\frac{1}{2}$ the difference, of the angles I_6 and I_7, and thence the value of each, in the following manner.

1. *To find $\frac{1}{2}$ their sum.*

$$\text{tang. } \tfrac{1}{2}(I_6 + I_7) = \frac{\text{cot. } 52^\circ\, 32'\, 30''\ \text{cos.} 4^\circ\, 15'\, 33'}{\text{cos. } 28^\circ\, 10'\, 25''}$$

log. cot. $52^\circ\, 32'\, 30''$	=	9·8843264
log. cos. $4^\circ\, 15'\, 32$	=	9·9987989
		19·8831253
log. cos. $28^\circ\, 10'\, 25''$	=	9·9452316
log. tang. $\frac{1}{2}\,(I_6 + I_7$	=	9·9378937

Therefore $\frac{1}{2}\,(I_6 + I_7) = 40^\circ\, 55'\, 2''$.

2. *To find ½ their difference.*

$$\text{tang.}\,\tfrac{1}{2}(I_6 - I_7) = \frac{\text{cot.}\,52^\circ\,32'\,30''\ \text{sin.}\,4^\circ\,15'\,33''}{\text{sin.}\,28^\circ\,10'\,25''}$$

$$\begin{array}{lcr}
\text{log. cot. } 52^\circ\,32'\,30'' & = & 9{\cdot}8843264 \\
\text{log. sin. } \ \ 4^\circ\,15'\,32'' & = & 8{\cdot}8707728 \\
\hline
 & & 18{\cdot}7550992 \\
\text{log. sin. } 28^\circ\,10'\,26'' & = & 9{\cdot}6740791 \\
\hline
\text{log. tang. } \tfrac{1}{2}(I_6 - I_7) & = & 9{\cdot}0810201 \\
\hline
\end{array}$$

Therefore $\tfrac{1}{2}(I_6 - I_7) = 6^\circ\,52'\,18''$

Therefore

$$I_6 = 40^\circ\,55'\,2'' + 6^\circ\,52'\,18'' = 47^\circ\,47'\,20''$$

$$I_7 = 40^\circ\,55'\,2'' - 6^\circ\,52'\,18'' = 34^\circ\,2'\,44''$$

and the inclination of the plane *efg* is consequently

$$\text{on P} = 180^\circ - 47^\circ\,47'\,20'' = 132^\circ\,12'\,40''$$

$$\text{P}' = 180^\circ - 34^\circ\,2'\,44'' = 145^\circ\,57'\,16''$$

which are very nearly the angles we are supposed to have found by measurement of the crystal, as given in p. 384, and from which we deduced the law of decrement we have here supposed to be known.

The instances here selected to illustrate the methods of calculation previously described, are among the most complicated that are likely to occur; and they have been so selected, because they contain more of the varieties of formulæ than will commonly present themselves during our researches.

ON THE DIRECT DETERMINATION OF THE LAWS OF DECREMENT FROM THE PARALLELISM OF THE SECONDARY EDGES OF CRYSTALS.

The resources of crystallography for determining the laws of decrement by which secondary planes are produced, are not limited to the methods already explained. In certain cases those laws may be determined, independently of the angle at which the secondary plane inclines on the primary, by means of the parallelisms which are observed to exist between two of the edges of the secondary plane, and two other known edges of the crystal; or sometimes one known edge, and a diagonal of the primary form.

One of the simplest instances of the distinctive character conferred on a secondary plane by the parallelism of its edges, is that by which the planes produced by simple or mixed decrements are distinguished from those produced by intermediary decrements.

We may also refer for an illustration to the tables of modifications of the right square prism. The parallelism of the lateral edges of the plane *d*, to the lateral edge of the prism, implies that plane *d* is produced by a simple or mixed decrement on a lateral edge of the prism;* and the parallelism of the diagonal of the terminal plane, to the edge at which the secondary plane *d* intersects that plane, indicates a decrement by 1 row on the lateral edge of the prism; for it is obvious that the edges of the defect are here proportional to the primary edges.

Cases however of a much more complicated nature may be determined by means of the parallelism of the edges of crystals.

We are indebted to the Abbé Haüy for the earliest observations which occur on this subject. He remarked, among other instances, that whenever the terminal edges, and the solid angles, of the hexagonal prism, were replaced at the same time, if the decrement in breadth on the edge happened to be double that on the angle, the opposite edges of the plane replacing the solid angle would be parallel, and the figure of that plane would be a rhomb.

A memoir by Mr. Monteiro, on the determination of the law of decrement producing a new variety of carbonate of lime, was inserted in No. 201 of the Journal des Mines for September 1813, and an outline of it is given here as an example of the method used to determine that law. This memoir illustrates the utility of this method of determining the laws of decrement, by its application to a case where the angle could not be measured at which the secondary plane whose law of decrement was required, inclined on the primary, or on any other plane, whose relation to the primary was known.

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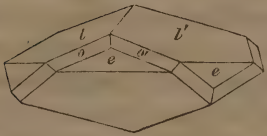
* It is apparent that the plane *d* is parallel to the edge it replaces; and it may be observed generally, that whenever the edges of a secondary plane are parallel, the plane itself is parallel to the edge it replaces.

Fig. 363.

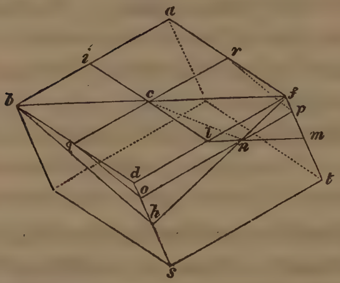


Mr. Monteiro had undertaken to describe a new crystal of carbonate of lime, as exhibited in fig. 363, the planes *o* of which were so imperfect, that they could not be subjected to the goniometer, and consequently the law of decrement by which they were produced, could not be determined by the ordinary methods. Two parallelisms of their edges, however, enabled Mr. M. to determine that law geometrically, without knowing the inclination to any other plane, of the secondary planes in question.

The planes *l*, *l'*, were found to correspond with mod. *l* of the rhomboid, and were observed to be striated, as those planes frequently are, in the direction of their oblique diagonals; this direction being parallel to the superior edges of the primary rhomboid.

The position of the planes *l*, and *l'*, relatively to the edge of the primary form being thus known, it was readily perceived that the plane *e* corresponded with mod. *e* of the same rhomboid, and the planes *o*, with some particular modification belonging to class *o*.

Fig. 364.



Let fig. 364 represent the primary rhomboid of carbonate of lime. The plane *l′*, fig. 363, is known to result from a decrement by 1 row on the superior edges of that rhomboid. The lines *i l*, *l m*, would therefore represent the intersections of the plane *l′* with the primary planes, the points *l*, and *m*, being the middle of the edges *d f*, and *f t*.

From the middle of the edge *d s*, draw *h b*, *h f*, and the triangle *b h f* would represent the position on the primary form, of the plane *e*, fig. 363.

The planes *l′* and *e* are thus observed to intersect each other at the points *c* and *n*, and consequently the line *c n*, would correspond with the common edge of the planes *l*, and *e*, if that edge were visible in fig. 363.

The first parallelism observed by Mr. Monteiro was between the line of intersection of the planes *o′* and *l*, and the striæ on the plane *l*. The plane *o′* on some of the crystals he examined, was so much broader than the plane *o*, as to exhibit this parallelism distinctly.

But as the striæ are parallel to an edge of the primary form, the edge at which the plane *o′* intersects the plane *l*, must be parallel to an edge of the primary form, and evidently to the edge *a b* of fig. 364.

The second parallelism observed was between those edges of the plane o', which are produced by its intersection with the planes l' and e. From this parallelism, as it has been already remarked, it is known that the plane o' is itself parallel to the edge at which the plane l and e meet.

We have thus obtained two conditions, which enable us to place the plane o' on the primary form.

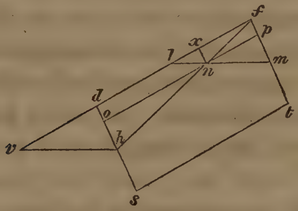
First an edge of that plane is parallel to the edge $a\ b$ of fig. 364, and secondly the plane itself is parallel to, and consequently may coincide with, the line $c\ n$, which represents the edge at which the planes l and c meet.

If therefore we draw the line $q\ r$, parallel to $a\ b$, and passing through the point c, and the line $o\ p$, parallel to $q\ r$, and passing through the point n, we shall, by joining $o\ q$, and $p\ r$, obtain the position on the primary form of the plane o'.

And the ratio of $q\ d$ to $d\ o$, will evidently give the law of decrement by which the plane o' has been produced.

This ratio Mr. Monteiro says is easily deduced, but he does not point out the method of discovering it; it is however very obvious.

Fig. 365.



Let the plane $d\,f\,s\,t$ be represented by fig. 365. Produce $d\,f$, and from the point h, draw $h\ v$, parallel to $l\ m$, whence $d\ v = l\,f$; and because the triangles $v\,f\,h$, $l\,f\,n$, are similar, $n\,f$ is evidently $\frac{1}{3}$ of $h\,f$; and if we draw $n\,x$, parallel $d\,h$, the triangles $x\,f\,n$, $d\,f\,h$ are similar, and $x\,n = d\,o$ is $\frac{1}{3}$ of $d\,h$.

But $d\,h = d\,g$, fig. 364. The plane o' is produced therefore by a decrement, consisting of 3 rows in breadth, on the plane $d\,b\,a\,f$.

This demonstration, it may be remarked, is purely geometrical, and limited in its application, to this particular case. The same method might however be adapted to other cases; but the problem would frequently become extremely complicated, and difficult of solution by the aid of geometry alone.

Perceiving this difficulty, and the limited nature of the method itself, Mr. Levy has generalised the problem by giving it an algebraical form, and has published an interesting paper on the subject in the 6th vol. of the Edinburgh Philosophical Journal, p. 227. In this paper, Mr. L. has given formulæ for determining the law of decrement, by which any secondary plane, modifying any parallelopiped, is produced, whenever two of the edges of that plane, not being parallel to each other, are parallel to two known edges of the crystal.

The following brief abstract of Mr. Levy's paper is inserted here, for the purpose of affording the reader a more immediate reference to the formulæ it supplies; and as an additional example of a method of investigation, which may be advantageously applied to other points of crystallographical research.

To derive these formulæ, Mr. Levy has first supposed the edges of the primary form to be represented by three co-ordinate axes, and the primary planes, consequently, to correspond to the three co-ordinate planes. He has then found the equations of all the planes concerned in the solution of the problem; and by combining these equations, has obtained the equations of the projections upon one of the co-ordinate planes, of those intersections of the known planes, to which the edges of the new plane are respectively parallel. And from the necessary relation subsisting among the co-efficients of some of the terms of these equations, the following equations are derived.

Let p_5, q_5, r_5, be the unknown indices of the new plane, which we chall call plane 5. Let p_1, q_1, r_1, and p_2, q_2, r_2, be the known indices of two planes to whose intersection one edge of plane 5 is parallel. And let p_3, q_3, r_3, and p_4, q_4, r_4, be the known indices of two other planes, to whose intersection another edge of plane 5 is parallel. The particular values of the indices of the planes 1, 2, 3, and 4 being substituted for the general indices of those planes in the following equations, the particular values of the indices of plane 5 will be obtained.

$$(1)\quad \frac{p_5}{r_5} = \cfrac{\left(\frac{1}{r_1\, p_2} - \frac{1}{r_2\, p_1}\right)\left(\frac{1}{q_3\, r_4} - \frac{1}{r_3\, q_4}\right)}{\left(\frac{1}{r_1\, p_2} - \frac{1}{r_2\, p_1}\right)\left(\frac{1}{q_2\, p_4} - \frac{1}{p_3\, q_4}\right) \cfrac{+\left(\frac{1}{q_1\, r_2} - \frac{1}{q_2\, r_1}\right)\left(\frac{1}{p_3\, r_4} - \frac{1}{p_4\, r_3}\right)}{+\left(\frac{1}{q_1\, p_2} - \frac{1}{q_2\, p_1}\right)\left(\frac{1}{p_3\, r_4} - \frac{1}{p_4\, r_3}\right)}}$$

$$(2)\quad \frac{q_5}{r_5} = \cfrac{\left(\frac{1}{r_1\, q_2} - \frac{1}{r_2\, q_1}\right)\left(\frac{1}{p_3\, r_4} - \frac{1}{r_3\, p_4}\right)}{\left(\frac{1}{r_1\, q_2} - \frac{1}{r_2\, q_1}\right)\left(\frac{1}{p_3\, q_4} - \frac{1}{q_3\, p_4}\right) \cfrac{+\left(\frac{1}{p_1\, r_2} - \frac{1}{p_2\, r_1}\right)\left(\frac{1}{q_3\, r_4} - \frac{1}{q_4\, r_3}\right)}{+\left(\frac{1}{p_1\, q_2} - \frac{1}{p_2\, q_1}\right)\left(\frac{1}{q_3\, r_4} - \frac{1}{q_4\, r_3}\right)}}$$

The two preceding equations are the most general that can be imagined.

If the known planes 1 and 2, be parallel to a diagonal of the terminal plane of the primary form, the plane 5 will be parallel to the same diagonal; in this case $p_1 = q_1$, and $p_2 = q_2$; and the values of $\frac{p_5}{r_5}$ and $\frac{q_5}{r_5}$ become equal; and by reducing the above equations, after the necessary substitutions are made, the following will result.

$$(3)\ \frac{p_5}{r_5} = \frac{q_5}{r_5} = \frac{\dfrac{1}{q_3\,p_4 - p_3\,q_4}}{\left(\dfrac{1}{q_3\,r_4} - \dfrac{1}{r_3\,q_4}\right) + \left(\dfrac{1}{p_3\,r_4} - \dfrac{1}{r_3\,p_4}\right)}$$

The indices of the planes I and 2, it will be remarked, have disappeared from this formula, since the condition of planes 5 being parallel to a diagonal of the primary form, does not depend upon any secondary plane.

If we now suppose the planes 1 and 2 parallel to a lateral edge of the primary plane, the plane 5 will be parallel to the same edge; then r_1, and r_2, become infinite, and the values of $\frac{p_5}{r_5}$, $\frac{q_5}{r_5}$, become infinite also. But if, instead of substituting the infinite in equations 1 and 2, for the indices of planes 1 and 2, we divide the first equation by the second, we shall obtain a new equation which does not contain the indices of planes 1 and 2, and which gives the values of the indices p_5 and q_5, in function of the indices of planes 3 and 4.

$$\frac{p_5}{q_5} = \frac{\dfrac{1}{q_3\,r_4} - \dfrac{1}{r_3\,q_4}}{\dfrac{1}{p_3\,r_4} - \dfrac{1}{p_4\,r_5}}$$

The first, second, and third of these formulæ are complicated from the generality which has been given to them. They are all remarkable too, for not containing the values of the primary edges, which were used in the preliminary equations.

The linear dimensions of the primary form are not therefore necessary to determine the laws of decrement of such planes as those we have been considering. Nor can the dimensions of the primary form be deduced from any observation of parallelism between the edges of a secondary crystal.

It is necessary to add that if any of the planes 1 to 5, should cut one of the co-ordinate axes on the negative side, its index referring to that axis must be taken negatively in the preceding formulæ.

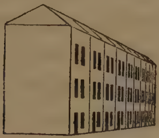
As this short abstract is given merely to introduce the formulæ, the reader will more thoroughly comprehend the author's views, by consulting the paper itself.

ON THE METHODS OF DRAWING THE FIGURES OF CRYSTALS.

The representation of surfaces, or of solid bodies, upon a plane, is the object of the art of perspective. The theory upon which this art has been founded, supposes an imaginary transparent skreen to be interposed between the eye of the observer and the object to be represented; and it supposes also that the rays of light which pass from the object to the eye through the skreen, should become, as it were, fixed at its surface, so that when the object is removed, its figure or representation should still remain apparent on the skreen. And the rules of perspective teach the methods of delineating the figures of objects upon a plane, in such a manner, as to resemble the appearance they would present to the eye if seen through the plane on which they are delineated, supposing that plane to be transparent, and held between the object represented and the eye.

A more familiar conception of the nature of a perspective representation may be derived from looking at a building, or along a street, through a piece of glass, and marking lines on the surface of the glass coinciding with the lines of the object we are observing through it. These lines, if accurately traced, will evidently represent the object to the eye, such as it appeared when seen through the glass.

Fig. 366.



If we look along a street, and imagine that we are seeing it through a transparent skreen, the upper and lower edges of the fronts of the houses, which we know to be nearly if not accurately parallel, appear to converge at the remote end of the street, forming a series of lines on the skreen something like that shewn in fig. 366.

And it is obvious that if this mass of houses were a single solid body, and even if it were very much reduced in dimensions, it must still be represented on a plane surface by lines some of which must converge, as those representing the upper and lower edges of the supposed fronts of the houses do in the above figure.

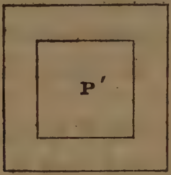
But a representation of the figures of crystals in this manner would not convey a sufficiently precise notion of their forms, and it would be extremely difficult to understand the figures of complicated secondary crystals, if they were thus traced.

In order to retain in the drawings of crystals the apparent symmetry of their forms, another kind of perspective has been used, which is known by the name of orthographic or geometrical projection, or simply by that of projection.

In this kind of perspective, the object to be represented is supposed to be removed to an infinite distance from the eye; in consequence of which *all* the lines which are *parallel* in the figure would appear *parallel* upon our supposed transparent skreen, and not converging as they do in the above diagram.*

The method of representing crystals in projection may be thus explained. Let us for a moment forget the abstract notion of the object being removed to an infinite distance from the eye, and let us imagine it distinctly within our view.

Fig. 367.



Let the figure to be represented, be a cube; and let us imagine this cube to be resting upon a *horizontal* surface, and the eye to be placed opposite one of its planes, and in the direction of a line drawn perpendicularly through the centre of that plane.

In these relative positions of the eye and the crystal, only that plane opposite to the eye will be visible; and if a transparent skreen were interposed between the eye and the crystal, and held parallel to the plane which is seen, the only linear traces which could be marked on the skreen would be the edges of the observed plane, as represented in fig. 367.

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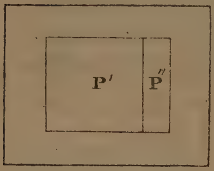
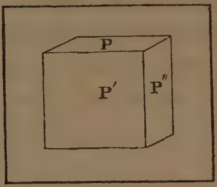
* This theoretical notion of the *infinite distance* of the object, is borrowed from mathematical considerations of the nature of infinite lines; and may be taken here merely to imply what is stated in the text, that the edges, or other lines, which are parallel on the crystals, are to be represented by parallel lines in the drawing.

Fig. 368.



If we now suppose the eye and the skreen to be moved horizontally toward the right of the spectator, the skreen retaining its parallelism to the plane P′, the rays proceeding from the edges of that plane, may be conceived to pass obliquely towards the skreen in its new position, and the edges of the plane P″ will now be visible, and may be traced on the skreen as in fig. 368. If we suppose the eye and the interposed skreen to move round the crystal, the skreen retaining its perpendicular position, but ceasing to be parallel to any plane of the cube, excepting at some particular points of its progress, it will be obvious, that while the eye and the skreen continue to move in the *same horizontal plane*, the vertical planes of the crystal, and those only, will become visible in succession; but the terminal plane will not be perceived. To see the terminal plane we must suppose the eye and the skreen to be raised; or, if the eye retain its position, the back of the crystal must be elevated.

Fig. 369.



It will be more consistent with most of the following explanations of the methods of drawing the figures of crystals, to suppose the position of the eye fixed, and the back of the crystal to be elevated.

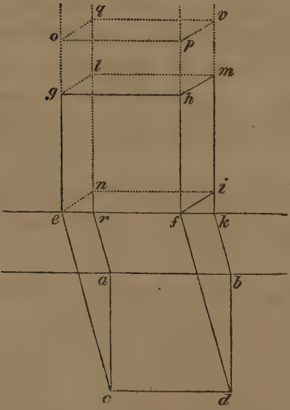
In this new relative position of the crystal and the eye, the figure traced on the interposed skreen, would resemble that exhibited in fig. 369.

It is evident from the preceding explanation, that the relative positions of the eye, the object, and the skreen, may be varied at pleasure, so as to produce in the drawing, such a representation of the object as best suits the illustration it is intended to afford. And although, as it has been stated in p. 102, an advantage will generally attend the placing the figures of crystals belonging to the same class of primary forms always in the same position, there may nevertheless be exceptions to this rule when the position of the modifying planes on the secondary crystal, is such, as to require some new position for their more perfect exhibition. The position chosen by the Abbé Haüy for the crystal of felspar, is perhaps the best that could be adopted for exhibiting advantageously the secondary planes of the crystals of that substance; yet the front lateral planes of his figure correspond to the back planes of the doubly oblique prism as it is given in the tables of modifications.

Having thus given a brief outline of the theory of geometrical projection, we shall proceed to shew how the forms of crystals may be accurately delineated, without entering into any further general explanation of the means which will be employed for this purpose.

To draw a Cube.

Fig. 370.



On the line *a b*, describe the square *a b c d*. Let the line *e k* be parallel to *a b*.

From the points *a b c d*, draw the lines *c e*, *a r*, *d f*, *b k*, and let these lines be more or less oblique, as the side *h f m i* is to be rendered more or less visible.

Draw the perpendiculars *e o*, *r q*, *f p*, *k v*. Take *e g* equal *e f*, and draw *g h* parallel to *e f*, and *e g h f* is consequently a square.

For the purpose of shewing the plane *g l m h* of the cube, the back of the figure is supposed to be a little elevated.

To represent this elevation take some portion, as *i k*, of the line *k v*, and draw *f i*; the portion *i k* may be greater or less according as more or less of the plane *g l m h* is required to be seen.

Draw *e n*, *g l*, *h m*, parallel to *f i*; and *l m*, *n i*, parallel to *g h*, and the figure represents a cube.

To draw a Square Prism.

The square prism differs from the cube only in its comparative height.

Let us suppose we have to represent a square prism, whose terminal edge is to its lateral edge as 2 to 3; we may divide the terminal edge *e f* in two parts, and make *e o* equal to three of those parts, and then complete the figure as represented in the diagram. And we may, by a similar proceeding, make the lateral edge of the prism bear any given proportion to the terminal edge.

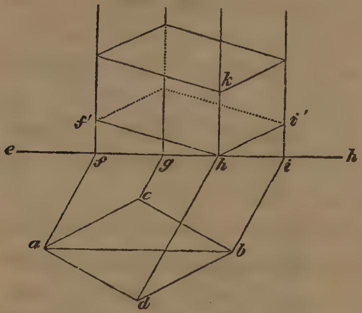
To draw a Right Rectangular Prism.

To draw a right rectangular prism, make the lines *a c*, *c d*, of the preceding figure, proportional to the corresponding terminal edges of the particular prism we wish to represent; and having proceeded to draw and elevate the base of the figure as for the cube, make *e g* proportional to the third dimension, or height of the prism, and then complete the figure by drawing the parallel lines as before.

It has been already stated that when the angles of the base of a prism, or octahedron, are right angles, the figure of the crystal is drawn with one of the *edges* of its base horizontal; but where those angles are not right angles, the *diagonal* of the base is horizontal in the figure, and the terminal edges are described by oblique lines.

To draw a Right Rhombic Prism.

Fig. 371.



Let *a b* represent the greater diagonal of a right rhombic prism, and let the rhomb *a c b d* represent the base of the particular prism we are about to delineate.

The angle *a d b* of the prism is supposed to be known, and that angle of the figure may be made equal to it, by adjusting the arms of the common goniometer to the required angle, and using them as a rule to draw the lines *a d*, *d b*.

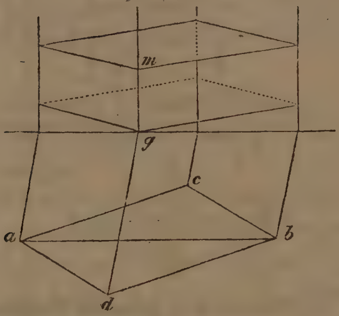
Draw *e h* parallel to *a b*; and also the oblique lines *a f*, *c g*, *d h*, *b i*, these being, as before, drawn more or less oblique, according as we wish to exhibit a greater or less difference between the two lateral planes of the prism shewn in the front of the figure.

To elevate the back part of the prism in order to exhibit the terminal plane, take *i i'* equal to *f f'*; and draw *h i'*, *h f'*, and their parallels.

Let the lateral edge *h k* be drawn in such proportion to the terminal edge *a d*, as it has been found by calculation on the particular crystal we are delineating; and draw the upper terminal edges parallel to the lower ones.

To draw a Right Oblique-angled Prism.

Fig. 372.



The right oblique-angled prism may be drawn in a similar manner, keeping the diagonal *a b* horizontal, and making the angle *a d b*, and the ratios of the edges *d a*, *d b*, and *g m*, such as they are found in the prism of which we propose to give the figure.

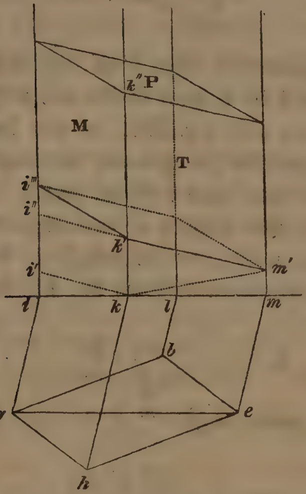
The oblique rhombic prism may be drawn in a manner similar to the two preceding prisms, but it should have a little more elevation given to the back of the figure, in order to render the character of obliquity of the prism more conspicuous.

From the necessity of elevating the back of right prisms for the purpose of shewing the terminal plane, it is apparent that the character of obliquity cannot be conferred on a figure so drawn, otherwise than by elevating the back of it rather more than that of the right prism.

To draw the Doubly-oblique Prism.

The double obliquity given to the figure of this primary form in the tables of modifications, is too slight to convey an accurate notion of its general character; that obliquity is therefore considerably increased in the following figure.

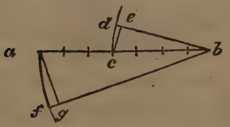
Fig. 373.



As the lateral angles of this form are not right angles, its base obviously cannot rest on a horizontal plane, while its lateral edges are perpendicular. To obtain its horizontal projection therefore, while its lateral edges are perpendicular, we may suppose those edges produced until they touch the horizontal plane *i m*, over which the figures appears to stand.

The area of the horizontal projection is clearly less than the base of the figure, and may be known from the ratio of the terminal edges, and from the plane angles of the lateral planes; which elements are supposed to have been previously determined.

Fig. 374.



Let us suppose the edge $k'\,m'$, fig. 373, to be to the edge $k'\,i'''$, as 7 to 4, and to the edge $k'\,k''$ as 7 to 6.

Let the line $a\,b$, fig. 374, be the length we may determine upon for the greater terminal edge of the prism; divide this into 7 equal parts, and 4 of those parts will be the length of the edge $k'\,i'''$, fig. 373, and 6 of them will give the height of the prism.

Let the inclination of P on M, fig. 373, be known and called I_1,
P . . T . . . . . . I_2,
M . . T . . . . . . I_3.

And let the plane angle $i'''\,k'\,k''$, be also known and called A_1,
$k''\,k'\,m'$, ... A_2.

And let us suppose A_1 an acute, and A_2 an obtuse angle.

It is evident that if the solid angle at m', of such a figure, be supposed to touch the horizontal plane $i\,m$, the lateral edges being kept perpendicular, the solid angle at k' must stand above the plane, and the solid angle at i''' still more above it. The elevation of the point at k' may be known by drawing the arc $a\,f$, fig. 374, with a radius $a\,b$, and drawing a second radius $b\,f$, making the angle $a\,b\,f = A_2 - 90°$.

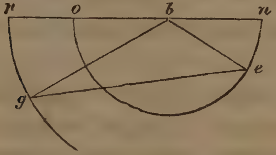
The perpendicular $a\,g$ dropped on the line $f\,b$, will be the required height of the point k' above the horizontal plane, and the line $g\,b$ will be the length of the horizontal projection of the greater terminal edge of the prism.

The increased elevation of the point at i''', fig. 373, may also be determined by drawing the arc $c\ d$, fig. 374, with a radius $c\ b$ equal to $\frac{4}{7}$ of $a\ b$, and making the angle $c\ b\ d$ equal to $90°—A_1$. The perpendicular $c\ e$ will be the increased elevation of the point at i''', fig. 373, and the line $e\ b$ will be the horizontal projection of the lesser terminal edge of the prism.

Having thus obtained the horizontal projections of the terminal edges of the prism, we may find the vertical projections of the lateral edges in the following manner.

Let the horizontal projection of the greater diagonal of the terminal plane be supposed parallel to the line $i\ m$, fig, 373, and the diagonal $g\ e$ of the plan $g\ b\ e\ h$, must be parallel to the same line.

Fig. 375.



The edges of this plan are known from the figure 374. The length of the line $g\ e$ may be determined by simply cutting a card so that the angle $g\ b\ e$, fig. 375, shall be equal to I_3, and making $b\ g$, equal to $b\ g$, fig. 374, and $b\ e$, equal to $b\ e$ of the same figure. The point b of the card being laid on the point b of fig. 373, the edge $g\ e$ may be made parallel to $i\ m$, by means of a parallel ruler, and the lines $g\ b$, $b\ e$, being traced by a pencil, their parallels $h\ e$, $h\ g$, may be drawn to complete the plan.

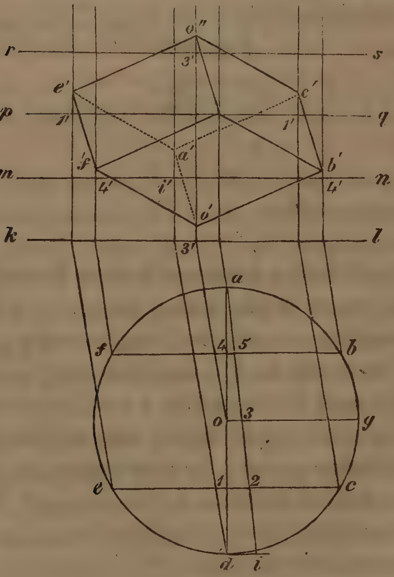
The length and position of the line *g e* may be also very easily determined geometrically.

Make *b p*, fig. 375, equal to *b g*, fig. 374, and *b n* equal to *b e* of the same figure. From *b* as a centre, describe the semicircle *n o*, and the segment *p g*. Take *n e* equal to the arc of $90^\circ - \frac{1}{2} I_3$, and *p g* equal to the same arc, and *g e* will obviously be the greater diagonal of the horizontal projection of the base of the prism.

Knowing the horizontal projection *g b e h*, we may proceed to the projection of the prism; elevating $i\,i'$, $m\,m'$, sufficient to exhibit the terminal plane, and taking $k\,k'$ equal to *a g*, fig. 374, and $i''\,i'''$ equal to *c e* of the same figure.

To draw a Rhomboid.

Fig. 376.



The ratio of the axis, to a perpendicular drawn upon it from one of the lateral solid angles of the particular rhomboid we are about to delineate, is supposed to have been ascertained.*

We should then determine the height our proposed figure is to be, which height will be the length of its axis. Our next step is to find a line which bears the same ratio to that which we have fixed on for the axis of our figure, as the perpendicular upon the axis of the crystal, does to the axis itself. This may generally be done with sufficient precision, by dividing the line we have assumed for our axis into such a number of equal parts, as will give the length of the required line in some other number of those parts. If, for example, we have found, that the perpendicular upon the axis, is to the axis itself, in the ratio of 7 to 10, and if we determine that our figure shall be an inch high, the required line will be evidently $\frac{7}{10}$ of an inch.

If, however, we are desirous of still greater accuracy, we may draw a perpendicular line equal to that which we have fixed on for the height of our rhomboid, and from the upper extremity of this line draw a second, inclining to it at the same angle that the axis does to a superior edge of the rhomboid we are about to represent; and if we now divide our first line into three equal parts, and from the upper point of section, draw a perpendicular to it which shall pass through the second line, the portion intercepted by the second line will be the required length of the perpendicular upon the axis.

With a radius equal to this line, which is, in the case we are supposing, $\frac{7}{10}$ of an inch, describe the circle *a b c d e f*, fig. 376.

---

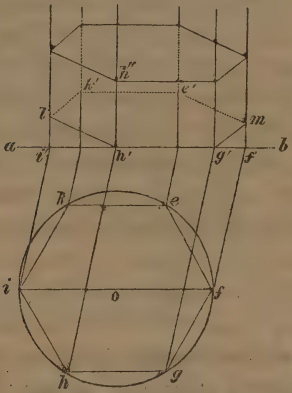
* The method of ascertaining this ratio has been already pointed out in p. 363.

Divide the circumference of this circle into six equal parts, by the points *a b c d e f*, as in the figure, and draw the lines *a d*, *f b*, *g o*, *e c*, within the circle. Draw *k l* parallel to *f b*. Draw the oblique lines shewn in the figure, from the several points on the circumference of the circle, and from its centre, to the line *k l*; and from the several points in that line where it is cut by the oblique lines, raise the perpendiculars as they appear in the figure. On the middle perpendicular line, take a portion 3′ 3′, equal to the length we have determined on for the axis of the rhomboid, and after dividing this portion into three equal parts, draw the lines *m n*, *p q*, *r s*, through the upper point 3, and through the points of division, and parallel to the line *k l*.

The oblique lines are to be drawn more or less obliquely, according as we would have the rhomboid appear more or less turned round. To elevate the back of the rhomboid, so as to render a plane truncating its terminal solid angles visible, draw *d i* parallel to *e c*, and join *a i*. The line *d i* is the quantity of elevation intended to be given to the solid angle *a*′ of the rhomboid; and the lines 1 2, 0 3, 4 5. are the proportional quantities which the other solid angles require to be elevated in order to preserve the symmetry of the figure. This imaginary elevation of the back of the figure, is thus produced in the drawing. On the perpendicular lines 4′, *i*′, and 4′, which pass through the line *m n*, take *i*′ *a*′ equal to *d i*; and 4′ *f*′, 4′ *b*′, each equal to the line 4 5. On the perpendiculars, 1′ and 1′, which pass through the line *p q*, take 1′ *e*, 1′ *c*′, each equal to the line 1 2. And on the perpendicular 3′, which passes through the lines *k l*, and *r s*, take 3′ 0′, and 3′ 0″, each equal to the line 3 0. From the several points thus obtained, and from the point d', draw the lines requisite to complete the figure.

To draw the Hexagonal Prism.

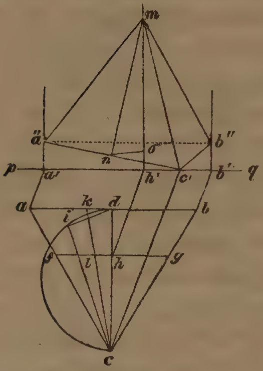
Fig. 377.



The size of the figure should be first determined. Draw the lines $a\,b$, and $i\,f$, parallel to each other. From the point o in the line $i\,f$, describe a circle with a radius equal to the line $h'\,g'$, which is to be the front edge of the prism. Draw the chords $f g$, $g\,h$, &c. and from the points e, f, g, h, i, k, draw the oblique lines to the line $a\,b$; and from the termination of the oblique lines at that line, raise the several perpendiculars shewn in the figure. To elevate the back of the prism, take some quantity $i'\,l$, $f'\,m$, on the perpendiculars at i' and f'; draw the lines $h'\,l$, and $g'\,m$, and complete the base by drawing parallels to these and to $k'\,e'$, as shewn in the figure. Let us suppose the height of the prism to be to its edge as 5 to 7; divide the edge $h'\,g'$ into 7 equal parts, and make $h'\,h''$ equal to 5 of those parts. Then complete the figure by drawing the upper terminal edges parallel to the edges of the base.

To draw the Tetrahedron.

Fig. 378.



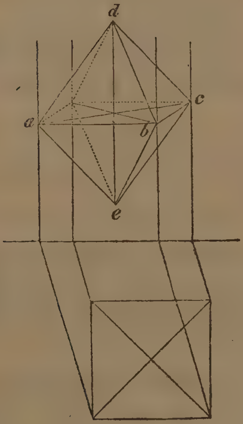
Draw $a\,b$ of the length which is intended to be given to the dotted edge $a''\,b''$ of the figure, and draw $p\,q$ parallel to $a\,b$.

On $a\,b$ construct the equilateral triangle $a\,b\,c$, and through its centre h draw $d\,c$, which will be perpendicular to $a\,b$. From the points a, b, c, and h, draw the oblique lines to $p\,q$, and from the points where they cut that line at a', b', h', draw the perpendiculars shewn in the figure. Having determined the quantity of elevation to be given to the back of the tetrahedron, make $k\,d$, on the line $a\,b$, equal to that quantity, and join $k\,c$; and through the centre h draw $f\,g$ parallel to $a\,b$. On the perpendiculars from a' and b', take $a'\,a''$, and $b'\,b''$, each equal to $k\,d$, and by drawing the lines $c'\,a''$, $c'\,b''$, and $a''\,b''$, the base of the figure will be delineated. To complete the figure we require the line $o\,m$, which is the height of the crystal. To obtain this we must have recourse to the following geometrical construction.

We may observe that the angle $m\ o\ n$ of the upper figure is a right angle, and that the line $m\ n$ is equal to the line $d\ c$ of the lower figure. But two lines drawn from the extremities of the diameter of a circle, and touching each other at the circumference, meet at a right angle. It is therefore obvious, that if we describe a semicircle on the line $d\ c$ as a diameter, and draw the chord $d\ i$ equal to $n\ o$, or, which is the same thing, to $d\ h$, the line $i\ c$ will be the required height of the figure. On the perpendicular from the point h' on the line $p\ q$, take $h'\ o$ equal to $l\ h$ of the lower figure, and take $o\ m$ equal to $i\ c$. Join $m\ a''$, $m\ c'$, $m\ b''$, and the figure is completed.

To draw the regular, or any of the irregular Octahedrons.

Fig. 379.

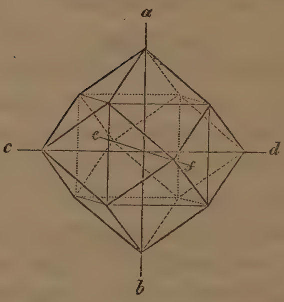


Draw the square, rectangular, or rhombic base, of the octahedron, in the same manner as the bases of the prisms of those forms are directed to be drawn. Then find the centre of the base $a\ b\ c$, by drawing the two diagonals; and through that centre draw a line *d e*, perpendicular to the edge, if the base be square or rectanglar, or perpendicular to the horizontal diagonal, if the base be a rhomb. On this perpendicular, and on each side of the base, take ½ the length of the axis of the particular octahedron we are delineating, and draw lines from the extremities of these semi-axes, to the angles of the base.

In the regular octahedron, which is the figure above represented, the semi-axis is equal to ½ the diagonal of the horizontal projection of the base.

To draw the Rhombic Dodecahedron.

Fig. 380.



The easiest method of drawing this figure is to project the cube as shewn above, and through its centre and perpendicular to its planes, to draw the lines *a b*, *c d*, *e f*.

Those lines are parallel respectively to the edges of the cube. Take on each line, and in each direction from the point in the centre of the cube, where the lines intersect each other, a quantity equal to that edge of the cube to which the particular line is parallel, and draw lines from the extremities of those portions of the lines, to the solid angles of the cube. The resulting figure will be the rhombic dodecahedron.

In the preceding pages the rules of projection have been applied to the delineation of only the primary forms of crystals; but they may also be applied to the delineation of some of the secondary forms; these may however be more easily drawn either by truncating the figures of the primary forms, or by circumscribing those primary forms with the planes of the secondary crystal.

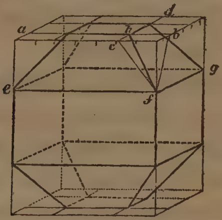
When the secondary form, whether it be simple or compound, is to be exhibited in its *entire* state, with all the primary planes effaced, the best method will be to delineate a small primary form, and to envelope that with the secondary planes; but when parts of the primary planes are also to be shewn in the figure of the secondary crystal, a larger primary form may be drawn, and then be truncated, or cut down, in the same manner as the modifications in the tables are drawn.

The fidelity of the representation of any secondary form, must obviously depend upon the accuracy of the *positions* of the secondary planes relatively to the primary and to each other. And as it is by the *intersections* of those planes with the primary or with each other, that their *positions* are rendered apparent, the accurate construction of a secondary form, must depend upon the accuracy with which those *intersections* are determined.

Hence the rules for drawing the secondary forms of crystals, will apply chiefly to the means of finding the intersections of the secondary planes.

There are two principal methods which may be used for this purpose; the one is to divide the edges of the primary figure we are about to truncate, into such a number of equal parts, as may enable us to construct the required secondary planes by finding the intersections within the same figure; the other, and, generally, the better method, is to produce the primary edges, and to obtain the intersections either within or without the figure, as may be most convenient.

Fig. 381.

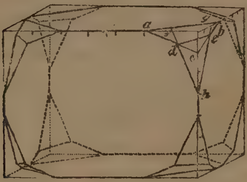


As an example of the first of these methods, let us suppose we have to represent a square prism modified on its terminal edges by planes belonging to *class c* of the tables of modifications, resulting from a decrement by one row of molecules, and whose symbol would consequently be $\overset{1}{B}$.

Let a square prism be drawn in pencil, of the relative dimensions of the prism to be represented. Divide the terminal edges into any number of equal parts, let us suppose into 7 parts, and at a distance from the angle, of two, or any other number of those parts, draw the lines *a b*, *c d*, and their parallels on the upper and lower planes of the figure. Draw *b f*, and *c f*, parallel to the diagonals of the planes on which they are drawn, and *e f*, *f g*, and their parallels, on the lateral planes. The line *h f*, is one of the intersections of the modifying planes, and is consequently a new edge of the secondary figure.

The other corresponding secondary edges being drawn, a secondary form will result, as exhibited by the dark lines in the above figure.

Fig. 382.



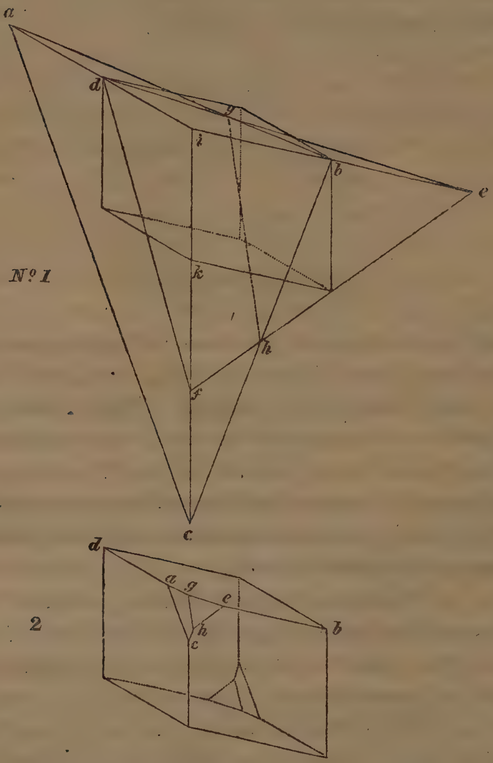
Let it be required to draw a right rectangular prism, with two planes belonging to class *a* modifying each of the solid angles. The dimensions of the prism are supposed to have been ascertained, and a prism is to be drawn in pencil corresponding in relative dimensions with our observed crystal.

We shall suppose the symbols of the two secondary planes to have been found,

(C2 G1 B3) and (C1 G3 B2).

Let each of the edges be divided into 7 equal parts; the plane represented by the first symbol would be traced by the lines *a b*, *a c*, *b c*, and that represented by the second would be traced by the lines *f h*, *h g*. But it is apparent that the two planes would intersect each other in the line *d e*, and that only the portions *a d e c*, and *d e h*, of the new planes, would be visible upon the secondary crystal. The same process being repeated on the other solid angles, the modified form, as shewn by the inner lines of the figure, would be produced.

Fig. 383.



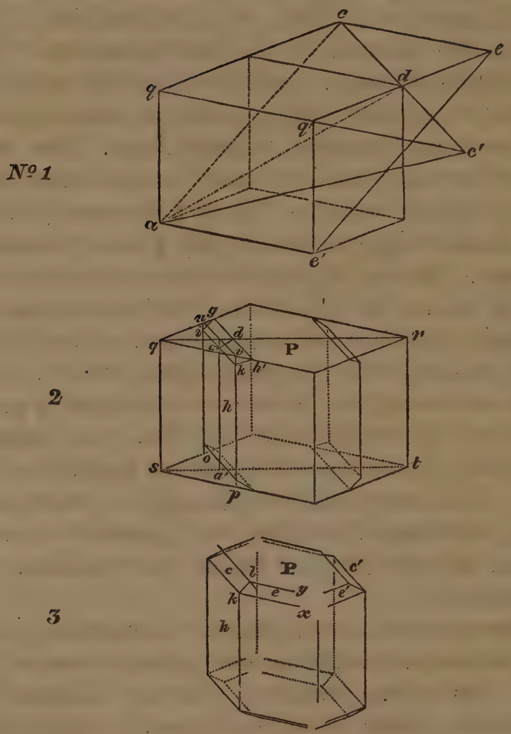
But it would frequently be inconvenient, if not impracticable, when several modifications are to be represented on the same crystal, to trace the requisite intersections within the crystal, on account of the minuteness of the secondary planes, and the number and proximity of the lines which must be drawn in order to exhibit them. We may, under such circumstances, resort to a construction, of the character of that exhibited in fig. 383. Let us suppose we have to draw a doubly-oblique prism modified on the solid angle at O of fig. 318, by two planes whose symbols are (D2 H3 F1), and O^{2}. If we produce the edge *i d* to *a*, so that *i a* is equal to twice *i d*, and make $i\ c = 3\ i\ k$, it is evident that *a b c* might be the position of the first plane upon an enlarged primary form. And if we make $i\ e = 2\ i\ b$, and $i\ f = 2\ i\ k$, *d e f* will represent the second plane upon the same enlarged primary form. But the edges of the two planes intersect each other at the points *g* and *h*; and the line *g h* is consequently the secondary edge at which the planes intersect each other. Having found this intersecting line on No. 1, as well as the positions of the intersections of the primary with the secondary planes, we may proceed to construct the secondary figure, No. 2. To do this, we should first draw a primary form in pencil, similar and parallel to *d i b k* of No. 1, and taking any point *a*, in the edge *i d*, draw *a c* parallel to *a c* No. 1. Take some point *e* in the edge *i b*, such that the part exhibited of the plane *d e f*, should be proportional in some measure to the part exhibited of the plane *a b c*.

This proportion must depend on circumstances, and on the particular illustration our figure is intended to afford. For we may evidently give any comparative extension we please to the two planes, by taking one of the points *a* or *e*, No. 2, nearer to *d* or *b*.

Having fixed on the points *a* and *e*, No. 2, we may draw *a g*, *g e*, *g h*, *c h*, parallel to the corresponding lines of No. I; and drawing the lines at the back of the figure parallel to those on the front, the secondary form will be completed.

In the future part of this section, the planes analogous to *a b c*, and *d e f*, will be termed *directing planes*; and their edges and intersecting lines, *directing lines*.

Fig. 384.



We shall now proceed to delineate the secondary form of a right rhombic prism, whose symbol would be

$$\underset{h}{{}_1G_1}\quad \underset{M}{M}\quad \underset{e}{\overset{2}{B}}\quad \underset{c}{\overset{2}{E}}\quad \underset{b_1}{(B_1\ H_1\ B'_2 : B_2\ H_1\ B'_1)}$$

$$\underset{b_2}{(B_1\ H_3\ B'_2 : B_2\ H_3\ B'_1)}\ \underset{P}{P}$$

This, although a more complicated figure than the receding, may be produced with equal accuracy, by finding successively, the intersections of all the secondary planes with each other and with the primary.

Let the primary forms, No. 1 and 2, fig. 384, be drawn in pencil,* and on No. 2 draw *u v* parallel to the short diagonal of the terminal plane, and from the points *u* and *v*, draw *u o*, *v p*, parallel to the lateral edges. The plane *h* would evidently be represented by *u v o p*. Let the line *a a′* be drawn on the plane *h*, and through the diagonals *q r*, *s t*.

The plane *c* should next be placed on the secondary figure, which plane must obviously lie between *h* and P. The law of decrement from which we have supposed this plane to result, is by 2 rows in breadth. If therefore we produce two of the primary terminal edges of No. 1 to *c* and *c′*, so that *q c*, and *q c′*, shall each be double the edge that is produced; and if we join *c c′*, the plane *a c c′*, will represent the plane *c*, the line *c c′* will touch the primary form at the solid angle *d*, and the line *d a* will pass through the middle of the plane *a c c′*.

We may call *d a*, therefore, the *directing line*, which enables us to place the plane *c* between *h* and P, by taking some point *d*, in the diagonal *q r*, No. 2, and drawing *d a* parallel to *d a*, No. 1. Through *d* and *a*, we may now draw the lines *g h′*, *i k*, parallel to $u\ v$; and as it is evident that the line $a\ d$ passes through the middle of the plane $g\ h'\ i\ k$, that plane will represent the required plane c. By drawing lines parallel to $g\ h'$, and $i\ k$, at corresponding distances from the diagonals $q\ r$, $s\ t$, and from the points q, r, s, and t, the figure with the planes h and c upon it may be completed, and may be traced separately in pencil as in No. 3, preparatory to the addition of the planes e.

---

* When two or more figures are to be drawn in the relation to each other in which these stand, their dimensions should be similar, and their corresponding edges or diagonals should either be parallel or in the same line, according to the relative positions of the figures.

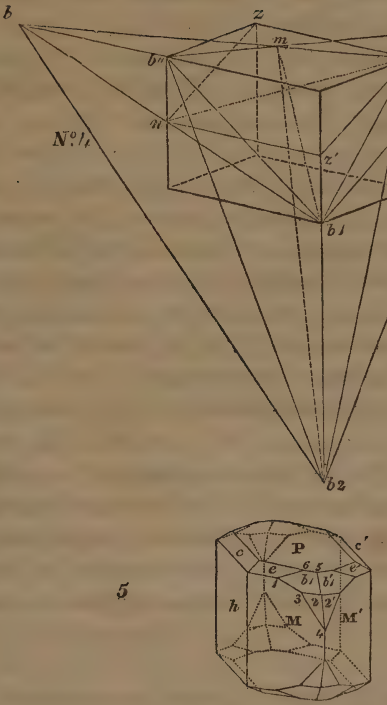
In the above figures, the corresponding lateral edges are in the same right lines, and the terminal edges are respectively parallel. These corresponding positions are always implied when two or more analogous figures are given.

To produce these planes, having first drawn $k\ x$, we require the position of the intersection $k\ l$, of the planes c and e. This intersection is shewn in No. 1 by the line $a\ c$; the points a and c being common to the plane $a\ c\ e\ e'$, which corresponds to e, and $a\ c\ c'$, which, as we have already seen, represents c.

If, therefore, from the point k, No. 3, we draw $k\ l$ parallel to $a\ c$, No. 1, and from the point l' we draw $l\ y$ parallel to $k\ x$, the plane $k\ l\ y\ x$ will be one of the required planes e. The other planes e may be drawn in a similar manner, or by parallels, or by finding the relation of the point l to some known point of the crystal.

The planes e are purposely left incomplete in the front and back of No. 3, where the planes b are to be placed.

Fig. 385.



Having thus produced the planes *d*, *c*, and *e*, we may add the planes b_1, and b_2, by tracing No. 3, fig. 384, on a separate paper, as No. 5, fig. 385, and above it draw an entire primary form, as shewn by No. 4.

To place the planes b_1 and ${}_2$ on the figure, we require their intersections with *e*, P, and M, and with each other.

The directing planes $b\ b'''\ b_1$, and $b'\ b''\ b_1$, No. 4, represent the 2 planes b_1, b'_1, No. 5; and $b\ b'''\ b_2$, $b'\ b''\ b_2$, No. 4, represent b_2, which are marked only by the number $_2$ in No. 5; and $n\ z\ b'''\ z'$, No. 4, is evidently parallel to plane e, No. 5.

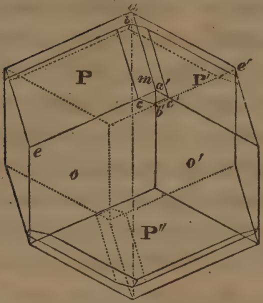
The intersection of $b\ b'''\ b_1$, with $n\ z\ b'''\ z'$, is the line $n\ b'''$; its intersection with P is $b\ b'''$; with M, it is $b\ b_1$; and with $b'\ b''\ b_1$, it is $b_1\ m$.

If, therefore, from the point 1, No. 5, we draw the line 1 6, parallel to $n\ b'''$, No. 4; 6 5, to $b\ b'''$; and 1 3, to $b\ b_1$, we shall obtain three of the edges of plane b_1. From the point 5, No. 5, draw the intersection of b_1 and b'_1, parallel to $m\ b_1$, No. 4; and by drawing on No. 5, 3 4, parallel to $b\ b_2$, No. 4, and the intersection of $_2$ and $_2'$, No. 5, parallel to $m\ b_2$, No. 4, we shall delineate the planes b_1 and $_2$; observing that those planes intersect each other in a line parallel to the intersection of b_1 and P. The positions of b'_1 and $_2$ may be obtained by a similar method of proceeding, and the other corresponding planes may be drawn by a similar process, or by parallel lines, or by finding the relation of their edges, or angles, to some known points on the crystal.

We shall give our next illustrative example from the rhomboid. Let us suppose an obtuse rhomboid is to be represented, modified by planes belonging to classes g, m, o, and p, and whose symbol is

$$\begin{array}{ccccc} {}_1D_1 & {}_2D_2 & {}_1B_1 & \overset{\frac{2}{3}}{O} & P. \\ o & p & m & g & P \end{array}$$

Fig. 386.



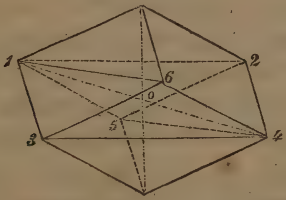
Let P, P′ P″, represent the primary planes.

The planes *o* may be added, by merely lengthening the primary axis, drawing at its two extremities the three upper and three lower primary planes of the rhomboid, and joining their angles by six vertical lines, which will then constitute the vertical edges of the plane *o*.

To add the planes *m*, take any points *c* *c*′, on the two adjacent edges of the rhomboid, such that a line passing through both should be parallel to the horizontal diagonal of the plane P″; and from these points draw lines on the planes P and P′, parallel to their common edge.

The intersection of the planes *m* with each other, is parallel to a line passing through two opposite solid angles, and through the axis.

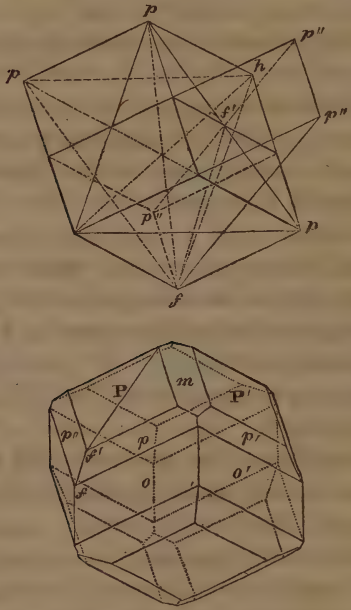
Fig. 387.



This is apparent from fig. 387, in which if the directing planes 1 2 3 4, and 1 6 5 4, be taken to represent two planes *m*, their intersection will be the line 1 4, bisecting the axis at *o*.

Having drawn the edges of the planes *m* on the primary planes in fig. 386, we may find their intersections with *o* and *o*′, by taking *a*′ *b*′, on the vertical edges of *o*, equal to the portion *a b* of the axis, cut off by the terminal edges of the planes *m*, and joining c *b*′, *c*′ *b*′.

Fig. 388.



Nº 1

2

Our next step should be to add the planes *p*. As we shall have subsequently to add the planes *g*, it is not strictly necessary to find any other intersections of the plane *p*, than those which correspond to *f f*′, fig. 388, No. 2. But for the sake of an additional illustration, we shall find the intersections of *p*, with *m*, and their adjacent intersection with each other.

If we recollect that the planes p result from a decrement by 2 rows in breadth, it will be apparent that the directing plane $p\ p\ p\ f$ of No. 1, corresponds to p of No. 2, and $p''\ p''\ p''\ f$ of No. 1, to p'' of No. 2. But the intersection of $p\ p\ p\ f$, and $p''\ p''\ p''\ f$, is the directing line $f\,f'$.

If therefore we take any point f in the vertical edge of No. 2, and draw $f\,f'$, parallel to $f\,f'$ No. 1, f' No. 2 being the intersection of the new edge with an oblique diagonal, and if from the points f and f' we draw lines parallel to the inferior primary edges, we shall obtain a representation of the planes p and p''.

The intersection of p and m, No. 2, is shewn by the directing line $p\ p$, No. 1; and that of p' and m, No. 2, by $p'\ h$, No. 1; and the intersection of p and p', No. 2, is parallel to the line $p\ f$, No. 1.*

It now remains only to add the planes g to our figure. For this purpose we may trace in pencil, as at No. 4, fig. 389, an accurate copy of No. 2, fig. 388; and above it draw the primary form, and the directing planes shewn by No. 3.

We observe here that $g\ g'\ g''$, No. 3, corresponds to plane g, No. 4. The intersection of this plane with m is parallel to the horizontal diagonal of P'', and its intersections with P and P' are parallel to $g\ g'$, and $g\ g''$, No. 3. Its intersections with p and p', are parallel to the directing lines $i\ l$ and $i\ l'$, No. 3; the points l, and l', being the only ones at which the edges of the planes p, p', and g, intersect each other, and the point i being common to the three planes.

The intersections of g with o and o' are the lines $i\ k$, $i\ k'$. For the planes o, and o', might evidently intersect the primary planes P′ and P, parallel to their oblique diagonals. If therefore two planes corresponding to *o* and *o*′, No. 4, be conceived to pass through the oblique diagonals of No. 3, and to be produced until they cut the edges *g g*′ and *g g*″, it is obvious that they would cut those edges at the points *k* and *k*′, and the point *i* would then be common to the three planes.

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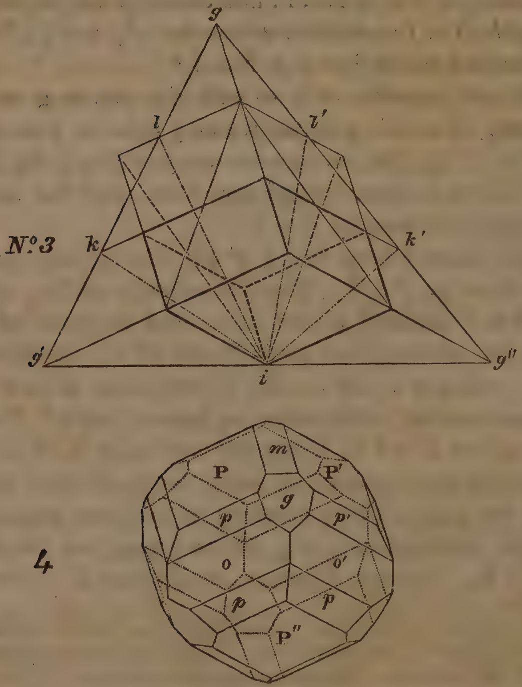
* It may be observed that there are three dotted lines terminating at h, No. 1, fig. 388; one of these has p at its other extremity, another has f, and the reader is requested to add p' to the third.

Assuming any point in the front vertical edge of No. 4, we may draw those lines of plane *g* which intersect *o* and *o*′, then those which intersect *p* and *p*′, and P and P′, and finally the intersection with *m*, to complete the figure.

Fig. 389.

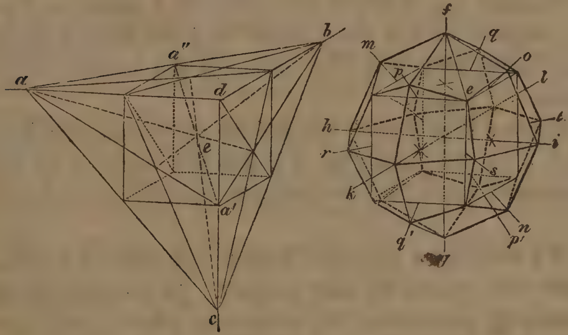


In drawing the secondary forms of crystals, it very frequently happens that the law of decrement will suggest some relation between the position of the secondary edges or angles, and some known points or lines of the primary form, which will supercede the necessity of any directing diagram. One instance of this will be seen if we turn to p. 420, where the rhombic dodecahedron is derived from the cube, through a previous knowledge of the relation of the two forms to each other.

And many expedients will probably occur to those who are accustomed to draw crystals, which will greatly abridge the laborious processes just described. These will, however, form particular cases, and will depend on the degree of attention and ingenuity employed in framing the diagrams.

The following figure will supply another example of the delineation of a secondary form, from ascertaining its relation to the primary.

Fig. 390.



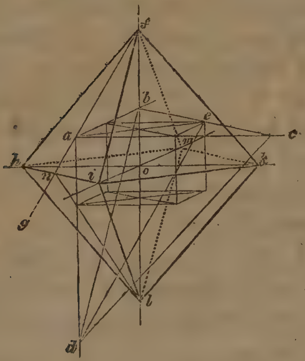
Let it be proposed to circumscribe a cube with a figure contained within 24 trapezoidal planes, belonging to mod. class *b* of the cube, the law of decrement being expressed by ${}^{2}\overset{2}{A}{}^{2}$. The fig. No. 1, has the necessary directing planes drawn upon it, from which it appears that the lines *e a*, *e b*, *e c*, represent three intersections of the secondary planes with each other. If on No. 2 we draw the lines *p p′*, *q q′*, &c. through the middle of the diagonally opposite edges, and from the solid angle at *e*, draw lines parallel to *e a*, *e b*, *e c*, of No. 1, those lines will be the edges of the new figure, and they will cut the lines *p p′*, &c. at a distance from the edges of the cube equal to $\frac{1}{6}$ of its diagonal. This will be apparent if we suppose the central point *e* of No. 1, to represent the solid angle *e* of No. 2; for the line *e a* evidently cuts a diagonal of the cube at a distance from its middle point equal to $\frac{1}{6}$ of its whole length.

The plane *a a′ b*, No. 1, represents one of the secondary planes, through the middle of which, if we draw the line *a′ a″*, that line will pass through the centre of the cube, and will consequently bisect its prismatic axes. A line drawn, therefore, from the solid angle at *e*, fig. 2, through the produced prismatic axis *f g*, will cut that axis at a distance from the surface of the cube equal to $\frac{1}{2}$ the length of the axis itself, and will pass through the middle of the secondary plane.

Having thus found the points at which the lines *p p′*, *q q′*, &c. and *f g*, *h i*, and *k l*, are cut by the secondary edges, we may readily complete the figure.

We shall give only one further illustration to complete this branch of our subject. In several of the preceding examples, the directing planes and lines have been drawn on separate, parallel, figures, for the purpose of exhibiting more distinctly the described methods of drawing. We may, however, in very many cases dispense with the second figure, and draw the directing planes in pencil, on the same figure which we purpose either to build upon, or to truncate. If there should be many planes to be placed on the secondary form, it will be found expedient sometimes to draw the directing lines with the point of a needle only, as the thickness of even a fine pencil line may become a source of error.

Fig. 391.



Let us suppose an octahedron with a square base, derived from a square prism, mod. *a*, required to be drawn, so as to envelope the prism from which it is derived; this being the method, as it has been already explained, by which nature is supposed to build up secondary forms. And let the law of decrement producing the plane, be by 2 rows in breadth, and 3 in height, on the terminal angle. Its symbol would consequently be $\overset{\frac{2}{3}}{A}$.

First draw a small prism of the proportional dimensions of the prism we propose to represent; and through the middle of the lateral planes draw the lines *h k*, *i m*, parallel to the terminal edges, and also draw the prismatic axis, *f l*. The dimensions of the circumscribing figure should be such, that its planes should touch the solid angles of the contained prism.

The directing plane, *b c d*, evidently represents a plane derived from the modification we have supposed; and the line *e d* passes through the middle of the plane. If, therefore, we draw *g f* parallel to *d e*, and touching the solid angle of the prism at *a*, we obtain the point *f*, at which the secondary plane cuts the prismatic axis. From the middle of the axis at *o*, take *o l*, equal to *o f*, and draw *l n*, touching another solid angle of the prism. The point *n* will evidently be on an edge of the secondary figure, which edge we know must be parallel to a diagonal of the terminal planes of the prism.

Through the point *n*, therefore, draw *h i* parallel to one of those diagonals; from the points *h* and *i* draw *h m*, *i k*, and join *k m* to complete the base of the figure. And the terminal edges being drawn, the entire figure will result, as shewn by the dark lines in the engraving.

ON MINERALOGICAL ARRANGEMENT.

There appears to be a degree of difficulty felt by most collectors of minerals, with regard to the arrangement of their cabinets, and particularly when new minerals occur, concerning which little more is known than their names.

This difficulty arises partly from the want of an accurate distribution of minerals into *natural species*, and partly from not attending sufficiently to a distinction, which has been hitherto regarded with less notice than it deserves, between this distribution into species, which constitutes the basis of a *natural classification* of the objects of any branch of natural history, and their *artificial arrangement for some purpose of illustration*, *of convenience*, or *as objects of taste*; which *artificial arrangement* may be regarded as analagous to the order in which words are placed in a dictionary for the convenience of reference.

This distinction will be rendered sufficiently apparent if we refer to some other branches of natural history for its illustration.

The botanist may perhaps place his specimens of dried plants in his portfolio, according to some preconceived notion of natural alliance; but when he arranges the plants themselves in his garden or his conservatory, their natural order is disregarded, the natural families are dispersed, and the situation assigned to each plant is determined by its habitudes, its necessities, or its peculiar character in reference to the pleasure it is capable of affording to some of the organs of sense.

Disparity in size also, among individuals belonging even to the same species of objects of natural history, will be another and a frequent cause of variance between their *arrangement* for purposes of amusement or use, and their *natural classification*. And examples will probably occur to the reader, of deviation from *natural classification* in the *cabinet arrangement* of minerals, where that *arrangement* has been intended to afford some particular illustration.

The cabinet of Leske, described by Kirwan, contained several separate collections, arranged for the illustration of distinct objects. One among these, exhibited in a regular series the distinctive external characters of minerals as taught by Werner; a second contained his systematic arrangement of most, if not of all, the mineral species then known; and a third exhibited the mineral substances used in various arts and manufactures, and was thence denominated the *economical* collection.

The collection of English minerals in the British Museum, is arranged according to counties, the subordinate arrangement of the minerals of each county being, however, systematic.

When, therefore, the question relates to the arrangement of a mineral cabinet generally, we should enquire into the object of the collector in forming his cabinet. In some few instances, it is possible that specimens may have been collected merely as objects of taste, and their selection may have depended merely on their rarity, or on the beauty of their forms or colours. The *arrangement* best adapted to a cabinet of this description, must evidently be such as would best exhibit the forms and colours of the specimens, and must be regulated by their size and character, according to the taste of the possessor.

But the views of those collectors by whom the greater number of cabinets are formed, are, probably, to derive from their specimens, an acquaintance with those *general external characters of minerals*, by which they are commonly discriminated from each other.

I would recommend to this class of collectors, an *arrangement of their cabinets* in nearly an *alphabetical order*, which, as it will greatly facilitate the reference to particular specimens, will afford them more ready means of comparing different specimens with each other; and every new substance that occurs, to which a name has been assigned, will also find an immediate place in the collection under its proper letter, if its precise station under any other leading name has not been previously determined.

The *alphabetical series* here recommended, is that which is distinguished by roman capitals in the *alphabetical list* which follows this section.

In this list I have endeavoured to collect and arrange all the mineral species at present known, with such of their synonyms as are not merely translations out of one language into another; and with the addition of such of the *primary forms* of those which are regularly crystallised, as appear to be accurately known.

Most of these forms have been determined from an examination of the substances themselves, and their angles have been measured, principally by the reflective goniometer, both by Mr. W. Phillips and myself; but from Mr. Phillips's greater precision in the use of that instrument, I have generally relied on his measurements where they have differed from my own; and in several instances, I have been indebted to

Mr. P. for the forms and measurements of crystals which I had not myself previously examined.

Following this *alphabetical* list, will be found a second table of primary forms, arranged according to their *classes.*

The synonyms have been collected chiefly from Leonhard's Handbuch der Oryktognosie, published in 1821, and corrected from such other sources as I have had an opportunity of consulting. The choice of a specific name among many synonyms, has been in some degree arbitrary, but I have generally been influenced in this choice by the previous adoption in this country of the name I have selected. On refering to the list, the word *Abrazite* will be found at its head, with a reference to *Zeagonite*, that being the name under which the mineral, also called *Abrazite* and *Gismondin*, had been previously known here. For this reason I have retained many of the old names, as *Chiastolite*, for example, instead of *Macle*, the name assigned to the same mineral by Haüy. In many instances, it will be perceived, I have adopted the names given by Haüy, either because they have already become familiar to the English mineralogist, as *Peridot*, instead of *Chrysolite*, or because they have comprehended several of the older species under a single name, as *Amphibole*, which includes the *Hornblende*, *Tremolite*, and *Actynolite* of the Wernerian school.

Although the basis of the proposed arrangement of minerals is alphabetical, it is to a certain extent founded on their chemical distinctions.

But a difficulty presents itself when we attempt a purely chemical classification of minerals, which arises out of the uncertainty of our knowledge relative to the *essential constituents* of many species. For however accurately these may have been analysed by the skill of modern chemistry, we are yet unable to determine which of their component parts are essential to the composition of the substance analysed, and which are but accidental mixtures. In the instance of the crystallised sandstone from Fontainbleau, no doubt can be entertained that either the carbonate of lime, or the grains of sand, must be regarded as accidental mixture, and foreign to the constitution of the species, accordingly as we chuse to consider the specimens, as *arenaceous quartz agglutinated by carbonate of lime*, or as *carbonate of lime inclosing grains of quartz.*

It would, however, present little difficulty to the chemist, to determine that the silex and lime are not chemically combined in the sandstone: but there are numerous other instances, in which even the sagacity of a Berzelius has probably failed in discriminating the matter accidentally present in several of the species of minerals which have been analysed, from that which is essential to the composition of each particular species.

These doubts are suggested by the observed fact, that the crystalline form of the Fontainbleau sandstone is similar to one of the secondary forms of carbonate of lime; and from remarking, that among the minerals which chemical analysis would raise into distinct species, there are several which appear to agree in their crystalline forms.

Now if we regard the Fontainbleau sandstone as a variety of carbonate of lime, enveloping grains of quartz; and as we observe that the crystalline form of the carbonate of lime is not altered by the presence of this siliceous mixture, we may infer that the crystalline character of minerals is not affected by the accidental presence of foreign matter in their composition, and consequently that minerals differing widely in their chemical character, may really belong to one species.

These considerations appear to confirm the propriety of the Abbé Haüy's definition of a mineral species, as given in p. 6. For although it might have been sufficient, theoretically, to comprise within the terms of this definition, such individual minerals *as are composed of similar particles united in equal proportions*, yet in the present imperfect state of our knowledge of the true constituent elements of many minerals, it appeared practically necessary to superadd to this definition the condition that, if they belong to the same species, *the form of their molecules*, or, which is the same thing in effect, *their primary forms should also be similar.*

Hence when we find different minerals agreeing in their crystalline forms, and varying in their chemical composition, we shall *probably* determine their species more accurately from their crystalline than from their chemical characters.

I say *probably*, for the crystallographical character has its uncertainties also. The natural planes of crystals are generally too imperfect to give measurements which may be said to agree very nearly with each other; the differences among such as belong to the same species of mineral, amounting frequently to nearly a degree; and the cleavage planes, which generally afford better corresponding results, cannot always be obtained; but if they could, the angles of mutual inclination even of those, are not always alike, owing probably to an interposition of foreign matter between the laminæ of the crystal, and being there *unequally* dispersed. Nor do we know that the difference of the angles under which the primary planes of different species of minerals meet, is not less than our best goniometers can distinguish. It demands great precision of hand and eye to obtain the true measurements of angles to a minute only, and we cannot say that a difference of species may not exist, with a difference of only a few seconds in the angles of inclination of their planes.

We know that the greater angle of a right rhombic prism must lie somewhere between 90° and 180°. If the angle were 90°, the prism would be square, and 180° would reduce the prism to a plane. But between 90° and 180°, we do not know how many different prisms may exist.

If they differed by degrees only, their number could not exceed 89. If the difference consisted of minutes, there might be 5399 such prisms, all distinguishable by the goniometer; but if the differences consisted of seconds only, there might evidently be 323999 rhombic prisms, of which no more than 5399 could be distinguished by the ordinary goniometric instruments.

But with all the uncertainties and difficulties attendant upon the crystallographical determination of a mineral species, the goniometer is probably the most accurate guide we at present possess to lead us to that determination. And it is almost the only one of which the practical mineralogist can at all times avail himself.

It appears almost unnecessary to state, that where a mineral is defective in crystalline character, or its chemical composition is unknown, it must be provisionally distinguished from other minerals by some other of its physical characters, as its specific gravity, hardness, fracture, &c.

Instances have been already alluded to where chemistry would separate minerals from each other, which, crystallographically, belong to the same species; of which the Amphiboles, and the Pyroxenes, afford examples. But there are a few cases also, in which minerals differing in their crystalline form, are similar in their chemical composition; as appears to be the case with the common, and white, iron pyrites. These anomalies will however, probably, be reconciled by the future investigations of science.

Dr. Brewster has, with that attachment which we usually evince towards a favorite pursuit, given a preference to the optical characters of minerals, as the surest means of determining their species. See a memoir by Dr. Brewster, in the Edinb. Phil. Journ. vol. 7. p. 12.

This memoir relates to a difference in the optical characters of the Apophyllites from different localities, upon which Dr. Brewster proposes to erect a particular variety into a new species under the name of Tesselite. Berzelius, as it appears from a paper, preceding that by Dr. Brewster, in the same volume of the Journal, has, at Dr. Brewster's desire, analysed the Tesselite, and found it agreeing perfectly in its chemical composition with the Apophyllites from other places. Chemically, therefore, the Tesselite does not appear a distinct species.

A few days before Dr. Brewster's paper was published, it happened that I had been measuring the angles of the Apophyllites from most of the localities in which they occur, all of which I found to agree with each other more nearly than different minerals of the same species frequently do. The Tesselite is not therefore, crystallographically, a separate species.* But when chemistry and crystallography concur so perfectly as they do in this instance, in determining the species to which a mineral belongs, it will be difficult to admit a variation of optical character, as a sufficient ground to alter that determination.

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* I have found several crystals of this substance corresponding in a remarkable manner in their general form of flattened four-sided prisms, terminated by four-sided pyramids with truncated summits, *but with their corresponding planes dissimilar.* The planes which appear as the summits of some of these prisms, being only the *lateral planes of very*

A paragraph published by Dr. Brewster in the 6th volume of the same Journal, p. 183, relative to the crystalline form of the *sulphato-tri-carbonate* of lead, furnishes an additional motive to believe that the connection between the optical characters of minerals and their crystalline forms, is not yet sufficiently understood.

Dr. Brewster admits what I believe is not liable to question, that "*the crystals of this substance are* "*acute rhomboids.*" But he adds, "Upon examining "their optical structure, I find that they have two "axes of double refraction, the principal one of "which is coincident with the axis of the rhomb. "The sulphato-tri-carbonate, therefore, *cannot have* "*the acute rhomboid for its primitive form, but must* "*belong to the prismatic system of Mohs.*"

But it appears from the "Outline of Professor Mohs's new system of Crystallography," published in vol. 3 of the same Journal, that *a rhomboid cannot belong to his prismatic system.* For it is stated in p. 173, that "*The rhomboid, and the four-sided oblique-* "*based pyramid,*" (the fundamental form of the prismatic system) "*are forms which cannot by any means* "*be derived from each other; the (two) groups of*

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short and otherwise disproportioned crystals; so that a line passing through these, in the direction of their greatest length, would in fact be perpendicular to the axis of the primary form. Sections perpendicular to the axes of these apparently similar prisms, would certainly present very different optical phenomena. But it is not probable that the practised eye of Dr. Brewster should have been misled by their apparent similarity, and the differences he has observed will still remain to be explained.

"*simple forms, as well as their combinations, must each be always distinct from the (other).*"

If therefore in the hands of Dr. Brewster, the use of optical characters cannot at present be relied upon for the determination of a mineral species, it may be doubted whether they can be successfully employed by less accurate and less intelligent observers.

The proposed arrangement in the following *alphabetical list*, is, as it has been already observed, to a certain extent, chemical; several species, to which separate specific names have been given, being arranged frequently under one head or *genus*, in the alphabet. And there are, probably, many other species which now stand singly in their alphabetical order, which in the opinion of some of my readers might, with equal propriety, be collected into other chemical groups.

This collection of species into groups or *genera*, has not been regulated by any very precise rule. The leading principles, however, upon which they have been formed, are either the simplicity of composition of the species of which they consist, or the apparent certainty with which that composition has been determined; some few species may, however, be considered as rather arbitrarily included under particular genera.

In most of the genera, the first and second of these principles are apparent; an example of the third may be seen under the head of Cerium, where the Yttrocerite is placed, although it contains a greater proportion of Yttria than it does of the oxide of Cerium.

The species which are left in their alphabetical order, are generally those which are denominated earthy minerals, and are composed of Alumine, Lime, Magnesia, Silex, &c. in various proportions, which are probably not as definite as they have been sometimes considered.

The proposed *alphabetical arrangement* will appear to deviate the less from *natural classification*, if we recollect that there is not any one strictly exclusive *natural order* to supercede this arrangement, and requiring that Zircon should be placed before or after the siliceous genus; or that Lead should precede, or follow, Iron or Copper. There may be conceived to be as many *natural classifications* of minerals, as there are natural properties common to the substances which are to be arranged. Thus, the metals (not including the bases of the alkalies and earths) might be arranged according to their fusibility, or their specific gravity, or their ductility, &c. Either of these characters might be adopted as the basis of a *natural classification*, and the order of the substances thus classed, would vary according to the generic character we might adopt.

The primary forms of most of the crystallised minerals contained in the following alphabetical list, are indicated in *italics*. The measurements there given, are the most accurate I have been able to obtain; but although they have been taken with much care, and probably do not vary much from the truth, they are to be regarded in strictness only as approximations to the true angles at which the planes of the crystals incline to each other.

I have added where I could, to the square, rectangular, and hexagonal prisms, the measurement of a primary plane on some modifying plane, which frequently occurs on the crystals; and the class of modifications to which the modifying plane belongs, is indicated by its appropriate letter.

It does not fall within the scope of my plan to give more than a mere *list* of minerals and of their *primary forms*. Descriptions of the minerals themselves, and figures of their secondary forms, as they occur in nature, will form the substance of a volume on Mineralogy by Mr. W. Phillips, which is now in the press.

AN ALPHABETICAL ARRANGEMENT

OF

MINERALS,

WITH THEIR

SYNONYMES AND PRIMARY FORMS.

Abrazit, see Zeagonite.
Achirite, see Copper, carbonate, siliceous.
Actinolite, see Amphibole.
Actinote, see Amphibole.
Adamantine spar, see Corundum.
Adularia, see Felspar, crystallised.
Aehrenstein, see Barytes, sulphate.
Aequinolite, see Spherulite.
Aerolite, see Iron, native, meteoric.
Aerosite, see Silver, sulphuret, antimonial.
AGALMATOLITE; Bildstein; Figure-stone; Koreite; Lardite; Pagodite.
Agaphite, see Alumine, hydrate, compact.
Agaric mineral, see Lime, carbonate, spongy.
Agate, see Quartz.
Agustite, see Emerald, var. Beryll.
Akanticone, see Epidote.
Alabaster, see Lime, sulphate.
Alalite, see Pyroxene.
Albin, see Apophyllite.
Albite, see Cleavelandite.
Allagite, see Manganese, carbonate, siliceous.
Allanite, see Cerium, oxide, ferriferous.
Allochroite, see Garnet.
Allophane, see Alumine, silicate.
Almandine, see Garnet.
Alum, see Alumine, sulphate.

ALUMINE.
  hydrate.
    crystallised; Diaspore. *A doubly oblique prism*, P on M, 108° 30′; P on T, 101° 20′; M on T, 65°, as measured and described by W. P.
    stalactitic; Gibbsite.
    compact; Agaphite; Calaite; Johnite; Turquoise.
    earthy.
  phosphate.
    crystallised; Devonite; Hydrargillite; Lasionite; Wavellite. *A right rhombic prism*, M on M′, about 122° 15′.
  silicate; Allophane. Cleavage parallel to the planes of a *square* or *rectangular prism*.
  sub-sulphate; Aluminite; Hallite; Websterite.
  sulphate of, and potash; Alum.
    crystallised.
    fibrous. *A regular octahedron.*
  sulphate of Alumine and Potash; Alum.
    crystallised.
    fibrous. *A regular octahedron.*
  ...., siliceous. Alum-stone.
    crystallised. *An obtuse rhomboid*, P on P′, 92° 50′, as measured by W. P.
    amorphous.
Aluminite, see Alumine, sub-sulphate.
Amalgam, see Mercury, argentiferous.
Amausite, see Felspar, compact.
Amazon-stone, see Felspar, green.
AMBER; Bernstein; Karasé; Succin.
AMBLYGONITE. Cleavage parallel to the lateral planes of a *prism* of about 105° 45′, with indistinct traces of cleavage oblique to the axis of the prism. I am indebted to Mr. Heuland for the loan of the specimen I have measured.
Amethyst, see Quartz.
Amianthinite, see Amphibole, Amianthoide.
Amianthoide, see Amphibole.
Amianthus, see Asbestus.
AMMONIA.
  muriate; Sal ammoniac.
    crystallised. *A regular octahedron.*
    stalactitic.
    earthy.
  sulphate; Mascagnin.
    stalactitic.
    earthy.

AMPHIBOLE.

crystallised. *An oblique rhombic prism*, P on M or M′, 103° 15′; M on M′, 124° 30′.

fibrous.

amorphous.

The following varieties appear, from the measurement of their angles, to belong to this species.

Common Hornblende, colour dark green or greenish black; Keraphyllite; Keratophyllite.

Carinthin, in colourless, yellowish, and greenish crystals.

Basaltic Hornblende.

The foliated Augite of Werner.

The blue Hyperstene of Giesecke.

The green Diallage of Haüy; Smaragdite; Lotalite.

Pargasite, in short green crystals.

Actynolite; Actinote; Strahlite; the crystals green, slender, and sometimes radiating.

Tremolite; Grammatite; the crystals colourless or green, or pink, or brownish and generally imbedded in Dolomite; frequently fibrous; and sometimes radiated.

A transparent and colourless variety occurs with the white Pyroxene at New York.

Several specimens sent me as *white pyroxene* were all amphibole except one, which contained two or three imbedded crystals of pyroxene.

Amianthoide; Amianthinite; Asbestinite; Byssolite; two separate fibres of this substance have afforded the measurement of 124° 30′.

Amphigene, see Leucite.

ANALCIME; Cubicite. *A cube.*

the crystals red; Sarcolite.

Anatase, see Titanium, oxide.

ANDALUSITE; Micaphyllite; Stanzaite. *A right rhombic prism*, M on M′, 91° 20′, as measured by W. P.

Andreasbergolite, see Harmotome.

Anhydrite, see Lime, sulphate, anhydrous.

ANTHOPHYLLITE. Cleavage parallel to the lateral planes of *a rhombic prism* of 125°, and to both its diagonals; and another cleavage apparently perpendicular to the axis of the prism. The bright plane which is generally visible in the specimens, is parallel to the greater diagonal of the prism.

Anthracite, see Coal.

Anthracolite, see Coal.

Anthraconite, see Lime, carbonate, columnar.

ANTIMONY.

native. Cleavage parallel to the planes of *an obtuse rhomboid*; P on P′, about 117°, but the measurement of different fragments does not agree within more than 2 degrees, owing to the dulness of the planes, and apparently to their being more or less curved. The rhombic planes are striated horizontally, and the bright planes, which are generally conspicuous in the specimens of this substance, are perpendicular to the axis of the rhomboid.

…… arseniferous.

oxide; White antimony.

crystallised. *A right rhombic prism*; M on M′, 137°; the broad bright planes of the crystals being parallel to the short diagonal of the prism.

fibrous, radiating.

earthy.

……, sulphuretted; Red antimony.

crystallised; probably *a right square prism*; Mr. Phillips having found one of the thin fibrous crystals measure on the lateral planes 90°, and 135°.

fibrous.

earthy; Tinder ore. (Leonhard.)

sulphuret; Grey antimony.

crystallised. *A right rhombic prism*, M on M′, very nearly 90°. Its secondary planes shew that the prism belongs to the rhombic class and is not square.

fibrous.

compact.

Apatite, see Lime, phosphate.

Aphrite, see Lime carbonate, nacreous.

Aphrizite, see Tourmaline.

APLOME. *A cube.*

APOPHYLLITE; Albin; Fish-eye-stone; Icthyopthalmite. *A right square prism*; M on a plane belonging to mod. class *a*, 128° 10′.

Aquamarine, see Emerald, var. Beryll.

Arendalite, see Epidote.

ARFWEDSONITE; Ferriferous hornblende; see Annals of Philosophy for May 1823. Cleavage parallel to the lateral planes and to both the diagonals of *a rhombic prism* of 123° 55′.

Argentine, see Lime, carbonate, nacreous.
Arkticite, see Scapolite.
Armenite, is said to be either Quartz or Carbonate of lime coloured by blue Carbonate of copper.
Arragonite, see Lime, carbonate.
ARSENIC.
- native.
- oxide. *A regular octahedron.*
- sulphuret.
  - red; Realgar.
    - crystallised. *An oblique rhombic prism*; P on M or M′, 104° 6′; M on M′, 74° 14′.
    - amorphous
  - yellow; Orpiment.
    - crystallised. *A right rhombic prism*; M on M′, 100°. Form determined and measured by W. P.
    - foliated.

Asbestinite, see Amphibole.
Asbestoide, see Amphibole.
ASBESTUS.
- common, the fibres parallel,
  - .... lying in many directions, and as it were matted together, forming
    - mountain paper
    - .... leather
    - ...... cork
    - .... wood.
- flexible; Amianthus.

Asparagus-stone, see Lime, phosphate.
Asphaltum, see Bitumen.
Astrapyalite, see Quartz, sand-tubes.
Atacamite, see Copper, muriate.
Atlaserz, see Copper, carbonate, green.
Atramentstein, see Iron, sulphate, decomposed.
Augite, see Pyroxene.
Augustite, see Lime, phosphate.
Automolite, see Zinc, oxide, aluminous.
Avanturine, see Quartz, amorphous.
Axe-stone, see Jade.
AXINITE; Thumerstone; Thumite; Yanolite. This substance does not readily yield to cleavage, so as to afford a determination of its primary form from cleavage planes. The primary form best agreeing with those secondary forms under which it generally occurs, is *a doubly oblique prism*, P on M, 134° 40′; P on T, 115° 17′; M on T, 135° 10′, as measured by W. P.

Azabache, see Coal, Jet.

AZURITE; Klaprothite; Tyrolite; Voraulite.
- crystallised. *A right rhombic prism*; M on M′, 121° 30′. I am indebted to the kindness of Mr. Heuland for the very rare specimen which has enabled me to determine this form.
- compact.

B

Baikalite, see Pyroxene.

Baldogée, see Green-earth.

Bardiglione, see Lime, sulphate, anhydrous.

BARYTES.
- carbonate; Barolite; Witherite.
  - crystallised. *A right rhombic prism*, M on M′, 118° 30″, as measured by Mr. W. Phillips. The ordinary hexagonal crystals probably result from the intersection of three of the primary crystals.
  - fibrous.
- sulphate; Baroselenite.
  - crystallised. *A right rhombic prism*, M on M′, 101° 42′.
  - columnar.
  - radiated; Bolognian spar; Litheosphore.
  - granular.
  - compact; Cawk.
    - acicular, diverging, and imbedded in some other substance; Aehrenstein.
  - earthy.
  - hepatic.
- sulphate of Barytes and Strontian.

Basanite, see Quartz.

Baudisserite, see Magnesia, carbonate, siliceous.

Beilstein, see Jade.

Bell-metal ore, see Tin, sulphuret of Copper and Tin.

BERGMANITE; Spreüstein. No crystalline form discoverable, nor any analysis, that I can find published. Is referred by Leonhard to Scapolite, but on what authority does not appear.

BERGMEHL, Mountain meal.

Bernstein, see Amber.

Beryll, see Emerald.

Berzelite, see Petalite.

Bildstein, see Agalmatolite.

Bimstein, see Pumice.

Bismuth.
  carbonate.
  native.
  oxide.
  sulphuret.
  .... cupriferous.
  .... plumbo-cupriferous; Needle-ore.
  .... plumbo-argentiferous; Bismuthic silver.
Bitter-spar, see Lime, carbonate, magnesian.
Bitumen.
  liquid; Naphtha.
  viscid; Petroleum.
  solid, elastic; Elaterite.
    compact; Asphaltum.
    earthy; Maltha.
  Dapeche, brought by Humboldt from South America, and probably does not belong to the mineral kingdom.
  Fossil copal; Highgate resin.
  Retinasphaltum.
Blattererz, see Tellurium, native, plumbo-auriferous.
Bleiniere, see Lead arseniate.
Bleischweif, see Lead, sulphuret, compact.
Blende, see Zinc, sulphuret.
Blizsinter, see Quartz, sand-tubes.
Bloedit, see Magnesia, sulphate of and Soda.
Bloodstone, see Quartz, calcedony.
Bohnerz, see Iron, oxide, hydrous.
Bole; Lemnian earth; Terra de Siena; Terra sigillata.
Bolide, see Iron, native meteoric.
Bolognian spar, see Barytes, sulphate.
Boracic Acid; Sassolin.
Boracite, see Magnesia, borate.
Borax, see Soda, borate.
Borech, see Soda, carbonate.
Botryolite, see Lime, borate, siliceous.
Bournonite, see Lead, triple sulphuret.
Breislakite.
Brewsterite. *A right oblique-angled prism*, M on T, about 93° 40′.
Brongniartin, see Soda, sulphate of Soda and Lime.
Bronzite; fibrous Diallage metalloide. Cleavage parallel to the planes and to both the diagonals of *a rhombic prism*, of about 93° 30′, with indications of another cleavage perpendicular to the axis of the prism. See Hypersthene.

Brown spar; see Iron, carbonate. And see Pearl-spar.
Brucite, see Condrodite.
Brunon, see Titanium, oxide, siliceo-calcareous.
Bucholzite; two different substances appear to have been included under this name, viz. a mineral from the Tyrol, called Fibrous Quartz by Werner, and a fibrous substance frequently found accompanying Andaluzite.
Byssolite, see Amphibole.

C

Cacholong, see Quartz, opal.
Cadmium, a metal found in combination with Zinc in several of its ores.
Calaite, see Alumine, hydrate.
Calamine, see Zinc, carbonate.
oxide.
silicate.
Each of these species having passed under the common appellation of Calamine.
Calcedony, see Quartz.
Cantalite, see Quartz, yellowish-green.
Carbonic Acid.
Carinthin; see Amphibole.
Carnelian, see Quartz, calcedony.
Cascalhao, clay indurated by Iron and Quartz, and frequently inclosing grains of Quartz, found in rolled fragments at the diamond mines in Brazil.
Catseye, see Quartz.
Cawk, see Barytes, sulphate, compact.
Celestine, see Strontian, sulphate.
Cerauniansinter, see Quartz, sand-tubes.
Ceraunite, see Jade, nephrite.
Cereolite.
Cerin, see Cerium.
Cerite, see Cerium.
Cerium.
fluate.
sub-fluate.
fluate of Yttria and Cerium.
fluate of Yttria Cerium and Lime; Yttro-cerite. Cleavage parallel to the planes of *a right rhombic prism* of about 97°, by the common goniometer.
oxide, siliceous, red; Cerite; Ochroite.
. . . . . ferro siliceous, black; Cerin; Allanite. *A right square prism*, as determined by W. P.
Chabasie. *An obtuse rhomboid*, P on P′, 94° 46′.

Chalcolite, see Uranium, phosphate.
Chalcosiderite, see Iron, green earth, fibrous.
Charcoal, mineral, see Coal.
Chelmsfordite, possesses the external characters of Scapolite, and crystallises in square prisms. See Scapolite.
CHIASTOLITE; Crucite; Macle. Probably *a right square* or *rectangular prism.*
Chlorite, see Talc.
Chlorophane, see Lime, fluate.
CHLOROPHÆITE, described by Dr. Mac Culloch.
Chondrodite, see Condrodite.
CHROME, oxide.
CHRYSOBERIL; Cymophane. *A right rhombic prism*, M on M′, 97° 12′. The plane P is generally bright and striated.
Chrysocolla, see Copper, carbonate, siliceous.
Chrysolite, see Peridot.
Chrysoprase, see Quartz, calcedony.
Chusite, see Peridot, granular, decomposing.
CIMOLITE.
Cinnabar, see Mercury, sulphuret.
CINNAMON-STONE; Essonite; Hyacinth; Romanzovite.
  crystallised. *A rhombic dodecahedron.* The cleavage planes afford measurements of about 90° in one direction, and about 120° in one or two others.
  amorphous.
CLEAVELANDITE; Albite; Siliceous spar from Chesterfield in Massachusetts. See Annals of Philosophy for May 1823.
  crystallised. Cleavage parallel to the planes of a *doubly oblique prism*, P on M, 119° 30′; P on T, 115°; M on T, 93° 30′.
  laminar.
COAL.
  carbon nearly pure; Anthracite; Anthracolite; Geantrace.
    compact.
    columnar.
    slaty.
  bituminous.
    compact; Cannel coal.
    columnar.
    foliated; Common coal.
    friable; Mineral charcoal.
  ligniform. Wood coal.
    compact; Azabache; Jet.
    fibrous; Bovey coal; Surturbrand.

    - foliated; Dysodile; Paper coal.
    - earthy.
  - peat.
- **Cobalt.**
  - arseniate. *A right oblique-angled prism.* M on T, 124°.
  - arsenical.
    - grey. *A cube* with regular modifications.
    - white. *A cube* with irregular modifications similar to those of Iron pyrites.
  - oxide, black.
    - ferriferous, brown.
    - yellow.
  - sulphate.
    - stalactitic.
  - sulphuret.
    - botryoidal.
    - amorphous.
- Coccolite, see Pyroxene, granular.
- Cockle, of the Cornish miners, see Tourmaline.
- **Collyrite**, or Kollyrite.
- Colophonite, see Garnet.
- Columbite, see Tantalite.
- **Comptonite.** *A right rectangular prism.* M on a plane belonging to mod. class *d*, 135° 30′.
- **Condrodite**, or Chondrodite. No crystalline form discoverable in any specimen I have seen.
- Conite, see Lime, carbonate, magnesian.
- Copal, fossil; Highgate resin, see Bitumen.
- **Copper.**
  - arseniate.
    - octahedral; Linsenerz. *An obtuse octahedron with a rectangular base*; P on P′, 72° 22; M on M′, 61°, as measured by W. P.
    - prismatic.
      - right; crystallised; Olivenit. *A right rhombic prism*, M on M′, about 111°.
        - fibrous; Wood copper.
        - compact.
      - oblique. *An oblique rhombic prism*, P or M or M′, 95°; M on M′, about 56°.
    - rhombic; Copper mica. *An acute rhomboid*, P on P′, 69° 30′, as measured by Mr. W. Phillips on some bright planes.
  - arseniate of Copper and Iron; Skorodite. *A right rhombic prism*, M and M′, 120°.

- carbonate.
  - blue.
    - crystallised. *An oblique rhombic prism*, P on M or M′, 91° 30; M on M′, 99°.
    - fibrous.
    - compact.
    - earthy.
  - green. Malachite.
    - crystallised; *an oblique rhombic prism*, P on M or M′, 112° 52′; M on M′, 107° 20′. All the crystals I have seen are hemitrope. The plane of junction of the reversed halves being parallel to the great diagonal of the prism.
    - fibrous; Atlaserz.
    - compact.
    - epigene, having the form of the crystals of blue carbonate, or of red oxide.
  - siliceous; Achirite; Dioptase. *An obtuse rhomboid*, P or P′, 126° 17′, as measured by W. P. on cleavage planes.
  - anhydrous, reddish brown, massive.
- hydrate.
  - .... siliceous; Chrysocolla.
- muriate.
  - crystallised. *A right rhombic prism*, M on M′, 97° 20′, as deduced from the measurement of secondary planes; the primary planes whose positions are indicated by cleavage, not appearing on the crystals.
  - arenaceous. Atacamite. Green sand of Peru.
- native.
  - crystallised. *A cube.*
  - fibrous.
  - laminar.
  - compact.
  - combined with Arsenic and Iron; White copper.
- oxide.
  - black.
    - earthy.
  - red.
    - crystallised. *A regular octahedron.*
    - fibrous.

      The longitudinal planes of the fibres are generally those which would result from the replacement of the edges of the base of the octahedron, the axis perpendicular to this base being very disproportionately lengthened.

earthy. Tile ore.

phosphate.

anhydrous.

crystallised. In the absence of distinct cleavage, *a right rhombic prism* may be regarded as the primary form, M on M′, 95° 20′.

compact.

hydrous.

crystallised. *An oblique rhombic prism*, P on M or M′, 97° 30′; M on M′, 37° 30′. The planes of the crystals I have measured are not sufficiently perfect to afford very accurate measurements.

The difference between the forms of these phosphates was I believe first observed by Mr. Levy.

fibrous.

seleneuret.

.... of Silver and Copper. Eukairite.

sulphate. *A doubly oblique prism*, P on M, 127° 30′; P on T, 108°; M on T, 123°.

sulphuret. Glance copper.

crystallised. The form under which the crystals usually occur is that of a *regular hexagonal prism*, with its terminal edges replaced. Mr. Levy found that the solid angles of the prism might be removed by regular cleavage, exhibiting thus an analogy to Quartz. The primary form may therefore be *an acute rhomboid*, P on P′, 71° 30′; as measured by W. P.

compact.

From the published analyses there appear to be two different sulphurets, in which the proportions of copper and sulphur differ. The form indicated here is that of the Cornish sulphuret.

.... of Copper and Bismuth.

.... of Copper and Silver.

.... of Copper and Iron. Copper pyrites; yellow copper ore.

crystallised. *An octahedron with a square base,* P on P′, 102° 14′; P on P″, 125° 15′.

mamellated.

amorphous.

purple Copper; Buntkupferez. Differing from yellow copper in the proportions of its constituent elements.

crystallised. *A regular octahedron*, as determined by W. P. from cleavage.

amorphous.

.... of Copper and Antimony.

.... of Copper, Iron, and Antimony. Colour, dark grey.

.... Arsenic. Colour, bright steel grey.

Both these varieties are termed Grey copper; Fahlore.

crystallised. *A regular tetrahedron.*

amorphous.

Platiniferous grey copper.

.... of Copper, Iron, and Arsenic, but differing in the proportions from the preceding. Tennantite.

crystallised. *A regular tetrahedron.* The regular octahedron is considered as the primary form by Mr. W. Phillips, but some of the modifications accord better with the tetrahedron.

undetermined species.

blue fibrous Velvet ore; Sammterz.

green foliated; Kupferschaum.

Corallenerz, see Mercury, sulphuret, hepatic.

Cordierite, see Dichroite.

CORUNDUM.

blue; Sapphire; Telesie.

red; Oriental ruby; Telesie.

yellow; Oriental topaz.

purple; Oriental amethyst.

common; Adamantine spar.

crystallised. *An acute rhomboid*, P on P′, 86° 4′.

granular.

compact.

ferriferous; Emery.

COUZERANITE. The crystalline form is said by Leonhard to be *a right rectangular prism.*

Crichtonite, see Titanium.
Crispite, see Titanium, Rutile.
Crocalite, see Mesotype, red, globular radiated.
Cronstedit, is said by Leonhard to crystallise in *hexagonal prisms.*
Crucite, see Chiastolite.
Cryolite. Cleavage parallel to the planes of *a square* or *rectangular prism.*
Cubicite, see Analcime.
Cyanite; Disthene; Kyanite; Rhetizite; Sappare. *A doubly oblique prism*, P on M, 93° 15′; P on T, 100° 50′; M on T, 106° 15′, measured by W. P. on cleavage planes.
Cymophane, see Chrysoberil.

D

Datholite, see Lime, borate, siliceous.
Daurite, see Tourmaline, red.
Dapeche, see Bitumen.
Delphinite, see Epidote.
Deodalite, see Pitchstone.
Desmine.
Devonite, see Alumine phosphate.
Diallage.
  geeen, see Amphibole.
  metalloide, foliated, see Schiller spar.
    fibrous, see Bronzite.
Dialogite, see Manganese, carbonate.
Diamond. *A regular octahedron.*
Diaspore, see Alumine, hydrate.
Dichroite; Cordierite; Jolite; Peliome; Steinheilite.
  crystallised. *A regular hexagonal prism.* M on a plane of mod. *c*, 137° 46′, as measured by W. P.
Diopside, see Pyroxene.
Dioptase, see Copper, carbonate, siliceous.
Dipyre; Leucolite. From a very minute crystal in my possession, I suppose the primary form to be *a regular hexagonal prism*, but the planes are too imperfect to determine this point by measurement.
Disthene, see Cyanite.
Dolomite, see Lime, carbonate, magnesian.
Dragonite, see Quartz, crystallised.
Dysodile, see Coal.

E

Edelite, see Mesotype, red, earthy.
Egerane, see Idocrase.
Egyptian pebble, see Quartz, Jasper.
Eisenkiesel, see Quartz, ferruginous.
Ekebergite, see Scapolite.
Elaeolite, see Fettstein.
Elaterite, see Bitumen, elastic.
Electrum, see Gold.
EMERALD
  green.
    transparent, precious Emerald.
    opaque, common.
  blue and yellow of various shades, and colourless; Beryll.
    blue from Siberia; Agustite.
    greenish blue from Brazil; Aquamarine.
  A regular hexagonal prism, P on a plane belonging to class *a*, 135°.
Emery, see Corundum.
Endellione, see Lead, triple-sulphuret of Lead, Copper, and Antimony.
EPIDOTE; Akanticone; Arendalite; Delphinite; Illuderite; Pistazite; Thallite.
  crystallised, *A right oblique-angled prism*, M on T, 115° 40′.
  amorphous.
  granular; Scorza.
Ercinite, see Harmotome.
Esmarkite, see Lime, borate, siliceous.
Essonite, see Cinnamon-stone.
EUCHYSIDERITE, see Pyroxene, fusible.
EUCLASE. *A right oblique-angled prism*, M on T, 130° 50′. The axis of the prism being perpendicular to the bright cleavage plane.
EUDYALITE.
  crystallised. *An acute rhomboid*, P on P′, about 74° 30′.
    I am obliged to Mr. Heuland for the loan of the crystal I have measured, which is large and nearly perfect; but the planes are not sufficiently brilliant to afford a very accurate measurement by the reflective goniometer.
  amorphous.
Eukairite, see Copper, selenuret.

F

Fahlore, see Copper, sulphuret, arsenical.

Fahlunite. Two substances differing in their external characters have passed under this name. The *Triklasite*, analysed by Hisinger, agrees nearly in its external character, and in its composition, with the *Gieseckite.* The hard Fahlunite, which has been analysed by Stromeyer, may remain here as a separate species, unless it be referred to Dichroite, to which Stromeyer appears to think it belongs.

Fassaite, see Pyroxene.

Felspar; Orthose.
- crystallised, *a doubly oblique prism*, P on M, 90°; P on T, 120° 15′; M on T, 112° 45′, as measured by W. P.
  - transparent, or translucent; Adularia.
    - with bluish opalescence; Moon-stone.
    - glassy; Sanidin.
  - opaque.
    - common.
    - blue.
    - green; Amazon stone.
    - variously opalescent; Labrador felspar.
    - fetid; Necronite.
- compact; Amausite; Felsite; Felstein; Hornstone, fusible; Lemanite; Lodalite; Saussurite.
- globular. Giving its peculiar character to the rock called Napoleonite, and to another termed Variolite.
- decomposd; Kaolin.

Felstein, see Felspar, compact.

Fettstein; Elaeolite; Sodaite. Cleavage parallel to the planes of a prism of 112° and 68°, but no transverse cleavage to determine the class to which the prism belongs: measured by W. P. A red variety has been called Lythrodes.
- blue, from Laurwig in Norway, see Glaucolite.

Fibrolite.

Figure-stone, see Agalmatolite.

Fiorite, see Quartz.

Fish-eye-stone, see Apophyllite.

Flint, see Quartz.

Flockenerz, see Lead, arseniate.

Flos-ferri, see Lime, Arragonite, coralloidal.

Fluor spar, see Lime, fluate.

Fossil copal, see Bitumen.

- Franklinite, see Iron oxide.
- FREISLEIBEN.
- Frugardit, reddish Idocrase containing Magnesia.
- Fulgurite, see Quartz, sand-tubes.
- FULLERS-EARTH.
- FUSCITE. *A square prism.* Is referred by Leonhard to Scapolite, but it is not stated upon what authority.

G

- GABBRONITE.
- GADOLINITE.
  - crystallised. *An oblique rhombic prism*, P on M or M′, about 96° 30′; M on M′, about 115°.
  - amorphous.
- Gahnite, see Zinc, oxide, aluminous.
- Galena, see Lead, sulphuret.
- Gallizenstein, see Zinc, sulphate.
- Gallizinit, see Titanium, oxide, rutile.
- GARNET.
  - crystallised. *A rhombic dodecahedron.*
    - black, Melanite.
    - ...., from the Pyrenees; Pyreneite.
    - greenish, from the Baikal; Grossularia.
    - yellow; Topazolite.
  - granular.
    - red; Pyrope.
    - yellow; Succinite.
    - brownish yellow; Colophonite.
  - amorphous.
    - transparent; Precious Garnet; Almandine; Greenlandite.
    - opaque and greenish; Allochroite.
  - Berzelius has, in his System of Mineralogy, chemically divided the Garnets into 13 species.
  - Almandine.
  - Garnet from Broddbo.
  - .... Finbo.
  - ...... oriental.
  - ...... from Syria.
  - .... Swappavara.
  - .... Thuringia.
  - .... Dannemora.
  - .... Longbanshyttan.
  - Melanite.
  - Grossularia.
  - Colophonite.

Allochroite.

But it is probable that most, if not all of these distinctions, may be referred to accidental mixtures, which chemistry cannot at present distinguish from the essential constituents of the pure garnet.

Geantrace, see Coal.

GEHLENITE; Stylobat. *A square* or *rectangular prism.* No secondary forms to determine which of these is the primary.

Geyserite, see Quartz.

Gibbsite, see Alumine hydrate.

GIESECKITE.

crystallised. *A regular hexagonal prism.* The Triklasite agrees very nearly with this species in its chemical composition, and possesses the same crystalline form, with nearly the same external characters.

Girasol, see Quartz, opal.

Gismondin, see Zeagonite.

Glance copper, see Copper sulphuret.

Glauberite; see Soda, sulphate of Soda, and Lime.

Glaucolite, a mineral so named by Fisher of Moscow, which resembles in colour and general appearance the blue Fettstein from Laurwig in Norway. The Norway mineral has cleavages parallel to the planes of *a rhombic dodecahedron.*

The only specimen I have seen of Glaucolite is in the possession of Mr. Heuland.

GOLD, native.

crystallised. *A regular octahedron.*
fibrous.
granular.
amorphous.
...... argentiferous; Electrum.
crystallised.
amorphous.

Gothite, see Iron, oxide, hydrous.

Grammatite, see Amphibole.

Grammite, see Lime, silicate.

Graphic ore, see Tellurium.

GRAPHITE; Plumbagine; Plumbago; occurs in thin hexagonal plates, or crystals, which are sometimes striated parallel to their edges.

Grenatite, see Staurotide.

GREEN-EARTH; Baldogee.

Greenlandite, see Garnet.

Gregorite, see Titanium, oxide, ferriferous.
Grossularia, see Garnet.
Gummistein, see Quartz, Hyalite.
Gurhofian, see Lime, carbonate, magnesian, compact.
Gypsum, see Lime, sulphate.

H

Hallite, see Alumine, sub-sulphate.
Hallotricum, see Magnesia, sulphate.
HARMOTOME; Andreasbergolite; Andreolite; Ercinite. *A right rectangular prism.* M on a plane belonging to class *b*, 150°.
HAÜYNE; Latialite. *A rhombic dodecahedron.* Another blue mineral from Vesuvius has been also called Haüyne, but it appears to be of a different species, to which, for the purpose of placing it in this alphabetical series, I have given the name of *Napolite.*
HAYDENITE.
HEDENBERGITE. Is said to have cleavages parallel to the planes of an obtuse rhomboid similar to that of carbonate of lime, of which it contains only about 5 parts in 100. If this be really so, it will afford an instance of the near approach of two rhomboids, belonging to different species of minerals, and will offer another example of the influence of a small portion of carbonate of lime to determine the form of the mass.
Heliotrope, see Quartz, calcedony.
Helvin; see Manganese, silicate.
Dr. Wollaston has kindly supplied me with the chemical character of this substance, and has thus enabled me to place it in its proper station in the list.
Hematite, brown, see Iron, oxide, hydrous.
red,...................., anhydrous.
Hepatite, see Barytes.
HEULANDITE; foliated Stilbite. *A right oblique-angled prism*; M on T, 130° 30′.
Highgate resin, see Bitumen.
HISINGERIT.
Hogauite, see Mesotype.
HOLMITE.
Honey-stone, see Mellite.
Hornblende, see Amphibole.
ferriferous, from Greenland, see Arfwedsonite.
Hornstone, fusible, see Felspar, compact.
infusible, see Quartz.
Humboldite, see Lime, borate, siliceous.

HUMITE. *A right rhombic prism*; M on M′, 120°.

Hyacinth, a name sometimes applied to a red variety of Zircon, and sometimes to Cinnamon-stone.

Hyalite, see Quartz.

Hydrargillite, see Alumine, phosphate.

Hydrophane, see Quartz, opal.

Hydropite, see Manganese, silicate.

HYPERSTENE, blue from Greenland, see Amphibole.

from Labrador; Paulite. Cleavage parallel to the planes, and to both the diagonals of *a rhombic prism*, of 93° 30′. The bright plane which is apparent in the specimens of this substance, is parallel to the short diagonal of the prism. There is no cleavage that I can perceive transverse to the axis of the prism, but I have a fragment of a crystal which indicates an oblique termination inclining upon the acute edge of the prism.

The Bronzite and Schiller spar have cleavages similar to the Hyperstene, and measure very nearly the same; but the Bronzite is much softer than the Hyperstene, and the Schiller spar softer than the Bronzite, and are probably therefore distinct minerals.

I J

JADE. Axe-stone. Beilstein.

Nephrite. Ceraunite.

Jargon, see Zircon.

Jasper, see Quartz.

ICE-SPAR. *A right oblique-angled prism*; M on M′, 129° 40′, as measured by W. P.

Ichthyopthalmite, see Apophyllite.

IDOCRASE; Egeran; Vesuvian; Wiluite.

crystallised. *A right square prism*; P on a plane belonging to mod. class *a*, 142° 50′.

red, containing Magnesia; Frugardite.

greenish yellow, containing Manganese; Loboite.

amorphous.

JEFFERSONITE.

crystallised. The crystals resemble one of the opaque varieties of Pyroxene. The cleavages are parallel to the terminal and lateral planes, and to both the diagonals of *an oblique rhombic prism*, of about 87° and 93°, oblique from an acute edge. These angles are nearly those of Pyroxene, to which species this mineral will probably be found to belong.

amorphous.

- **Jenite**; Ilvaite; Lievrit.
  - crystallised. *A right rhombic prism*; M on M′, 112°.
  - fibrous radiated.
  - amorphous.
- Jet, see Coal.
- Igloite, see Lime, carbonate, Arragonite.
- Illuderite, see Epidote.
- Ilvaite, see Jenite.
- **Indianite**, cleavage parallel to the planes of a prism of about 95° 15′, which is the angle of Silicate of lime.
- Indicolite, see Tourmaline, blue.
- Inolite, see Lime, carbonate, stalactitic.
- Johnite, see Alumine, hydrate.
- Jolite, see Dichroite.
- **Iridium**, native, alloyed with Osmium. *A regular hexagonal prism.*
- **Iron.**
  - arseniate. *A cube*, the modifications of which are sometimes defective, and such as might result from the tetrahedron as a primary form.
  - arsenical, see Sulphuret, arsenical.
  - carbonate; Brown spar; Stahlstein.
    - crystallised. *An obtuse rhomboid*; P on P, 107°.
    - fibrous.
      - the fibres parallel.
      - .... radiating and forming a mammellated surface. Spherosiderite.
  - chromate.
    - crystallised. *A regular octahedron.*
    - amorphous.
  - native.
  - ..... meteoric; Aerolite; Bolide; Meteorite.
  - ..... steel.
  - oxidulous; Magnetic.
    - crystallised. *A regular octahedron.*
    - fibrous.
    - amorphous.
      - with oxides of Manganese and Zinc. Franklinite. This mineral crystallises in *regular octahedrons*; and the Manganese and Zinc are probably only accidental mixtures with the Iron.
  - oxide.
    - anhydrous.
      - crystallised; Oligiste Iron; Specular Iron. *An acute rhomboid*; P on P′, 86° 10′.

foliated; Micaceous.
scaly; Iron froth.
fibrous; red hematite.
compact; red hematite.
earthy; red ochre.
red siliceous Iron-stone.
red clay-Iron-stone.
  compact.
  columnar.
  earthy.
hydrous.
  crystallised. *A right rhombic prism*; M on M′, 130° 40′. Cleaves easily in the direction of the short diagonal of the base. Crystals of this substance occur at St. Just in Cornwall, and with crystallized quartz at St. Vincent's rocks near Bristol. Those from St. Vincent's rocks have been formerly supposed to be Wolfram.
  red scaly; Gothite; Pyrosiderite; Rubinglimmer.
  .... fibrous; Lepidocrokite.
  fibrous brown hematite, containing a greater proportion of water than the crystallised variety.
  compact brown hematite; Stilpnosiderite.
  earthy; brown ochre.
  brown clay-iron-stone.
    globular; pea ore.
    lenticular.
    compact.
    earthy; Umber.
  yellow clay-iron-stone
    fibrous.
    compact.
    earthy; Yellow ochre.
  mixed with clay, and sometimes sand; Bog, Meadow, &c.; Iron ore. Limonite.
phosphate.
  crystallized; Vivianite. *A right oblique-angled prism*; M on T, 125° 15′.
  earthy.
sulphate; Melanteria.
  green crystallised. *An oblique rhombic prism*; P on M or M′, 99° 20′; M on M′, 82° 20′.
  fibrous.

red.
decomposed. The Atramentstein and Misy, and the red sulphate are, according to Leonhard, related to this species, but their composition is not given.
sub-sulphate, resinous; Pittizite.
earthy.
sulphuret; Iron pyrites.
common.
crystallised. *A cube*, the modifications of which are frequently defective, and such as might occur if the pentagonal dodecahedron were the primary form.
auriferous.
fibrous.
in *hexagonal prisms*, probably pseudomorphous.
arsenical; arsenical Iron; Mispickel; Marcasite.
crystallised. *A right rhombic prism*; M on M; 111° 12′.
amorphous.
........, argentiferous.
magnetic; cleavage parallel to the planes of *a regular hexagonal prism.*
white.
crystallised. *A right rhombic prism*; M on M′, 106°.
scheelate of Iron and Manganese; Wolfram.
crystallised. *A right oblique-angled prism*; M on T, 117° 22′ as measured by W. P.
fibrous.
amorphous.
undetermined species.
green Iron earth.
.... fibrous.
......................, containing Lime and Copper; Chalcosiderite.
Iserine, see Titanium, oxide, ferriferous.

K

Kali, see Potash.
Kaolin, see Felspar, decomposed.
KARPHOLITE. Occurs in fibrous crystals radiating from a centre, but those which I have seen are too imperfect to admit a determination of their forms.
Karstenite, see Lime, sulphate, anhydrous.

Karstin, see Schiller spar.
Keffekillite.
Keraphyllite, see Amphibole.
Keratite, see Quartz, hornstone.
Keratophyllite, see Amphibole.
Killenite.
Kil, see Magnesia, siliceous.
Killkeff, see Magnesia, siliceous.
Klaprothite, see Azurite.
Knebelite.
Kollyrite, see Collyrite.
Konite, see Conite.
Koreite, see Agalmatolite.
Koupholite, see Prehnite.
Kupferschaum, see Copper.
Kyanite, see Cyanite.

L

Lapis lazuli, see Lazulite.
Lardite, see Agalmatolite.
Lasionite, see Alumine, phosphate.
Latialite, see Haüyne.
Laumonite. *An oblique rhombic prism*; P on M or M′, 113° 30′; M on M′, 86° 15′.
Lava compact.
  vesicular.
  fibrous.
  earthy.
Lazulite; Lapis lazuli. *A rhombic dodecahedron.* The *Azurite* has also been termed Lazulite.
Lead.
  arseniate; Bleiniere; Flockenerz.
    crystallised. *A regular hexagonal prism.*
    acicular.
    filamentous.
    compact, mamellated.
  arsenite, fibrous.
  carbonate.
    crystallised. *A right rhombic prism*, M on M′; 117° 18′.
    columnar.
    acicular.
    compact, mamellated.
    earthy.

chromate. *An oblique rhombic prism*; P on M, 99° 10′; M on M′, 93° 30′.
........, of Lead and Copper: Vauquelinite.
molybdate. *An octahedron with a square base*; P on P′, 99° 46′; P on P″, 131° 15′.
murio-carbonate. *A right square prism.*
native.
oxide, red; Native minium.
 yellow.
 hydro-aluminous; Plombgomme.
phosphate; Polychrome; Pyromorphite.
 crystallised. *An obtuse rhomboid*; P on P′, 110° 5′, as measured by W. P.
 fibrous.
phosphato-arseniate. *A regular hexagonal prism.* This species occurs at Johangeorgenstadt in yellow hexagonal prisms with the terminal edges replaced, and in small yellow hexagonal prisms at Beeralston.
sulphate, crystallised. *A right rhombic prism*; M on M′; 103° 42′.
 earthy compact.
sulphato-carbonate. *A right oblique-angled prism*; M on T′, 120° 45′.
sulphato-tri-carbonate. *An acute rhomboid*; P on P′; 72° 30′.
cupreous sulphato-carbonate. *A right rhombic prism*; M on M′, 95°.
cupreous sulphate. *A right oblique-angled prism*; M on T, 102° 45′. This substance occurs at Linares in Spain, and has been described as cupriferous *carbonate.*
sulphuret; Galena.
 crystallised. *A cube.*
 lamellar.
 granular. Steel-grained.
.... of Lead and Antimony.
  crystallised. *A cube.*
  compact; Bleischweif.
........of Lead and Arsenic.
........of Lead, Antimony and Silver; white Silver ore.
..............................; grey Silver ore, containing less silver than the preceding.
........of Lead, Antimony, and Copper; Bournonite; Endellione.

crystallised. *A right rectangular prism*; M on a plane belonging to mod. class *d*, 136°. 50′.

........of Lead, Bismuth, and Silver. Bismuthic silver.

scheelate,

crystallised, apparently in *right square prisms*. The only crystallised specimen I have seen is in Mr. Heuland's cabinet.

Leelite.

Lemanite, see Felspar, compact.

Lemnian earth, see Bole.

Lenzinite; Wallerite.

Lepidolite; Lillalite.

crystallised. *A regular hexagonal prism.*

lamellar.

Lepidokrokite, see Iron, oxide, hydrous.

Leucite; Amphigene. *A cube.*

Leucolite, see Dipyre.

Lievrite, see Jenite.

Ligurite, said by Leonhard to be *an oblique rhombic prism*; P on M′, 146°; M on M′, 140°. I have not seen the substance.

Lillalite, see Lepidolite.

Limbilite.

Lime.

arseniate; Pharmacolite.

borate, siliceous.

crystallised.

from Norway; Datholite; Esmarkite. *A right rhombic prism*; M on M, 103° 40′.

from the Tyrol; Humboldite. *An oblique rhombic prism*; P on M, 91° 25′; M on M′, 115° 45′, as determined by Mr. Levy; see his paper in the Annals of Philosophy for Feb. 1823. This variety probably differs from the Norway Datholite in the proportions of its elements.

fibrous, botryoidal. Botryolite.

carbonate.

crystallised. *An obtuse rhomboid*; P on P′, 105° 5′.

nacreous; Scheiffer spar.

crystallised, in thin crystals belonging to mod. *a* of the Tables.

laminar; Aphrite.

scaly.

columnar; Madreporite.
fibrous.
laminar.
lamellar.
compact. Marble and common Limestone.
slaty, containing shells; Lumachella.
globular; Oolite.
earthy; Chalk.
spongy; Agaric mineral.
pulverulent; Fossil farina.
stalactitic; Inolite.
botryoidal.
tubercular.
globular; Pea-stone; Pisolite.
incrusting; Tufa; Ostrecolla.
sedimentary. Travertino.

arragonite; Igloite.
crystallised. *A right rhombic prism*; M on M′, 116° 10′, measured on cleavage planes.
acicular, radiated.
fibrous.
coralloidal; Flos ferri.
compact.

magnesian carbonate; Anthraconite; Bitterspar; Miemite; Muricalcite; Pearl spar; Picrite; Tharandite.
crystallised. *An obtuse rhomboid*; P on P′, 106° 15′; the crystals are frequently pearly with curved surfaces.
granular; Dolomite.
flexible.
compact; Conite; Gurhofian; Magnesian limestone.

fetid.

bituminous.

aluminous. Marl.
compact.
earthy.

fluate.
crystallised. *A regular octahedron.*
laminar, straight.
curved.
stalactitic.
compact.
earthy. Ratoffkitt.
quartziferous.
aluminiferous; in single cubes, Derbyshire.
chlorophane.

nitrate.
acicular.
earthy.
phosphate; Apatite; Augustite.
crystallised. *A regular hexagonal prism*; M on a plane belonging to mod. class *c*, 130° 10′.
the crystals yellow; Asparagus stone.
.... bluish; Moroxite.
fibrous; Phosphorite.
botryoidal; Phosphorite.
compact.
pulverulent; Terre de Marmarosch.
quartziferous.
silicate; Grammite; Schaalstein; Tabular spar; Wollastonite.
crystallised. *A doubly oblique prism*; P on M, 126°; P on T, 93° 40′; M on T, 95° 15′.
fibrous.
sulphate; Gypsum.
crystallised; Selenite. *A right oblique-angled prism*; M on T, 113° 8′.
foliated.
fibrous; Gypsum.
scaly; niviform Gypsum.
compact; Alabaster.
earthy.
calcareous; Plaster stone.
anhydrous; Anhydrite; Bardiglione; Karstenite; Muriacite.
crystallised. *A right rectangular prism*.
fibrous.
...... contorted, pierre de trippes.
compact.
quartziferous; Vulpinite.
scheelate; Tungsten.
crystallised. *An octahedron with a square base*; P on P′, 100° 40′; P on P″, 129° 2′.
Limonit, see Iron, oxide.
Linsenerz, see Copper arseniate.
Lipalite, see Quartz, flint.
Litheosphore, see Barytes, sulphate, radiated.
LITHOMARGE; Steinmark.
Liver ore, see Mercury.
Loboite, see Idocrase.
Lodalite, see Felspar, compact.
Lotalite, see Amphibole, var. green diallage.

Lumachella, see Lime, carbonate.
Lydian-stone, see Quartz.
Lythrodes, see Fettstein.

M

Macle, see Chiastolite.
Maclurite, probably Condrodite.
Madreporite, see Lime, carbonate.
MAGNESIA.
    borate; Boracite. *A cube.* Some of the secondary forms are however such as might result from the regular modifications of a tetrahedron.
    carbonate,
        crystallised, from New Jersey, but the crystals I have seen are too imperfect to admit of a determination of their forms.
        pulverulent.
    ....of Magnesia and Iron; yellow Bitterspar from the Tyrol.
        crystallised. *An obtuse rhomboid*; P on P′, 107° 30′. See Annals of Philosophy for May 1823.
    ....siliceous, compact; Baudisserite; Magnesite. The silica probably not essential to the species, which may be merely a carbonate mixed with silex.
        pulverulent; Razoumoffskin.
        fluate?
    hydrate, siliceous; Meerschaum; Myrsen; Kil; Killkeffe.
    ....of Magnesia and Soda; Bloedit.
    sulphate.
        crystallised. *A right square prism*; M on a plane belonging to class *c*, 129°.
        fibrous.
        earthy.
    ....of Magnesia and Soda.
    ....of Magnesia and Iron; Hallotricum.
Magnetic iron, see Iron, oxydulous.
Malachite, see Copper, carbonate.
Malacolite, see Pyroxene.
Maltha, see Bitumen.
MANGANESE.
    carbonate.
        crystallised. *An obtuse rhomboid*, P on P′, about 107° 20′, but the planes of the only specimen

I have seen are too much curved to admit of a very precise measurement.

foliated; Dialogite.

compact; Rhodochrosite.

siliceous.

anhydrous; Allagite; Photizite.

hydrous; Rhodonite.

hydrate.

oxide.

crystallised. *A right rhombic prism*, M on M′, 100°.

compact.

earthy.

Wad.

fibrous.

frothy.

earthy.

silicate; Red manganese ore.

foliated.

compact.

.... Helvin, which see.

.... Hydropite.

........, ferriferous, in octahedrons from Piedmont.

hydrous.

The precise differences between the preceding varieties cannot be accurately stated, there being no exact descriptions of the different minerals analysed, except of the Helvin.

phosphate of Iron and Manganese.

sulphuret.

Marble, see Lime, carbonate, compact.

Marekanite, see Obsidian.

Markasite, see Iron, sulphuret, arsenical.

Marl, see Lime, aluminous.

Mascagnin, see Ammonia, sulphate.

Meerschaum, see Magnesia, hydrate, siliceous.

MEIONITE. *A right square prism*, mod. *d*. on a plane belonging to class *a*, 122°. This species nearly corresponds in measurement and chemical composition with Scapolite.

Melanite, see Garnet, black.

Melanteria, see Iron, sulphate.

MELLILITE. *A right square prism*; determined by W. P. from measurement of lateral primary, and secondary planes.

MELLITE; Honeystone. *An octahedron with a square base*, P on P′, 93° 7; P on P″, 118° 31′.

Menachanite, see Titanium, oxide.
Menilite, see Quartz, opal.
MERCURY.
  muriate. *A right square prism*, M on a plane belonging to mod. class *c*, 158°.
  native.
  ......argentiferous; Native amalgam. *A rhombic dodecahedron.*
  sulphuret; Cinnabar.
    crystallised. *An acute rhomboid*, P on P′, 72°.
    fibrous.
    pulverulent.
    compact.
    slaty.
    hepatic; Corallenerz; Liver ore.
MESOLE. See Edinb. Phil. Jour. vol. 7. p. 7.
MESOLINE. See Edinb. Phil. Jour. vol. 7. p. 7.
MESOLITE.
MESOTYPE.
  crystallised. *A right rhombic prism*, M on M′, 91° 10′.
  red, globular radiated; Crocalite.
  ....earthy; Edelite.
  yellow, globular radiated, or reddish or white; Hogauite; Natrolite.
Meteorite, see Iron, native.
MIASZITE.
MICA. The crystalline form of the brown Mica from Vesuvius is *an oblique rhombic prism*, P on M or M′, 98° 40′; M on M′, 100°, as determined by W. P, from measurement of some brilliant crystals. From the analyses of different substances which have been denominated Mica, it appears probable that different species of minerals have been comprehended under that name, and that among these there may be different crystalline forms. One of these varieties appears, from the direction of some of its cleavages, to crystalline in *right prisms*, which are probably hexagonal.
Micaphyllite, see Andalusite.
Micarelle; Pinite and Scapolite have both passed under this name.
Miemite, see Lime, carbonate, magnesian.
Mispickel, see Iron, sulphuret, arsenical.
Misy, see Iron, sulphate, decomposed.
Mocha-stone, see Quartz, Agate, dendritic.
Molarite, see Quartz, buhrstone.

Molybdenum, oxide, fibrous.
  pulverulent.
  sulphuret. The form of the only crystals I have seen is *a regular hexagonal prism*, which is probably the primary form.
Moon-stone, see Felspar.
Moroxite, see Lime, phosphate.
Mountain, cork, leather, wood, see Asbestus.
Mountain meal, see Bergmehl.
Mountain soap.
Müllers glass, see Quartz, hyalite.
Muriacite, see Lime, sulphate, anhydrous.
Muricalcite, see Lime, carbonate, magnesian.
Mundic, a name given by the Cornish miners to Iron pyrites.
Muriatic acid.
Mussite, see Pyroxene.
Myrsen, see Magnesia, hydrate, siliceous.

N

Naphtha, see Bitumen.
Napoleonite, see felspar, globular.
Napolite. A blue mineral from Vesuvius, see Annals of Philosophy, vol. 7. p. 402. I have called it *Napolite* for the purpose of distinguishing it by name from *Hauyne* with which it has been classed, but to which species it appears not to belong.
Natrolite, see Mesotype.
Natron, see Soda.
Necronite, is probably Felspar. It has two cleavages producing bright planes at right angles to each other, and an indistinct oblique cleavage, and has the same lustre and hardness as Felspar.
Needle ore, see Bismuth, sulphuret.
Needle-stone; Scolezite.
  crystallised. *A right rhombic prism*, M on M′, 91° 20′. The Needle stone from Iceland, and that from Faroe, afford the same measurements by the reflective goniometer. Dr. Brewster regards them however as distinct species.
  acicular.
  pulverulent; Mealy stilbite.
Neopetre, see Quartz, hornstone.
Nepheline; Sommite. *A regular hexagonal prism*, M on a plane belonging to mod. class c, 134°

The Nephelines from Monte Somma and from Capo di Bove, afford an instance of chemical discordance in relation to minerals having the same crystalline form.

Nephrite, see Jade.
NICKEL, arseniate.
colouring clay or some other substances; Pimelite.
arsenical.
……., antimonial
oxide; black.
sulphuret of Nickel, Arsenic and Iron.
………. Nickel, Antimony and Arsenic.
Nigrine, see Titanium.
Nitre, see Potash, nitrate.
Nosin, see Spinellane.
NOVACULITE; Turkey stone.

O

OBSIDIAN; Volcanic glass.
in small rolled fragments; Marekanite.
fibrous.
amorphous.
Ochre, see Iron, oxide.
Ochroite, see Cerium, oxide.
Octahedrite, see Cerium, oxide.
ODERIT; probably Black mica.
Oisanite, see Titanium, oxide, anatase.
Oligiste Iron, see Iron, oxide.
Olivenit, see Copper, arseniate.
Olivin, see Peridot.
Omphazit, appears from the specimens sent here, to be a mixture of Garnet, and that variety of Amphibole called by Haüy Green diallage, and probably Cyanite.
Onegite; perhaps an ore of Titanium.
Oolite, see Lime, carbonate.
Opal, see Quartz.
Ophite, see Serpentine.
Orpiment, see Arsenic, yellow sulphuret.
ORTHITE.
Orthose, see Felspar.
OSMIUM; occurs alloyed with Iridium, which see.
Osteocolla, see Lime, carbonate, incrusting.
Otrelite, see Schiller spar.

P

Pagodite, see Agalmatolite.
PALLADIUM, native.
Paranthine, see Scapolite.
Pargasite, see Amphibole.
Paulite, see Hyperstene.

Pearl-spar, see Lime, carbonate, magnesian. There has been much uncertainty in the use of the terms Brown spar and Pearl spar; the first of these having been applied to carbonate of Iron, and also to those varieties of Pearl spar, or Magnesian carbonate of Lime, which are of a brown colour, and probably to some other of the carbonates, of Lime, and of Manganese.

PEARL STONE.

Peat, see Coal.

Pechuran, see Uranium, oxide, ferriferous.

Peliome, see Dichroite.

Pentaclasite, see Pyroxene.

PERIDOT.

crystallised; Chrysolite. *A right rectangular prism*, M on a plane belonging to mod. class *d*, 141° 30'.

granular; Olivine.

......., in a decomposing state; Chusite.

PETALITE; Berzelite. Cleavage parallel to the planes of a prism of 100° and 80°, and to both its diagonals; with indications of a cleavage oblique to its axis.

Petroleum, see Bitumen.

Petro-silex, a name applied sometimes to compact Felspar, and sometimes to a compact variety of Quartz.

Petuntzé, a Chinese name for one of the substances used in the manufacture of their porcellain, which is probably Quartz.

Pharmacolite, see Lime, arseniate.

Phengite, referred by Leonhard both to Anhydrite and Topaz.

Phosphorite, see Lime, phosphate, fibrous.

Photizite, see Manganese, carbonate, siliceous.

Physalite, see Topaz.

Picotite, see Tourmaline.

PICROLITE, a fibrous radiating substance found in the Serpentine at Taberg in Sweden.

Picrite, see Lime, carbonate, magnesian.

Pictite, see Turnerite.

Pimelite, see Nickel, oxide.

PINITE. *A regular hexagonal prism.*

A very soft substance from Pinistollen has passed under the name of Pinite, but from analysis as well as external character, it appears to be a separate species.

Pisolite, see Lime, carbonate.

Pistazite, see Epidote.

Pitchblende, see Uranium, oxide, ferriferous.

PITCHSTONE; Deodalite; Pyraphrolite; Retinite.

Pittizite, see Iron, sub-sulphate.
Plasma, see Quartz, calcedony.
PLATINA, native.
.... alloyed with other metals.
.... black, containing a larger quantity of the ore of Iridium than the common Platina does.
Pleonaste, see Spinelle.
Plombagine, see Graphite.
Plomb-gomme, see Lead, oxide, hydro-aluminous.
Plumbago, see Graphite.
Polishing slate, see Quartz, earthy.
Polychrome, see Lead, phosphate.
POLYHALLITE.
Porcellain Jasper, see Quartz, Jasper.
POTASH ; Kali.
    nitrate ; Nitre.
        fibrous.
Potstone, see Talc, compact.
Pounxa, see Soda, borate.
Prase, see Quartz.
PREHNITE.
    crystallised. *A right rhombic prism*, M on M′, 100°.
        the crystals tabular and very thin ; Kocepholite.
    fibrous.
    compact.
PUMICE ; Bimstein.
Pycnite, see Topaz.
PYRALLOLITE, from a small fragment with which I have been favored by M. Nordenskiold, its discoverer, I find that there are cleavages parallel to the lateral planes and to the diagonals of a rhombic prism. But the planes are too imperfect to determine the angles of the prism.
Pyraphrolite, see Pitchstone.
Pyrenaite, see Garnet, black.
Pyrgom, see Pyroxene.
Pyrites, see Iron, sulphuret ; and Copper, sulphuret.
PYRODMALITE ; Pyromalite. *A regular hexagonal prism.*
Pyromorphite, see Lead, phosphate.
Pyrope, see Garnet.
Pyrophysalite, see Topaz.
PYRORTHITE.
Pyrosiderite, see Iron, oxide, hydrous.
Pyrosmalite, see Pyrodmalite.
PYROXENE.
    crystallised ; Alalite ; Augite ; Baikalite ; Diopside ; Fassaite ; Malacolite ; Mussite ; Pentaclasite ;

Pyrgom; Sahlite; Vulcanite; the green prisms which accompany the Jenite from Elba, and which have been called Hornblende; the white Pyroxene from New York; and Bournon's yellow Topaz from Vesuvius. *An oblique rhombic prism*, P on M or M′, 101°; M on M, 87° 5.′

Several of these varieties, particularly the Sahlite, have a cleavage transverse to the axis of the prism, which the others have not. But this cleavage appears to take place only where some foreign matter is interposed between the laminæ of the crystals; for the same crystals which may be separated at one of these apparent junctions, cannot be cleaved in the same direction in other parts of the prism.

granular; Coccolite.

amorphous.

fusible. The cleavages and the angles of this variety are similar to those of Pyroxene, as nearly as can be determined by the reflective goniometer, from planes which are not very bright; yet from its ready fusibility, it may possibly be a distinct species. I have observed this variety differing in colour and external appearance from two localities, from Sweden, imbedded in Quartz; Euchysiderite.

Greenland, accompanying Eudyalite.

Q

QUARTZ.

crystallised. *An obtuse rhomboid*, P on P′, 94° 15′.

colourless; Rock crystal; Dragonite.

black.

brown; Smoky quartz.

red; Compostella quartz; Ferruginous quartz.

yellow; transparent.

.... opaque, ferriferous; Eisenkiesel.

violet to purple; Amethyst.

green; prase.

laminar; milky.

.... rose.

acicular radiated.

fibrous.

granular.

.... yellowish green; Cantalite.

arenaceous.

.... flexible.

- earthy, mixed with other substances.
  - slaty; Polishing slate.
  - compact; Rotten stone; Trepoli.
- amorphous; common.
  - blue; Siderite.
  - greasy.
  - avanturine, containing numerous minute fissures,
    - or scales of mica.
  - iridescent.
- pseudomorphous.
- penetrated by Asbestus; Cats-eye.
- fetid.
- black opaque; Basanite; Lydian stone.
- .... slaty.
- jasper; common.
  - red; Sinople.
  - spotted.
  - Ribbon jasper.
  - Porcellain jasper.
- agate.
  - calcedony.
    - crystallised. The crystals are pseudomorphous, and probably have taken the form of Fluate of lime. I have found the mutual inclination of the planes to be 90° by the reflective goniometer.
    - stalactitic.
    - blue.
    - pale green, coloured by Arseniate of nickel; Chrysoprase.
    - dark green; Plasma.
      - .... with red dots; Bloodstone; Heliotrope.
    - Carnelian.
      - white.
      - yellow.
      - red.
      - brown; Sardonyx.
  - spheroidal, in concentric layers, or bands; Agate.
  - in parallel layers or bands; Onyx.
  - veined.
  - dendritic; Moss agate; Mocha stone.
  - breccia agate.
  - wood agate.
  - mixed with clay; Jasper.
    - Egyptian pebble.

- flint.
- swimming Quartz.
- carious Quartz; Buhr stone; Molarite.
- hornstone, infusible; Keratite; Neopetre.
  - crystallised, in pseudomorphous crystals.
  - Woodstone.
- opal.
  - precious.
  - girasol.
  - hydrophane.
  - common.
  - semi-opal.
  - opal Jasper.
  - Wood opal.
  - aluminiferous; Cacholong.
- chloropal.
- menilite.
- hyalite; Gummistein; Müllersglass.
- fiorite.
- recent deposit from hot springs; Siliceous sinter.
  - Geyserite.
    - botryoidal.
      - compact.
      - pulverulent.
    - tufaceous, enclosing grass, leaves, &c.
    - earthy.
  - Azorite, from St. Michael's.
    - fibrous.
- sand tubes; Astrapyalite; Blizsinter; Ceraunian sinter; Fulgurite.

R

Rapidolite, see Scapolite.
Ratofkit, see Lime, fluate, earthy.
Razoumoffskin, see Magnesia, carbonate, siliceous.
Realgar, see Arsenic, sulphuret.
Retinasphaltum, see Bitumen.
Retinite, see Pitch-stone.
Reussite, see Soda, sulphate of and Magnesia.
Rhetizite, see Cyanite.
RHODIUM, native; alloyed with Platina.
Rhodochrosite, see Manganese, carbonate.
Rhodonite, see Manganese, carbonate, siliceous.
Romanzovite, see Cinnamon-stone.
Rotten-stone, see Quartz, earthy.
Rubellite, see Tourmaline.

Rubin glimmer, see Iron, oxide, hydrous.

Ruby, see Spinell.

oriental, see Corundum.

Rutile, see Titanium, oxide.

S

Sagenite, see Titanium, oxide, Rutile.

Sahlite, see Pyroxene.

Sal-ammoniac, see Ammonia, muriate.

Salt, common, see Soda, muriate.

Sanidin, see Felspar.

Sappare, see Cyanite.

Sapphire, see Corundum.

SAPPHIRINE, appears from analysis to be a distinct species, but I cannot ascertain its crystalline form.

Sarcolite, see Analcime.

Sardonyx, see Quartz agate.

Sassolin, see Boracic acid.

Saussurite, see Felspar, compact.

SCAPOLITE; Arktizit; Chelmsfordite? Ekebergite; Paranthine; Rapidolite; Wernerite.

crystallised. *A right square prism*, mod. *d.* on a plane belonging to mod. class *a*, 122° 5′.

amorphous.

Schaalstein, see Lime, silicate.

Scheelium; Tungsten.

oxide.

calcareous, see Lime, scheelate.

ferriferous, see Iron, scheelate.

plumbiferous, see Lead, scheelate.

Schieffer-spar, see Lime, carbonate, nacreous.

SCHILLER-SPAR; Diallage metalloide, foliated; Karstin; Otrelite. Cleavage parallel to the planes and to both the diagonals of *a rhombic prism* of about 93° 30′, and 86° 30′, but uncertain whether right or oblique. See Hyperstene.

Schorl, see Tourmaline.

Scorza, see Epidote, granular.

Selenite, see Lime, sulphate.

Selenium, see Copper, selenuret.

Semeline, see Titanium, oxide, siliceo-calcareous.

SERPENTINE.

precious; Ophite.

common.

Siberite, see Tourmaline, red.

Siderite, see Quartz, blue. The Siderite of Kirwan is Phosphate of Iron.

SIDEROCLEPTE.

SIDERO-GRAPHITE.

Silex, see Quartz.

Siliceous spar of Haussman, occurs with the Tourmaline at Chesterfield in Massachusetts, see Cleavelandite.

SILVER.
- carbonate.
- muriate.
  - crystallised. *A cube.*
  - mamellated.
  - amorphous.
- native.
  - crystallised. *A cube.*
  - capillary.
  - massive.
  - ....antimonial.
    - crystallised.
    - granular.
    - massive.
  - ....arseniferous.
- sulphuret.
  - crystallised. *A cube.*
  - amorphous.
  - ....of Silver and Antimony; Red silver.
  - impure or decomposed; Black silver.
    - Its colours are light red.
      dark red.
    - crystallised. *An obtuse rhomboid*, P on P′, 109° 56′.
    - amorphous.
    - scaly, from Colivan in Siberia; Aerosite.
  - ....of Silver and Antimony, but probably differing in the proportions from those which constitute Red silver. This variety was described by Romé de l'Isle under the name of "Mine d'Argent grise antimoniale." It has since been called Bournonite from Freyberg. *A right rhombic prism*, M on M′, 100°, as measured by W. P. on cleavage planes.
  - ....of Silver, Antimony and Iron; Brittle silver. All the crystallised specimens denominated Brittle silver, which I have seen, appear to be Red silver. Some of those specimens have been transparent and red, and others distinctly red

in the fracture, although opaque; and the measurements of all agree with those of Red silver. The Iron would appear therefore to be an accidental ingredient of the specimen analysed.

........of Silver and Iron; Flexible sulphuret. *A right oblique-angled prism*; M on T, 125°.

Sinople, see Quartz, jasper.
Skolezit, see Needle-stone.
Skorodite; see Copper, Arseniate of Copper and Iron.
Skorza, see Epidote, granular.
Smaragdite, see Amphibole.
SOAPSTONE.
SODA; Natron.
Borate; Borax; Pounxa; Swaga; Zala; Tincal.
crystallised. *An oblique rhombic prism*, P on M or M′, 101° 30′; M on M′, 133° 30′, as measured by W. P.
carbonate; Borech.
fibrous.
earthy.
muriate; common Salt.
crystallised. *A cube.*
fibrous.
amorphous.
nitrate.
sulphate.
fibrous.
earthy.
.... of Soda and Lime; Brongniartin; Glauberite. *An oblique rhombic prism*; P on M or M′, 104° 15′; M on M′, 83° 30′.
.... of Soda and Magnesia; Reussite.
Sodaite, see Fettstein.
SODALITE, from Greenland. *A rhombic dodecahedron.*
from Vesuvius. *A rhombic dodecahedron*, but is probably a distinct species, as it is ranked by Berzelius.
Sommite, see Nepheline.
SORDAWALITE.
Speckstein, see Steatite.
Specular iron, see Iron, oxide, anhydrous.
SPHEROLITE.
Aequinolite, supposed to belong to this species.
Sphero-siderite, see Iron, carbonate.
Sphene, see Titanium, oxide, siliceo-calcareous.

SPINELLANE; Nosin. *A rhombic dodecahedron.* W. P.

SPINELLE; Ruby. *A regular octahedron.*
if black, dark blue, greenish; Pleonaste.

Spinelline, see Titanium, oxide, siliceo-calcareous.

Spinthere, see Titanium, oxide, siliceo-calcareous.

SPODUMENE; Triphane. Cleavage parallel to the planes, and to both the diagonals, of *a rhombic prism* of 93° and 87°; the bright cleavage plane being parallel to the short diagonal of the prism. No cleavage planes to determine whether the crystal is right or oblique.

Two substancos from the Tyrol have been called Spodumene; one of these resembles the Spodumene from Uto, the other is Zoizite.

Spreüstein, see Bergmanit.

Stahlstein, see Iron, carbonate.

Stanzaite, see Andalusite.

Staurolite, see Staurotide.

STAUROTIDE; Grenatite; Staurolite. *A right rhombic prism,* M on M′, 129° 30′.

STEATITE; Speckstein.
in Pseudomorphous crystals.
amorphous.

Steinheilite, see Dichroite.

Steinmark, see Lithomarge.

STILBITE.
foliated, see Heulandite.
radiated. *A right rhombic prism,* M on M′ 101° 36.

This mineral cleaves parallel to the lateral planes of a right rectangular prism, but there is no cleavage parallel to the terminal planes of such a prism. There are, however, in some crystals, indications of cleavage parallel to the planes of a rhombic prism, which have induced me to adopt that as the primary form.

Stilpno-siderite, see Iron, oxide, hydrous.

Stralite, see Amphibole.

Stromnite, see Strontian, carbonate, barytiferous.

STRONTIAN.
carbonate.
crystallised. *A right rhombic prism,* M on M′, 117° 32′.
fibrous.
barytiferous; Stromnite.
sulphate; Celestine.
crystallised. *A right rhombic prism,* M on M′, 104°.
fibrous.

Stylobat, see Gehlenite.
Succin; Amber.
Succinite, see Garnet, yellow, granular.
SULPHURIC ACID.
SULPHUR.
crystallised. *An octahedron with a rhombic base*, P on P′, 106° 20′; P on P″, 143° 25′.
stalactitic.
amorphous.
Surturbrand, see Coal.
Swaga, see Soda, borate.
Sylvan, see Tellurium.

T

Tabular spar, see Lime, silicate,
TALC.
crystallised, in hexagonal plates.
amorphous.

From the analyses of the different minerals which have been brought together under this name, it appears that several have been included which probably do not belong to the same species. It has been made to comprehend

Chlorite.
Potstone.
Venetian talc.
French chalk, and other substances.
TANTALITE; Columbite.
crystallised. *A right rectangular prism*; T on a plane belonging to mod. class *c*, 150°. It is through the liberality of Mr. Heuland that I am in possession of the crystal which has afforded me the measurement here given.
Yttro-tantalite.
black.
dark brown.
yellow.
Telesie, see Corundum.
TELLURIUM; Sylvan ore.
native. *A regular hexagonal prism*; M on a plane belonging to mod. class *c*, 147° 15′.
...... auro-argentiferous; Graphic ore. *A right rhombic prism*; M on M′, about 107° 44′, as measured by W. P.
...... auro-plumbiferous; White tellurium. *A right rhombic prism*; M on M′, 105° 30′.

native plumbo-auriferous; Blättererz; foliated Tellurium. *A right square prism*; P on a plane belonging to mod. class *c*, 110°.

Tennantite, see Copper, sulphuret of Copper, Iron and Arsenic.

Terra de Siena, see Bole.

Terra sigillata, see Bole.

Thallite, see Epidote.

Tharandite, see Lime, carbonate, magnesian.

THOMSONITE. *A right rectangular prism*; M on a plane belonging to class *d*, 135° 10′.

THULITE. Cleavage parallel to the planes of a prism of 92° 30′, and 87° 30′, but not any distinct cleavage transverse to the axis of this prism.

Thumerstone, see Axinite.

Thumite, see Axinite.

TIN.

oxide, crystallised. *An octahedron with a square base.* P on P′, 133° 30′; P on P″, 67° 52′, as measured by W. P.

compact.

fibrous; Wood tin.

sulphuret of Tin and Copper; Bell-metal ore.

Tinder-ore, see Antimony.

Tinkal, see Soda, borate.

TITANIUM, is found pure, and crystallised in small copper-coloured cubes, in the iron slag from Merthyr Tydvill. Discovered by Dr. Wollaston.

oxide; Anatase; Octahedrite; Oisanite. *An octahedron with a square base*; P on P′, 98°; P on P″, 136° 12′.

......Rutile; Crispite; Gallizinite; Saginite. *A right square prism*, with a cleavage parallel to its diagonals and to its lateral planes. P on a plane belonging to mod. class *c*, 132° 32′. It is from a very perfect crystal belonging to Mr. Heuland, that I have been able to ascertain this form. Haüy gives it as a rectangular prism; but the measurement of Mr. Heuland's crystal, and the modifications it contains, leave no doubt of its being what I have described.

chromiferous.

ferriferous; Gregorite; Iserine; Menachanite; Nigrine.

crystallized. *A regular octahedron.*
granular.
amorphous.
siliceo-calcareous; Semeline; Sphene; Spinthere; Spinelline *An oblique rhombic prism*; M on M′, 76° 2′; P on M, 93° 1′, according to the measurements of Rose.
chrichtonite; *An acute rhomboid*; P on P′ 61°.20′.
Topaz; Physolite; Pycnite; Pyrophysalite. *A right rhombic prism*; M on M′, 124° 23′.
yellow of Bournon, from Vesuvius, see Pyroxene.
Topazolite, see Garnet, yellow.
Torberite, see Uranium, phosphate.
Touch-stone, see Quartz.
Tourmaline; Electric schorl. *An obtuse rhomboid*; P on P′, 133° 20′.
black from the Hartz; Aphrizite.
blue; Indicolite.
red to purple, and sometimes colourless; Apyrite; Daurite; Rubellite; Siberite.
dark brown acicular crystals are termed Cockle by the Cornish miners.
acicular crystals from the Pyrenees; Picotite.
Travertino, see Lime, carbonate, sedimentary.
Tremolite, see Amphibole.
Triklasite; Fahlunite.
crystallised. *A regular hexagonal prism.*
amorphous.
Is probably the same substance as Gieseckite.
Triphane, see Spodumene.
Tripoli, see Quartz, earthy.
Tungsten, see Lime, scheelate. Both the metal, and the ore in which it is combined with Lime, have been called Tungsten, which tends to confuse the description of one or the other of those substances. As an oxide has been lately discovered, I have preferred adopting the term Scheelium for the metal.
Turkey-stone, see Novaculite.
Turquoise, see Alumine, hydrate.
Turnerite; Pictite. *An oblique rhombic prism*; P on M or M′, 99° 40′; M on M′, 96° 10′. See Annals of Philosophy, April 1823.
Tyrolite, see Azurite.

V

Variolite, see Felspar, globular.
Vauquelinite, see Lead, chromate of, and Copper.
Vesuvian, see Idocrose.
Vivianite, see Iron, phosphate.
Umber, see Iron, oxide, hydrous.
Volcanic glass, see Obsidian.
Voraulite, see Azurite.
Uranite, see Uranium, phosphate.
URANIUM.
    oxide, ferriferous; Pechuran; Pitch blende; Uran Pitch ore.
    phosphate; Chalcolite; Torberit; Uranit. *A right square prism.*
Vulcanite, see Pyroxene, var. Augite.
Vulpinite, see Lime, sulphate, anhydrous.

W

Wad, see Manganese, oxide, earthy.
Wallerite, see Lenzinite.
Wavellite, see Alumine, phosphate.
Websterite, see Alumine, subsulphate.
Wernerite, see Scapolite.
Wiluite, see Idocrase.
Witherite, see Barytes, carbonate.
Wolfram, see Iron, scheelate of Iron and Manganese.
Wollastonite, see Lime, silicate.
Wolnyn, is probably crystallised Alum-stone.

Y

Yanolite, see Axinite.
YELLOW-EARTH.
YTTRIA, see its combinations with Cerium, &c.
Yttro-cerite, see Cerium.
Yttro-columbite, see Tantalite.
Yttro-tantalite, see Tantalite.
Yu, supposed to be Prehnite, which see.

Z

Zala, see Soda, borate.
ZEAGONITE; Abrazite; Gismondin. *An octahedron with a square base*; P on P′, 122° 54′; P on P″, 85° 2′.
Zeolite, see Analcime.
    Chabasie.
    Heulandite.
    Mesotype.

Natrolite.
Needle-stone.
Stilbite.

Zinc.
carbonate.
crystallised. *An obtuse rhomboid*; P on P′, 107° 40′; measured by Dr. Wollaston.
botryoidal.
earthy.
........, cadmiferous.
oxide.
.....manganesian; red oxide. *A regular hexagonal prism*, as determined by W. P. from cleavage.
.....aluminous; Automolite; Gahnite.
crystallised. *A regular octahedron.*
granular.
silicate. *A right rhombic prism*, M on M′, 102° 35′.
sulphate; Gallizenstein.
crystallised.
fibrous.
earthy.
sulphuret; Blende.
crystallised. *A rhombic dodecahedron.*
amorphous.
cadmiferous.
fibrous.
..... compact.

Zircon; Hyacinth; Jargon. *An octahedron with a square base*; P on P′, 123° 20′; P on P″, 84° 20′.

Zoizite, has been considered by the Abbé Haüy as a variety of Epidote, an error into which he has probably been led by the crystals of Epidote which are found in the Zoizite from Carinthia. It is however a distinct species, having for its primary form *a rhombic prism*, M on M′, 116° 30′. There is apparently a cleavage transverse to the axis of the prism, but not sufficiently distinct for measurement, which indicates that the prism is oblique from an obtuse edge.

Zurlite, or Zurlonite.

Zurlonite.

TABLE OF THE PRIMARY FORMS OF MINERALS,

ARRANGED ACCORDING TO THEIR CLASSES.

Cube.

Analcime.
Aplome.
Arsenical Cobalt.
Native Copper.
Arseniate of Iron.
Iron Pyrites.
Leucite.
Galena.
Boracite.
Muriate of Silver.
Native Silver.
Sulphuret of Silver.
Muriate of Soda.

Regular Octahedron.

Alum.
Muriate of Ammonia.
Oxide of Arsenic.
Red Oxide of Copper.
Purple Copper.
Diamond.
Native Gold.
Chromate of Iron.
Magnetic Iron.
Fluate of Lime.
Spinelle.
Menachanite.
Automolite.

Regular Tetrahedron.

Grey Copper.
Tennantite.
Helvin.

Rhombic Dodecahedron.

Cinnamon-stone.
Garnet.
Glaucolite ?
Haüyne.
Lazulite.
Native Amalgam.
Sodalite
Spinellane.
Blende.

Octahedron with a square base.

	P on P′.
Anatase	98°
Molybdate of Lead	99 46′
Tungsten	100 40
Copper Pyrites	102 15
Zeagonite	122 55
Zircon	123 20
Oxide of Tin	133 30

Octahedron with a rectangular base.

	P on P′.
Arseniate of Copper	72° 22′.

Octahedron with a rhombic base.

	P on P′.
Sulphur	106° 20′.

Right Square prism.

Red Antimony ?
Apophyllite.
Allanite.
Chiastolite ?
Fuscite.
Idocrase.
Murio-carbonate of Lead.
Scheelate of Lead.
Sulphate of Magnesia.
Meionite.
Mellilite.
Muriate of Mercury.
Scapolite.
Foliated Tellurium.
Rutile.
Phosphate of Uranium.

Right Rectangular prism.

Comptonite.
Couzeranite ?
Harmotome.
Bournonite.
Peridot.
Sulphate of Potash.
Tantalite.
Thomsonite.
Anhydrite.

Uncertain whether *square* or *rectangular* prism.

Gehlenite.

Cleavage parallel to the planes of a Square or Rectangular prism.

Allophane.
Cryolite.

Right Rhombic prism.

	M on M′.
Sulphuret of Antimony	nearly 90°
Mesotype	91° 10′
Needle-stone	91 20
Andalusite	91 20
Cupreous Sulphato-carbonate of Lead.	95°
Anhydrous Phosphate of Copper	95 20
Yttro-cerite	97
Yellow Sulphuret of Arsenic	100
Oxide of Manganese	100
Prehnite	100
Sulphuret of Silver and Antimony	100
Stilbite	101 36
Sulphate of Barytes	101 42
Silicate of Zinc	102 35
Datholite	103 40
Sulphate of Lead	103 42
Sulphate of Strontian	104
White Tellurium	105 30
White Iron Pyrites	106
Graphic Tellurium	107 44
Arseniate of Copper	111
Arsenical Iron Pyrites	111 12
Jenite	112
Arragonite	116 10
Carbonate of Lead	117 18
Carbonate of Strontian	117 32
Carbonate of Barytes	118 30
Humite	120
Arseniate of Copper and Iron	120
Azurite	121 30
Wavellite	122 15
Topaz	124 23
Staurotide	129 30
Hydro-oxide of Iron	130 40
White Antimony	137

Right Oblique-angled prism.

	M on T.
Brewsterite	93° 40′
Cupreous Sulphate of Lead	102 45
Sulphate of Lime	113 8
Epidote	115 40
Wolfram	117 22

Sulphato-carbonate of Lead	120° 45′
Arseniate of Cobalt	124
Flexible Sulphuret of Silver	125
Phosphate of Iron	125 15
Heulandite	130 30
Euclase	130 50

Oblique Rhombic prism.

Oblique from an acute edge.

	M on M′.
Hydrous Phosphate of Copper	37° 30′
Arseniate of Copper	56
Realgar	74 14
Sphene	76 2
Sulphate of Iron	82 20
Glauberite	83 20
Laumonite	86 15
Pyroxene	87 5

Oblique from an obtuse edge.

	M on M′.
Chromate of Lead	93° 30′
Turnerite	96 10
Blue Carbonate of Copper	99
Mica from Vesuvius	100
Green Carbonate of Copper	107 20
Gadolinite	115
Humboldite	115 45
Zoizite	116 30
Amphibole	124 30
Ligurite?	140

Doubly Oblique prism.

Diaspore	P on M	108° 30′
	P on T	101 20
	M on T	65
Axinite	P on M	134 40
	P on T	115 17
	M on T	135 10
Cleavelandite	P on M	119° 20′
	P on T	115
	M on T	93 30
Sulphate of Copper	P on M	127 30
	P on T	108
	M on T	123
Cyanite	P on M	93 15
	P on T	100 50
	M on T	106 15
Felspar	P on M	90
	P on T	120 15
	M on T	112 45
Silicate of Lime	P on M	126
	P on T	93 40
	M on T	95 15

Cleavage parallel to the planes of a Rhombic prism, but uncertain whether right or oblique.

	M on M′.
Spodumene	93°
Bronzite	93 30′
Hyperstene	93 30
Schiller spar	93 30
Petalite	100
Anthophyllite	125

Cleavage parallel to the planes of a prism whose other characters are not known.

	The greater angles.
Thulite	92° 30′
Indianite	95 15
Amblygonite	105 45
Fettstein	112 —

Regular hexagonal prism.

Cronstedit?
Dichroite.
Dipyre?
Emerald.

Giesechite.
Graphite.
Iridium and Osmium.
Magnetic Pyrites.
Arseniate of Lead.
Phosphato-arseniate of Lead.
Lepidolite?
Phosphate of Lime.
Sulphuret of Molybdenum.
Nepheline.
Pinite.
Pyrodmalite.
Talc?
Native Tellurium.
Triklasite.
Red Oxide of Zinc.

Acute Rhomboid.

	P on P′.	
Crichtonite	61°	20′
Arseniate of Copper.	69	30
Sulphuret of Copper.	71	30
Cinnabar	72	
Sulphato-tri-carbonate of Lead	72	30
Eudyalite	74	30
Corundum	86	4
Oligiste Iron	86	10

Obtuse Rhomboid.

	P on P′.	
Crystallised Alum-stone	92°	50′
Quartz	94	15
Chabasie	94	46
Carbonate of Lime ..	105	5
Bitter Spar	106	15
Carbonate of Iron ..	107	
Carbonate of Manganese	107	20
Carbonate of Iron and Magnesia	107	30
Carbonate of Zinc ..	107	40
Red Silver	109	56
Phosphate of Lead ..	110	5
Dioptase	126	17
Tourmaline	133	20

Cleavage parallel to the planes of an Obtuse Rhomboid.

	P on P′.	
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